Linear Euthymorphic Functions of the First Order.

By E. M. Blake. (Abstract.)

Euthymorphic functions are those monogenic functions which satisfy an equation of the form

$$\phi(z) + p_1(z)\,\phi(f_1(z)) + \ .\ .\ .\ + p_n(z)\,.\,\phi(f_n(z)) + p(z) = o$$

where $f_1, \ .\ .\ .\ , f_n, p_1, \ .\ .\ .\ , p_n, p$ are given functions of which $p_1, \ .\ .\ .\ ,$ p_n, p are algebraic. The *order* of $\phi(z)$ is n and it is *linear* if all of $f_1, \ .\ .\ .\ ,$ f_n are of the form $\frac{\alpha z + \beta}{\gamma z + \delta}$.

The paper gives a systematic compilation of the investigations of Babbage, Rausenberger, Koenigs and others upon functions defined by an equation of the form

$$\phi(z) = p(z)\,.\,\phi\left(\frac{\alpha z + \beta}{\gamma z + \delta}\right) \qquad (1)$$

(where $p(z)$ is algebraic) in so far as relates to their existence and analytical expression. The theorems of Koenigs relate to more general functions but they are only defined within a limited circle of convergence. The application of these theorems to euthymorphic functions and their continuation over the entire z-plane are believed to be new.

A tabulation of the results contained in the paper is as follows:

Every equation (1) can be reduced by a linear transformation to one of the three forms:

$$\phi(z) = p(z)\,\phi(z+1) \qquad \text{I.}$$

$$\phi(z) = p(z)\,\phi(e^{i\theta} z) \qquad \text{II.}$$

$$\phi(z) = p(z)\,\phi(\alpha z), \ |\,\alpha\,| < |\,. \qquad \text{III.}$$

Sub-forms and their solutions, (f is any function),

I*a*. $\phi(z) = \phi(z+1)$; $f(e^{2\pi i z})$

I*b*. $\phi(z) = b\,\phi(z+1)$; $b^{-z}\,.\,f(e^{2\pi i z})$

I*c*. $\phi(z) = \frac{(z-a_1)\ .\ .\ .\ .\ (z-a_m)}{(z-b_1)\ .\ .\ .\ .\ (z-b_n)}\,\phi(z+1)$

$$;\ \frac{\Gamma(z-b_1)\ .\ .\ .\ .\ \Gamma(z-b_n)}{\Gamma(z-a_1)\ .\ .\ .\ .\ \Gamma(z-a_m)}\,.\,f(e^{2\pi i z})$$

I*d*. $\phi(z) = p(z)\,\phi(z+1)$; $p(z)$ irrational is unsolved.

II*a*. $\phi(z) = \phi(e^{i\theta} z)$; $f(z^{\frac{2\pi}{\theta}})$

II*b*. $\phi(z) = b\,\phi(e^{i\theta} z)$; $z^{-\frac{\log b}{i\theta}}\,.\,f(z^{\frac{2\pi}{\theta}})$

II*c*. $\phi(z) = p(z)\,\phi(-z)$; $(p(z)\,.\,p(-z) = 1)$; $(1 + p(z))\,.\,f(z^{\frac{2\pi}{\theta}})$

For $p(z)$ not a constant II*c*. is the only solved form.

III*a*. $\phi(z) = \varphi(a z)$; $f(z^{\frac{2 \pi i}{\log a}})$

III*b*. $\phi(z) = b \phi(a z)$; $z^{-\frac{\log b}{\log a}} \cdot f(z^{\frac{2 \pi i}{\log a}})$

III*c*. $\phi(z) = z \phi(a z)$; $\prod_{0}^{\infty} (1 + a^{m} z) \cdot \prod_{1}^{\infty} (1 + \frac{a^{n}}{z}) \cdot f(z^{\frac{2 \pi i}{\log a}})$

III*d*. $\phi(z) = p(z) \phi(a z)$; $(p(o) = 1)$; $\tau(z) \cdot f(z^{\frac{2 \pi i}{\log a}})$.

The $\tau(z)$ has the same number of branches as $p(z)$. It may be algebraic. When transcendental ∞ is its only essential singular point.

The solution of any equation of form III. consists of a product of solutions of the four types given.

New Mechanical Computer. By Fred Morley.

A New Apparatus for Photographic Surveying. By Fred Morley.

Crushing Strength of Wrought Iron Cylinders. By W. K. Hatt and L. Fletemeyer.

Tests of a Wrought Iron Car Axle. By W. F. M. Goss.

While much has been written concerning the variety and intensity of the stresses which service conditions impose upon car axles, there have been presented but few descriptions of the behavior of such axles when under stresses that are simple and definite in character. Interesting material of the latter class is supplied by a recent test of a 60,000-pound axle made in the Engineering Laboratory of Purdue University.

The axle tested was supplied by the Bass Foundry and Machine Works, of Fort Wayne. It is said to have been made of No. 1 wrought railroad scrap, and to have been selected at random from a lot of 100 which were being shipped to a railroad company, and with it there was delivered to the laboratory a small test specimen which had been drawn down from the crop end of the axle. As prepared for the tests the axle carried two 33-inch cast wheels, and it was tested under transverse stresses, while the small specimen was subjected to tensional tests. The work was executed by Mr. J. H. Klepinger, who perfected details in the general plan and was painstaking in the manipulation of the apparatus.