It may be observed that the velocity is the same as that acquired by a body falling through the height h, and is independent of the radial distance, r. The time of descent is directly proportional to r; and both are independent of the weights. That is, we have the theorem:

Motion on the helix surface is equivalent to that on the incline plane, when r is constant.

A Generalization of Fermat's Theorem.

Jacob Westlund.

Consider the function

$$(1)\quad F_{(\alpha,\ A)} = \alpha^{n(A)} - \left(\alpha^{\frac{n(A)}{n(P_1)}} + \dots + \alpha^{\frac{n(A)}{n(P_i)}}\right) + \left(\alpha^{\frac{n(A)}{n(P_1P_2)}} + \dots\right) - \dots + (-1)^i\, \alpha^{\frac{n(A)}{n(P_1P_2\dots P_i)}},$$

where α is any algebraic integer and A any ideal in a given algebraic number field, $P_1, \dots P_i$ are the distinct prime factors of A, and n(A) denotes the norm of A. The theorem which we shall prove is that $F(\alpha, A)$ is always divisible by A.

For the case when α and A are rational integers several proofs of the divisibility of $F(\alpha, A)$ by A have been given*.

When A is a prime ideal the function $F(\alpha, A)$ reduces to $\alpha^{n(A)} - \alpha$, which, as we know, is divisible by A.

Let us first consider the case when $A = P_1^{S_1}$, where P_1 is a prime ideal of degree f, and p_1 the rational prime divisible by P_1. Then

$$F(\alpha,\ P_1^{S_1}) = \alpha^{p_1^{fs_1}} - \alpha^{p_1^{f(s_1-1)}}.$$

But

$$\alpha^{p_1^{f(s_1-1)}(p_1 f - 1)} \equiv 1, \text{ mod } P_1^{S_1}$$

and

$$\alpha^{p_1^{fs_1}} \equiv \alpha^{p_1^{f(s_1-1)}}, \text{ mod } P_1^{S_1}$$

hence

$$(2)\quad F(\alpha,\ P_1^{S_1}) \equiv 0, \text{ mod } P_1^{S}.$$

Now, suppose $A = B \,.\, P_1^{S_1}$ where B is any ideal not divisible by P_1. Then we can easily derive the following relation:

$$F(\alpha^{p_1^{fs_1}},\ B) - F(\alpha,\ B \,.\, P_1^{S_1}) = F(\alpha^{p_1^{f(s_1-1)}},\ B),$$

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* Dickson, Annals of Mathematics, 2d Series, Vol. 1, 1899, p. 31.

or

(3) $F(a, BP_1^{S_1}) = F(a^{p_1^{fs_1}}, B) - F(a^{p_1^{f(s_1-1)}}, B)$.

If we let $B = P_2^{S_2}$ we get from (3)

$$F(a, P_2^{S_2}P_1^{S_1}) = F(a^{p_1^{fs_1}}, P_2^{S_2}) - F(a^{p_1^{f(s_1-1)}}, P_2^{S_2})$$

and hence by (2)

(4) $F(a, P_2^{S_2}P_1^{S_1}) \equiv 0, \bmod P_2^S$

By a similar reasoning we also get,

(5) $F\left(a, P_2^{S_2} P_1^S\right) \equiv 0 \bmod P_1^{S_1}$ and hence by (4) and (5).

(6) $F\left(a, P_2^S P_1^{S_1}\right) \equiv 0 \bmod P_2^{S_2} P_1^{S_1}$.

We now assume that for an arbitrary a the function $F(a, A)$ is divisible by A, then if P be any prime ideal not contained in A we have by (3)

$$F(a, AP^s) = F(a^{p^{fs}}, A) - F(a^{p^{f(s-1)}}, A) \text{ and hence,}$$

(7) $F(a, AP^s) \equiv 0 \bmod A$.

Now let $A = CQ^t$ where Q is a prime ideal and C prime to Q. Then,

$F(a, AP^s) = F(a^{q^{ft}}, CP^s) - F(a^{q^{f(t-1)}}, CP^s)$ where q is the rational prime divisible by Q and t the degree of Q, and since by our assumption the two terms on the right side are divisible by CP^s it follows that,

(8) $F(a, AP^s) \equiv 0 \bmod CP^s$, and hence,

(9) $F(a, AP^s) \equiv 0 \bmod AP^s$.

Hence if $F(a, A)$ is divisible by A when A contains n distinct prime factors it is also divisible by A when A contains n+1 distinct prime factors. Making use of (4) we then find that $F(a, A)$ is divisible by A for any A.

On the Class Number of the Cyclotomic Numberfield $K\left(e^{\frac{2\pi i}{p^n}}\right)$

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[By title.]

[Will appear in Transactions American Mathematical Society, Vol. IV; 2.]