clear that if $\triangle$ XYZ were to be given the rank of rt. $\angle$ $A''B''C''$ and a new one were to be formed from it, as it is formed from $\angle$ $A''B''C''$, then the point S'' would fall upon H. Therefore, since H is the in-center of $\triangle$ $H_aH_bH_c$, S'' must be the in-center of $\triangle$ $M_{a''}M_{b''}M_{c''}$.

Therefore we see that the six Simson's lines, three with reference to one $\triangle$ and three with reference to the other, meet in the same point.

32. This, at the same time, establishes another even more interesting proposition, namely: If the Simson's lines of the vertices of a first $\triangle$ with reference to a second $\triangle$ concur in a point S'', then the Simson's lines of the vertices of the second $\triangle$ with reference to the first $\triangle$ concur in the same point S''.

The broad scope covered by this proposition would enable me to double in number the points of concurrency of Simson's lines, but there would be little benefit in merely pointing them out, as the interested reader can easily see them for himself.

A Bibliography of Foundations of Geometry. By Morton Clark Bradley.

Euclid's treatment of parallels and angles and his definitions and axioms—particularly his twelfth—are the points of controversy that cause the most discussion. For nearly twenty centuries Euclid's work remained unquestioned. Since John Kepler's day, however, there have been new theories constantly advanced, theories built on axioms and definitions, a part of which, at least, are different from those of Euclid. The most important of the non-Euclideans are John Bolyai, Lobatschevski, Helmholtz, Riemann, Clifford, Henrici, Caley, Sylvester and Ball. The most prominent exponent of the non-Euclidean ideas in this country is Prof. Geo. Bruce Halsted, of Texas University. These mathematicians hold that Euclid's twelfth axiom is not, strictly speaking, an axiom—that it is not "a self-evident and necessary truth," but that it requires demonstration. They claim, too, that his definitions are not sufficient nor necessarily intelligible. Some of these men have built up new theories upon their substituted axioms and definitions, retaining those of Euclid that fit their theories. A few of these "reform" works are mere quibbles on words, but others deserve the serious consideration of all interested in pure geometry.

The list following is a complete list of English references to be found in the mathematical library of the University of Indiana or in the private library of Dr. Aley. Chrystal says the bibliography credited to Mr. Halsted contains all the references up to its time, save one, giving the non-Euclidean arguments. The list is not complete in arguments for Euclid, it being impossible to enumerate all the editions of Euclid, edited and upheld by the different mathematicians. The list is complete enough, however, to assure the reader that there are arguments for Euclid as well as against him.

1. T. S. Aldis: "Remarks on the Teaching of Geometry."
2. Isaac Barrow: "Mathematical Lectures," London, Stephen Austen, 1734.
3. Arthur Caley, Collected Papers of, Vol. II, pp. 604-6; Vol. V, p. 471; Vol. VIII, XXXIII-V, pp. 409-13; Vol. XII, pp. 220-38; Vol. XIII, p. 480.
4. H. W. Challis: "A Letter to John Stuart Mill on the Necessity of Geometry and the Association of Ideas," Oxford and London, James Parker & Co., 1867.
5. G. Chrystal: "Non-Euclidean Geometry," Edinburg, David Douglas, 1880; "Presidential Address," Bedford, W. J. Robinson, 1887.
6. Thos. Cullorin: "A paper on Parallels," Quar. Jour. of Math., Vol. 27, pp. 188-225.
7. Edward T. Dixon: "The Foundations of Geometry," London, Geo. Bell & Sons, 1891.
8. Charles L. Dodgson: "Euclid and His Modern Rivals," Supplement to "Euclid and His Modern Rivals," London, Macmillan & Co., 1885; "A New Theory of Parallels," London, Macmillan & Co., 1890.
9. Geo. B. Halsted: "A Bibliography of Hyper-space and Non-Euclidean Geometry," Amer. Jour. of Math., Vol. I, pp. 261-276, 384, 385; Vol. II, pp. 65-70; "Elements of Geometry," New York, John Wiley & Sons, 1885. (See Lobatschewski.)
10. Henrici: "Presidential Address," Bedford, W. J. Robinson, 1887.
11. J. Larmor: "On the Geometrical Method," The Math. Gazette, No. 7, April, 1896.
12. Nicolai Ivanovich Lobatschewski: "New Principles of Geometry," translated from the Russian by G. B. Halsted, Austin, The Neomon, 1897; "Geometrical Researches on the Theory of Parallels," translated by Halsted.

13. J. N. Lyle: "Euclid and the Anti-Euclideans," St. Louis, Frederick Printing Co., 1890.
14. F. S. Macaulay: "John Bolyai's 'Science of Absolute Space,'" Math. Gazette, Nos. 8 and 9, July and October, 1896.
15. Simon Newcomb: "Elements of Geometry" (appendix), New York, Henry Holt & Co., 1894.
16. Francis Wm. Newman: "Difficulties of Elementary Geometry," London, Longman, Brown, Green & Longmans, 1841.
17. Riemann: "On the Hypotheses which Lie at the Bases of Geometry," translated by Wm. K. Clifford, Nature, Vol. VIII, No. 183, pp. 14-17; No. 184, pp. 36, 37.
18. A. W. Russel: "The Foundations of Geometry," Cambridge, University Press, 1847.
19. Robert Simson: "Euclid," Philadelphia, Conrad & Co., 1810.
20. W. E. Story: "On the Non-Euclidean Trigonometry," Amer. Jour. of Math., Vol. IV, p. 332; "On Non-Euclidean Geometry," Amer. Jour. of Math., Vol. V, p. 80; "On Non-Euclidean Properties of Conics," Amer. Jour. of Math., Vol. V, p. 358.

Point-Invariants for the Lie Groups of the Plane.

By David A. Rothrock.

Among the many interesting and important applications of Lie's Theory of Transformation Groups none deserves more prominent mention than the application to invariant theory. Whether the invariants dealt with be functions or equations, surfaces and curves or points, equally interesting results are obtained. The present paper has to do with the determination of the point-invariants for the finite continuous groups of the plane as classified by Lie in Vol. XVI. of the Mathematische Annalen. In the first part of the paper is sketched a brief outline of the Lie theory leading up to the point-invariant, then follow the calculations of the invariant functions.

An infinitesimal point-transformation gives to x and y the increments

$$\delta x = \xi\,(x, y)\,\delta t,\quad \delta y = \eta\,(x, y)\,\delta t,$$