$$\cdot \prod_{\mu = 0}^{n^2 -} \sigma \left(u - \frac{\alpha\, \omega_1 + \mu\, \omega_2}{n_2} \,\middle|\, \omega_1, \omega_2 \right)$$

Whence $X_{\frac{\alpha}{n^2}}(u)$ is a σ-product of n^2 factors whose residue sum,

$S \cdot \alpha\, \omega_1 + \frac{n^2 - 1}{2} \omega_2$ and hence can be expressed as a linear homogeneous function of x i defined in equation (4)

By repetition of the argument made in case (a) it follows that $X_{\frac{\alpha}{n^2}}(u)$ can be expressed as a linear homogeneous function of $\sigma_{\frac{\lambda}{u}, \frac{\mu}{n}}(nu \mid \omega_1\ \omega_2)$.

Hence our proposition is proved for all integral values of n.

A Formula for the Deflection of Car Bolsters.* By W. K. Hatt.

The body bolster of a car is a beam which carries the weight of the car and its loading and transfers this weight to the center of the truck bolster, which, in turn, transfers the weight to the wheels.

The bolsters are either of trussed form or of beam form. In the latter case they are of I section or else with one flange and web plates.

It is quite important to construct the body bolster so that it may be stiff enough to prevent contact at the side bearings. These side bearings are placed between the truck and body bolster to limit the oscillations of the car. Evidently if the side bearings come into contact the consequent friction will offer additional resistance when the car goes around curves.

The problem is to compute the deflection of a beam of variable depth.

In case of beam bolsters the moment of inertia of the cross section may be taken to be a linear function of the distance of the cross section from the free end of the beam.

Referring to Fig. 1, let AB be one-half of a body bolster and OB the curve into which the half-bolster is bent. Any point of this curve is located with reference to O by its co-ordinates x y; mn is a section of the bolster distant x from O;

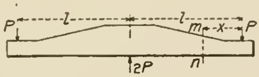

Fig. 4.

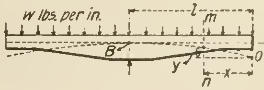

Fig. 1.

*The following is an abstract of a paper which is given in complete form in the Railroad Gazette for December 23, 1898.

l is half the length of the bolster. Let I_0 be the moment of inertia at A, I^1 the moment of inertia at B and I the moment of inertia at the invariable section mn.

Assume the bolster to be uniformly loaded, to be supported at a point and also assume that the moment of inertia I at mn

$$= I_0 + \left\{ \frac{I^1 - I_0}{l} \right\} x :$$

that is, the moment of inertia increases directly as x increases.

E is the modulus of elasticity of the material.

Equating the moment of the elastic forces to the moment of the external forces about the neutral axis of section mn, we find

$$+ E I \frac{d^2 y}{d x^2} = - \frac{w x^2}{2}$$

$$\text{or, } E \frac{d^2 y}{d x^2} = - \frac{w}{2} \cdot \frac{x^2}{I} = - \frac{w}{2} \cdot \frac{x^2}{I_0 + \left\{ \frac{I^1 - I_0}{l} \right\} x}.$$

Dividing the enumerator of the fraction by the denominator, integrating twice and determining the value of the constants, we find

$$E y = \tfrac{1}{2} \frac{w l^4}{I^1 - I_0} \left\{ n \left[\tfrac{1}{2} - C + C^2 \left\{ 1 + \log._e \left(\frac{1 + C}{n + C} \right) \right\} \right] \right.$$

$$\left. - C^3 \log._e \left(\frac{C}{n + C} \right) + \frac{C n^2}{2} - \frac{n^3}{6} \right\} \ldots\ldots\ldots\ldots\ldots\ldots (A)$$

$$\text{where } n = \frac{x}{l}, \text{ and } C = \frac{I_0}{(I^1 - I_0)}$$

To obtain the deflection at the side bearing, it would be necessary to substitute for n its proper numerical value along with other values, and compute the resulting value of y. The deflection at the side bearing is not equal this value of y; but is equal to the end deflection minus this value of y.

When n = 1,

$$E y = \frac{w l^4}{2 (I^1 - I_0)} \left\{ \tfrac{1}{3} - \frac{C}{2} + C^2 - C^3 \left[\log._e I_0 - \log._e I^1 \right] \right\}$$

When n = 1 and $I_0 = 0$,

$E y = \frac{w l^4}{6 I^1}$, and when n = 1 and $I_0 = I^1$ the expression becomes indeterminate and evaluates to $E y = \frac{w l^4}{8 I^1}$.

The truck bolster may be treated as a cantilever, with a terminal load equal to one-half the center load and a length one-half the length of the bolster as shown in Fig. 4.

If $I = I_0 + \left\{ \frac{I^1 - I_0}{l} \right\} x$

then, $E\frac{d^2y}{dx_2} = -\frac{Px}{I} = -\frac{Px}{I_0 + \left\{\frac{I^1 - I_0}{l}\right\}}$

Integrating this equation twice and determining the value of the constants, it becomes

$$Ey = \frac{Pl_3}{I^1 - I_0}\Big\{ n\left[1 - C\left(\log_e\left(\frac{1+C}{n+C}\right) + 1\right)\right]$$

$$- C^2 \log_e \frac{C}{C+n} - \frac{n^2}{2}\Big\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (B)$$

$$\text{Where } n = \frac{x}{l} \text{ and } C = \frac{I_0}{I^1 - I_0}.$$

When $n = 1$,

$$Ey = \frac{Pl^3}{I^1 - I_0}\left\{\frac{1}{2} - C + C^2(\log_e I^1 - \log_e I)\right\}.$$

If $I_0 = 0$ and $n = 1$

$$Ey = \frac{1}{2}\,\frac{Pl^3}{I^1}$$

When $I^1 = I_0$ and $n = 1$, the expression becomes an indeterminate form which evaluates to $y = \frac{1}{3}\frac{Pl^3}{EI_0}$; which is a well-known formula.

Applying formula (A) to a body bolster, uniform load, when $I^1 = 115$, $I_0 = 28$, $l = 53$ in.; n for side bearing $= \frac{23}{53}$, $E = 30{,}000{,}000$, $w = 750$ pounds per running inch, we find that the deflection of side bearing below center $= 0.117$ inches. This same bolster subjected to actual test showed a deflection at side bearing, under above conditions, of 0.115 inches.

In this case, a close approximation to the deflection at side bearing will be given by the expression

$$d = \frac{1}{15}\,\frac{wl^4}{EI^1}$$

The method of loading a body bolster for the purpose of a laboratory test used in Purdue University laboratory, may be worth noting.

A wire is stretched between the side bearings, and the bolster is loaded near the ends. The movement of the wire with reference to the center is noted. The bolster is next loaded at points between the ends and the center. Successive loadings are thus applied and the consequent deflections of side bearings noted. Since the deflections are all elastic deflections, the sum of the individual deflections for part loadings may be taken without error to be the total deflection under the sum of the loadings.