Hammond and other flourishing cities in the northwest part of the State would experience the most direct benefits by the increase of business and manufacturing facilities and consequent increase in population.

The proposition is worth at least a passing thought and is commended to the State and general governments for further consideration.

Some Properties of the Symmedian Point.

By Robert J. Aley.

Monsieur Emile Lemoine, at the Lyons meeting of the French Association for the advancement of the Sciences in 1873, called attention to a particular point within the triangle, which he called the center of antiparallel medians. Since that time a number of mathematicians have studied the point and have discovered many of its properties. The point is such an interesting one that a brief collection of its more striking properties may be of some value. No claim is made to completeness.

DEFINITIONS OF THE POINT.

1. The point of concurrency of the bisectors of all lines antiparallel to the sides of the triangle.

2. The point of concurrency of the lines isogonal conjugate to the medians of the triangle; that is, the point of concurrency of the symmedians of the triangle.

3. The point within the triangle, the sum of the squares of whose distances from the three sides is the least possible.

4. The point within the triangle, whose distances from the sides is directly proportional to the sides.

NAMES OF THE POINT.

1. Center of antiparallel medians, proposed by Monsieur Emile Lemoine.

2. Symmedian point (symédiane, from symétrique de la médiane), proposed by Monsieur Maurice d'Ocagne. The Engish form "symmedian" was suggested by Mr. R. Tucker in 1884.

3. Minimum point, suggested by Dr. E. W. Grebe.
4. Grebe's point, proposed by Dr. A. Emmerich.
5. Lemoine's point, proposed by Professor J. Neuberg.

METHODS OF CONSTRUCTING THE POINT.

1. Draw the medians AM_a, BM_b of the triangle ABC. Then draw AK'_a, BK'_b, making the same angle with the bisectors of angles A and B, respectively, as are made by AM_a and BM_b. The intersection of AK'_a, BK'_b is K, the symmedian point.

2. Draw antiparallels to BC and CA. Join A and B, respectively, to the midpoints of these antiparallels, and the intersection of these joining lines is K, the symmedian point.

3. To the circumcircle of the triangle draw tangents at B, C and A, and let these intersect in X, Y, Z, respectively. Then AX, BY, CZ concur at K, the symmedian point.

SOME PROPERTIES OF THE POINT.

1. K is the point isogonal conjugate to G, the centroid.

2. If K_a, K_b, K_c are the feet of the perpendiculars from K to the three sides respectively, then

$$\left.\begin{aligned} KK_a &= \frac{2\triangle a}{a^2+b^2+c^2}. \\ KK_b &= \frac{2\triangle b}{a^2+b^2+c^2}. \\ KK_c &= \frac{2\triangle c}{a^2+b^2+c^2}. \end{aligned}\right\}$$

Where $\triangle$ is the area of the triangle ABC, and a, b, c are three sides of the same triangle.

3. Area of $\triangle$ BKC $= \frac{\triangle a^2}{a^2+b^2+c^2}$

Area of $\triangle$ CKA $= \frac{\triangle b^2}{a^2+b^2+c^2}$

Area of $\triangle$ AKB $= \frac{\triangle c^2}{a^2+b^2+c^2}$

$\triangle BKC : \triangle CKA : \triangle AKB = a^2 : b^2 : c^2.$

4. Antiparallels to sides of the triangle through K are equal. Such antiparallels cut the sides of the triangle in six points which lie on a circle whose centre is K. This circle is called the *Cosine* Circle.

5. K is the median point of the triangle $K_aK_bK_c$.

6. The line KM_a (M_a is the mid point of BC) passes through the mid point of the altitude AH_a.

7. The sides of the K-pedal triangle $K_aK_bK_c$ are perpendicular to the medians of ABC, respectively.

8. The sides of the G-pedal triangle $G_aG_bG_c$ are perpendicular to the symmedians AK, BK, CK, respectively.

9. $a . GA . KA + b . GB . KB + C . GC . KC = a . b . c.$

10. If the symmedian lines AK, BK, CK meet the circumcircle of ABC in A′, B′, C′, then the triangles ABC and A′B′C′ are co-symmedian, that is they have the same symmedian point K.

