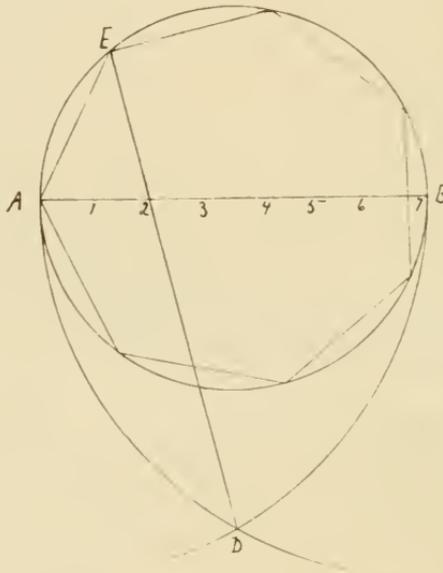


## NOTE ON ANGEL'S METHOD OF INSCRIBING REGULAR POLYGONS.

BY ROBERT JUDSON ALEY.

On page 47, "Practical Plane and Solid Geometry," by Henry Angel, the following method of inscribing a regular polygon in a circle is given:

"Let  $ACB$  be the given circle, and let the required figure be a heptagon. Draw the diameter  $AB$ , and divide it into seven equal parts. (The number of parts is regulated by the required number of sides.) With  $A$  and  $B$  as centers—radius  $AB$ —describe two arcs intersecting in  $D$ . From  $D$  draw the line  $D 2$ , passing through the second division of the diameter, and produce it, to meet the circle in  $E$ . The distance,  $AE$ , will divide the circle into seven equal parts; and if the points of division be joined, a heptagon will be inscribed in the circle."



The method has the merit of seeming to succeed. When applied to circles of short radii, no noticeable error is found in the drawing. I have not attempted to give a geometric demonstration of the error which arises in this and all similar rule of thumb methods of inscribing regular polygons. Let the diameter  $AB$ , for convenience, be fourteen units in length; then, by obvious trigonometric processes, we find  $AE$  to be 6.09212 units in length, while in a true heptagon the side would be 6.07436 units long.

Take a circle whose diameter is thirty-six units; Angel's method makes the side of a 36-gon equal to 3.33982 units, while the true length is 3.13776 units. The larger number of sides makes the error of the method more apparent.

CONCURRENT SETS OF THREE LINES CONNECTED WITH THE TRIANGLE.

BY ROBERT JUDSON ALEY.

To the student of the pure geometry of the triangle, few subjects are more interesting than the concurrency of lines. The following collection of concurrent sets of three lines has been made in the hope that it may prove of value to geometric students. No claim is made to completeness. The list is as complete as the author could make it with the material to which he had access. Many of the notes, and a large number of the propositions have been taken from the published papers of Dr. J. S. Mackay, of Edinburgh, perhaps the foremost student of the geometry of the triangle. No classification of the propositions seems possible and so none has been attempted.

1. The median lines of a triangle are concurrent. The point of concurrency, usually denoted by  $G$ , is called the *median point* or centroid.

2. The in-symmedian lines of a triangle are concurrent. The point of concurrency is called the symmedian point or Grebe's point, and is generally denoted by  $K$ . (For a history of this point, see J. S. Mackay, in Proceedings of Edinburgh Mathematical Society, Vol. XI.)

3. The altitudes of a triangle are concurrent. The point of concurrency, usually denoted by  $H$ , is called the ortho centre. (This proposition occurs in Archimedes's Lemmas and in Pappus's Mathematical Collection.)

4. The internal angle bisectors of a triangle are concurrent. The point of concurrency is the center of the inscribed circle and is usually denoted by  $I$ . (Euclid IV, 4.)

5. The internal bisector of any angle of a triangle and the external bisectors of the other two angles of the triangle are concurrent. The points of concurrency, denoted by  $I_1$ ,  $I_2$ ,  $I_3$  are the centers of the three escribed circles.

6. The perpendiculars to the sides of a triangle at the midpoints of the sides concur at the center of the circumscribed circle. This point of concurrence is usually denoted by  $O$ . (Euclid.)

7. Lines drawn from the vertices to the points of contact of the in-circle with the opposite sides are concurrent. (The point of concurrency,  $\Gamma$ , is called the Gergonne Point. It was named by J. Neuberger after J. D. Gergonne.)