22. If from K'_a perpendiculars p an q are drawn to CA, AB respectively, then

$$\frac{p}{b}=\frac{q}{c}=\frac{2\triangle}{a^2+b^2}$$

23. $AK:KK'_a = b^2+c^2:a^2$

24. $BK'_a:K'_a C = c^2:b^2$

$CK'_b:K'_b A = a^2:c^2$

$AK'_c:K'_c B = b^2:a^2$

$BK'_a = \frac{ac^2}{b^2+c^2}$ etc

25. The tangent to the circumcircle at A, and the symmedian AK are harmonic conjugates with respect to AB and AC.

26. The angles AMK, BMK, CMK are equal respectively to the angles (BC, B_1C_1) (AC, A_1C_1), (AB, A_1B_1), that is the respective angles between the sides of Brocard's first triangle and the corresponding sides of the fundamental triangle.

27. The sides of the $\triangle K_aK_bK_c$ are proportional to the medians of the $\triangle ABC$, and the angles of the $\triangle K_aK_bK_c$ are equal to the angles which the medians make with each other.

28. The sum of the squares of the sides of $K_aK_bK_c$ is less than the sum of the squares of the sides of any other triangle inscribed in ABC.

29. The ratio of the area of ABC to that of its co-symmedian triangle $A'B'C'$ (See No. 10) is $(-a^2+2b^2+2c^2)\ (2a^2-b^2+2c^2)\ (2a^2+2b^2-c^2):27a^2b^2c^2$.

Note On McGinnis's Universal Solution.

By Robert J. Aley.

The full title of the book is, "The Universal Solution for numerical and literal equations by which the roots of equations of all degrees can be expressed in terms of their coefficients, by M. A. McGinnis, Kansas City, Missouri, the Mathematical Book Company, 1900."

In his preface the author announces that the book appears at "the request of many able mathematicians, teachers and scholars throughout the United States." He also modestly states that the imaginary is for the first time put upon a true basis, that bi-quadratics are more thoroughly treated than in any prior work and that it is the only work in which general equations beyond the fourth degree are solved. It is also the only book that shows the fallacies in Abel's proof that equations of higher degree than the fourth can not be solved by radicals.

That the book is interesting goes without saying. No one who promises so much can fail to write in an interesting manner. One follows breathlessly to see the kind of a paradox that will be produced.

A number of simple theorems in the theory of numbers and the theory of equations are stated as though they were new.

On page 53, article 164, we read: "The roots of quadratics represent the sides of *right triangles* when Real Quantities; the sides of *isosceles triangles* when *Real Imaginaries;* and when *Pure Imaginaries* may be represented by *lines*." His argument for the latter part of the statement, it is needless to say, is not convincing.

A number of special numerical problems in equations of various degrees are solved. In many of these some very ingenious special methods are exhibited.

One chapter is devoted to the discussion of Wantzel's modification of Abel's proof of the impossibility of an algebraic solution of equations of higher degree than the fourth. The character of the discussion can be best understood by quoting the conclusion. "If we should accept his (Wantzel's) demonstration as true, we would be forced to the conclusion that the general equation of a degree higher than four was destitute of roots. The concluson of Wantzel that the roots can not be indicated in algebraical language is equivalent to saying that there are no roots, since it is absurd to say that finite quantities exist which can not be expressed in any function of other finite quantities, which are themselves symmetrical functions of the first, however complicated."

The author's notion of the imaginary is summed up in a general theorem, as follows: "An Imaginary Quantity is the indicated square root of the difference of the squares (with its sign changed) of the bases of two right triangles having a common perpendicular which is the radius of a circle; two of such triangles lying wholly within the semicircle, and two partly within and partly without the semicircle." What the theorem or the demonstration means would be hard to tell.

Of his so-called universal solution I will consider only that of the sixth degree. He assumes that—

$$x^6 + mx^5 + nx^4 + bx^3 + px^2 + tx + q = 0$$

$$\left\{x^2 + \frac{m}{a}x + y\right\}\left\{x^2 + \frac{m}{b}x + z\right\}\left\{x^2 + \frac{m}{c}x + w\right\} = 0$$

He then puts

$$(1)\quad n - \frac{m^2}{A} = \frac{A_o}{2m} - \frac{m^2}{2A^2} = y + z + x$$

$$(2)\quad p - \left\{\frac{m^2n}{B^2} - \frac{m^4}{B^3}\right\} = \frac{Bt}{m} = yz + yw + zw.$$

$$(3)\quad q = yzw$$

$$(4)\quad oA^3 - 2mnA^2 + 2m^3A - m^3 = 0$$

$$(5)\quad tB^4 - mpB^3 + m^3nB - m^5 = 0.$$

From (4) and (5) find A and B

Then x, y, z are found from 1, 2, 3 by means of a cubic equation.

The author incidentally remarks that the proper combination of the three values of A, and the four values of B are easily determined by a little practice. The author also says that it is evident that by comparing coefficients the values of 1/a, 1/b, 1/c can be obtained. The novice will find some difficulty in doing it. The real point of difficulty, however, is that we have eight unknown quantities, viz., a, b, c, x, y, z, A, B, and nine equations to be satisfied, viz., five by equating coefficients, and four from (1) and (2). So that the boasted solution is after all only a solution when there is some condition placed on the roots.

Graphic Methods in Elementary Mathematics.

By Robert J. Aley.

The Automatic Temperature Regulator.

By Chas. T. Knipp.

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