11. K and M (M is the circumcentre of ABC) are opposite ends of a diameter of Brocard's Circle.

12. Parallels to the sides of ABC through K, determine six points on the sides which lie on the Lemoine Circle.

13. If points A′, B′, C′ be taken on KA, KB, KC so that $KA' : KB' : KC' = KA : KB : KC =$ constant, then antiparallels to the sides through A′, B′, C′, respectively, determine six points on the sides of the triangle which lie on a Tucker Circle.

14. If $A_1 B_1 C_1$ is Brocard's first triangle, then

A_1 K is parallel to BC.

B_1 K is parallel to CA.

C_1 K is parallel to AB.

15. AK, BK, CK produced meet Brocard's circle again in A″, B″, C″ respectively, and these points form Brocard's second triangle A″ B″ C″.

16. If KA, KB, KC, meet the sides of ABC in X_1, X_2, Y_1, Y_2 and Z_1, Z_2 respectively, then the sides of the triangle $Z_1 X_1 Y_1$ are parallel to A Ω, B Ω, C Ω respectively, and the sides of $Y_2 Z_2 X_2$ are parallel to A Ω′, B Ω′, C Ω′ respectively, where Ω and Ω′ are the Brocard points of ABC. Ω and K are the Brocard points of $Z_1 X_1 Y_1$ and Ω′ and K are the Brocard points of $Y_2 Z_2 X_2$.

17. The point of concurrency D of AA_1, BB_1, CC_1 is the point isotomic conjugate to K.

18. The line MK is perpendicular to and bisects the line ΩΩ′.

19. The Simson line of Tarry's point is perpendicular to MK.

20. $\cot \angle KBC + \cot \angle KCA + \cot \angle KAB = 3 \cot \omega$ where ω is the Bocard angle.

21. If the symmedian AK cut BC in K'_a and the line MM_a in Q then (AK'_a, KQ) is a harmonic range.

22. If from K'_a perpendiculars p an q are drawn to CA, AB respectively, then

$$\frac{p}{b} = \frac{q}{c} = \frac{2\triangle}{a^2+b^2}$$

23. $AK : KK'_a = b^2 + c^2 : a^2$

24. $BK'_a : K'_a C = c^2 : b^2$

$CK'_b : K'_b A = a^2 : c^2$

$AK'_c : K'_c B = b^2 : a^2$

$BK'_a = \frac{ac^2}{b^2+c^2}$ etc

25. The tangent to the circumcircle at A, and the symmedian AK are harmonic conjugates with respect to AB and AC.

26. The angles AMK, BMK, CMK are equal respectively to the angles (BC, B_1C_1) (AC, A_1C_1), (AB, A_1B_1), that is the respective angles between the sides of Brocard's first triangle and the corresponding sides of the fundamental triangle.

27. The sides of the $\triangle K_aK_bK_c$ are proportional to the medians of the $\triangle ABC$, and the angles of the $\triangle K_aK_bK_c$ are equal to the angles which the medians make with each other.

28. The sum of the squares of the sides of $K_aK_bK_c$ is less than the sum of the squares of the sides of any other triangle inscribed in ABC.

29. The ratio of the area of ABC to that of its co-symmedian triangle $A'B'C'$ (See No. 10) is $(-a^2+2b^2+2c^2)\ (2a^2-b^2+2c^2)\ (2a^2+2b^2-c^2) : 27a^2b^2c^2$.

Note On McGinnis's Universal Solution.

By Robert J. Aley.

The full title of the book is, "The Universal Solution for numerical and literal equations by which the roots of equations of all degrees can be expressed in terms of their coefficients, by M. A. McGinnis, Kansas City, Missouri, the Mathematical Book Company, 1900."

In his preface the author announces that the book appears at "the request of many able mathematicians, teachers and scholars throughout the United States." He also modestly states that the imaginary is for the first time put upon a true basis, that bi-quadratics are more thoroughly