In these eight functions we must express y^i in terms of y_i and x^i, and then eliminate the differentials x', x'', ... This work of elimination is quite tedious, but may be briefly indicated. We construct three functions.

$$A \equiv 3\,R_3 - 12\,R_4 - 5R_2{}^2 = \frac{3y_2y_4 - 5y_3{}^2}{y_2{}^2}\,(x')^2,$$

$$B \equiv 15\,R_6 + 3R_5 + \frac{40}{3}\,R_2{}^3 - 15\,R_2\,R_3 + 30\,R_2\,R_4$$

$$\equiv \frac{3y_2{}^2y_5 - 15y_2y_3y_4 + \frac{40}{3}\,y_3{}^3}{y_2{}^3}\,(x')^3,$$

$$C \equiv 18R_8 + 3R_4 - 60\,R_9 - 21\,R_2\,R_5 - \tfrac{35}{3}\,R_2{}^4 + 35\,R_2{}^2R_3 + 70\,R_2{}^2R_4 + 210R_4{}^2$$

$$\equiv \frac{3y_2{}^3y_6 - 21y_2{}^2y_3y_5 + 35\,y_2\,y_3{}^2y_4 - \frac{35}{3}\,y_3{}^4}{y_2{}^4}\,(x')^4,$$

and eliminate from these x', giving the differential invariants

$$\Phi_1 = (3y_2{}^2y_5 - 15\,y_2y_3y_4 + \tfrac{40}{3}\,y_3{}^3) : (3y_2y_4 - 5y_3{}^2)^{\frac{3}{2}}$$

$$\Phi_2 = (3y_2{}^3y_6 - 21\,y_2{}^2y_3y_5 + 35y_2y_3{}^2y_4 - \tfrac{35}{3}\,y_3{}^4) : (3y_2y_4 - 5y_3{}^2)^2.$$

Mathematical Definitions. By Moses C. Stevens.

Performance of the Twenty-Million-Gallon Snow Pumping Engine of the Indianapolis Water Company. By W. F. M. Goss.

The fact that a pumping engine recently installed within the State of Indiana has given a duty performance higher than that previously reported for any pumping engine in any country is deemed of sufficient moment to merit the attention of the Academy.

This engine was built by the Snow Steam Pump Works of Buffalo, N. Y., and its installation at the Riverside station of the Indianapolis Water Company was completed in season for an acceptance test in July, 1898. It is a triple-expansion, fly-wheel engine, having a single acting pump below and in line with each of the three steam cylinders. Its principal dimensions are as follows:

Diameter of cylinders:	*Inches.*
High pressure	29
Intermediate	52
Low pressure	80

Diameter of piston rods:	*Inches.*
High pressure	6
Intermediate	7
Low pressure	8
Diameter of pump plunger (three single acting)	33
Stroke of all pistons and plungers	60

The more important conditions prevailing during the test of July 5 were substantially those of every-day service, and were as follows:

Revolutions per minute	21.5
Steam pressure	155.6
Total pressure against which pumps were operated (water pressure against which pumps delivered, plus suction lift.) pounds	88.7
Indicated horse power	775.5

Under these general conditions it was found that the engine performed 11,725,000,000 foot-pounds of work at the pump, on a consumption of 79,-093,000 British thermal units, giving a duty per million B. T. U. of 150.1 million foot-pounds, a performance which, as already noted, exceeds by a liberal margin that obtained in any test, the results of which have thus far been published. A comparison of the performance of this engiue with that of two other famous engines is as follows:

Name of designer or builder	E. P. Allis Co.	E. D. Leavitt, Jr.	Snow Steam Pump Works.
Locality	Milwaukee, Wis.	Chestnut Hill, Mass	Indianapolis, Ind.
Type	Triple expansion.	Triple expansion.	Triple expansion.
Name of expert conducting test	Prof. R. C. Carpenter	Prof. E. F. Miller.	Prof. W. F. M. Goss.
Capacity—million gals. in 24 hours	18.	20.	20.
Indicated horse power, I. H. P.	573.9	575.7	775.5.
Duty based on 1,000,000 heat units, expressed in million foot-pounds	137	141.9	150.1.

The work incident to the test was advanced by careful, painstaking and conservative methods, and it was believed at the time that the results obtained were as worthy of confidence as those ordinarily derived from such work. In view, however, of the remarkable performance obtained, and to avoid any possible questioning concerning the performance of the engine, the Indianapolis Water Company, with great liberality, arranged for a second test which should be so complete as to admit of a thorough analysis of its action. This second test was run early in the present month (December 3, 1898), and, while all the facts to be derived from it have not yet been determined, enough is known of them to make certain the accuracy of the previous work. The exceptional performance of the engine having, therefore, been carefully established, it is evident that the engine represents a very high standard of engineering practice. It marks the engineering progress of the day. This makes it not only a machine in which its owners may take just pride, but one which lends lustre to the whole State.

Tests to Determine the Efficiency of Locomotive Boiler Coverings.

By W. F. M. Goss.

The extent of heat losses occurring by radiation from a modern locomotive boiler under service conditions has long been a matter of speculation. There have been many investigations to determine the radiation from pipes and other steam heated surfaces, usually within buildings, but until recently no tests have been made which would disclose the effect of the air currents which, at speed, circulate about a locomotive boiler.

During the past summer (1898), however, Mr. Robert Quayle, Superintendent of Motive Power of the Chicago and Northwestern Railroad Company, in co-operation with manufacturers of boiler coverings, and, with the assistance of the undersigned, undertook to determine both the heat losses from a boiler and the relative value of several different makes of boiler coverings designed to reduce such losses. The following is a brief abstract of a report of results submitted to Mr. Quayle:

In carrying out the tests, two locomotives were employed; one to be hereafter referred to as the "experimental locomotive" was subject to the varying conditions of the test; the other being under normal conditions and serving to give motion to the experimental locomotive, and, also, as a source of supply from which steam could be drawn for use in maintaining the experimental boiler at the desired temperature. The experimental locomotive was coupled ahead of the normal engine, and, consequently,