From this expression the following table has been computed for a 3-inch shaft running at 100 and 250 revolutions per minute:

Diameter of shaft in inches.	Revolutions per minute.	Percentage of loss when length in feet.				
		100	200	400	800	1600
3	100	5.1	9.9	19.6	38.7	77
3	250	5.8	10.6	20	39	77½

It is worthy of remark that in long lines of shafting the influence of belt pull on the bearings is very slight compared to the weight of shaft and pulleys, so that the loss in friction is but little more than that due to weight alone.

With better alignment and better lubrication the loss will be less than that here given; in long continuous lines of shafting the bearings are always more or less out of line, and for this reason the loss will be less if short lengths be employed.

Orthogonal Surfaces. By A. S. Hathaway.

It is well known that a given system of surfaces $f(x, y, z) = c$ has in general no pair of orthogonal conjugate systems, *i. e.*, such that the surfaces of the three systems through any point are mutually orthogonal at that point. It has been shown by Cayley [Salmon's Three Dimensions, p. 447] that $f(x, y, z)$ must satisfy a differential equation of third order if it possess a pair of orthogonal conjugates. In the course of some recent investigations on fluid motion I was led to observe that a given system of surfaces might have two pairs of orthogonal conjugates, in which case it would have an infinite number of such pairs. In order that such may be the case $f(x, y, z)$ must satisfy a differential equation of second order which is a particular integral of Cayley's equation of third order. This differential equation is, in Cayley's notation,

$$[(a, b, c, f, g, h)(L, M, N)^2 - (a + b + c)(L^2 + M^2 + N^2)]^2$$
$$= 4(L^2 + M^2 + N^2)(A, B, C, F, G, H)(L, M, N)^2$$

where $L, M, N, a, b, c, f, g, h$, are the first and second differential coefficients of $f(x, y, z)$, and A, B, C, *etc.*, are the minors of a, b, c, *etc.*, in the matrix

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$$

A very general solution of this equation comes from $a = b = c$, $f = g = h = o$, which are the differential equations of the series of spheres that pass through a given fixed circle, including, as particular cases, concentric spheres, planes intersecting in a fixed line, and parallel planes.

It may be shown that the above equation factors into four factors of the form $\sqrt{b^1 - c^1}\, L^1 = \sqrt{c^1 - a^1}\, M^1 = \sqrt{a^1 - b^1}\, N^1$ where a^1, b^1, c^1, are the roots of the cubic found by replacing a, b, c in the above matrix by $a-x$, $b-x$, $c-x$. The differential equation may also, by the usual reciprocal transformation $X = L$, $Y = M$, $Z = N$, $U + u = L\,x + M\,y + N\,z$, be reduced to a simpler form.

The preceding differential equation and the resulting theory of orthogonal surfaces were obtained by quaternion analysis. Briefly, if λ, $\lambda\sigma$, are two perpendiculars to the surface normal σ, that are also surface normals, then we have,

$$(1)\ S\lambda\sigma = o; \quad (2)\ S\lambda\nabla\lambda = o; \quad (3)\ S\lambda\sigma\nabla\lambda\sigma = o.$$

We may replace (3) by

(3^1) $S\lambda\sigma\nabla_1\lambda\sigma_1 = o$, or $S\lambda\phi V\sigma\lambda = o$, where $\phi\lambda = \frac{1}{2}(\nabla_1 S\lambda\sigma_1 + \sigma_1 S\lambda\nabla_1)$.

Thus ϕ is the self conjugate linear vector function, whose matrix is given above. From (1) and (3^1) we find

$$V\lambda V\sigma\phi V\sigma\gamma = o$$

This determines λ as one (and $\lambda\sigma$ as the other) of the two latent directions of the plane self-conjugate vector function $V\sigma\phi V\sigma\lambda$. There is therefore in general but one pair of normals that may satisfy the conditions of which (2) becomes a condition upon σ, or the differential equation satisfied by $f(x, y, z)$ in order that it may possess a pair of orthogonal conjugates. If, however, the above plane vector function have equal latent roots, then its latent directions become indeterminate. This means that (1) becomes a factor of (3^1) so that the only equations to be satisfied are (1), (2). These may be satisfied without other condition upon σ than the above equality of latent roots which is the differential equation that we have given at the beginning of the paper.

NOTE.—Since presenting the above I have noticed that the latent roots of the plane strain mentioned are proportionals to the principal radii of curvature of normal sections of the surface $f(x, y, z) = c$. The above differential equation of second order therefore expresses that every point of each of these surfaces is an umbilic. Hence the general solution consists of a system of spheres (or planes) with one variable parameter. $(u = A(x, y, z))$. The above quaternion method gives also the conditions that a system of lines may be the intersection of one pair of orthogonal systems of surfaces, or of an infinite number of such pairs.

THE CALENDAR GROUP. BY C. A. WALDO.