Decrement Measurements.

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In wireless work one of the important measurements is the logarithmic decrement of the aerial or decrement, as it is called in wireless. Decrement is an indication of the sharpness of the radiating wave. To liken radiation from a wireless station to the radiation from a light source: a station with a low decrement gives a line spectrum of a definite wave length while a large decrement means a band spectrum covering a large range of wave lengths. It is hard, or next to impossible to tune out a station with a large decrement. On this account the U. S. Government has outlawed stations with decrement greater than .2. Another advantage of small decrement is that all the radiated energy of the sending station is concentrated on one wave length, while the energy is scattered over a broad band when the decrement is large.

The solution of the differential equation of an oscillating circuit containing resistance, inductance, and capacity may be put into the form $I = I_0 e^{-\alpha t} \sin wt$ where I_0 is the initial or maximum value of the current, I is the value of the current at any time, t; w, is the angular velocity or $2\pi n$, n being the frequency and, $\alpha = R/2L$, R being the resistance and L the inductance of the circuit.

The equation can be represented by the curve of figure 1.

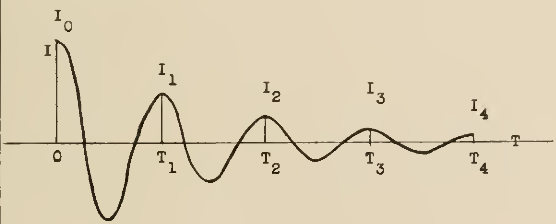

Fig. 1 .

The amplitudes are:

$$I_0 = I_0 e^{-\alpha o T}$$

$$I_1 = I_0 e^{-\alpha T} \qquad \text{and} \qquad \frac{I_1}{I_2} = \frac{I_0 e^{-\alpha T}}{I_0 e - \alpha 2T} = e^{\alpha T}$$

$$I_2 = I_0 e^{-2\alpha T} \qquad \frac{I_2}{I_3} = \frac{I_0 e^{-\alpha 2T}}{I_0 e^{-\alpha 3T}} = e^{\alpha T}$$

etc.

$$\frac{In}{In+1} = e^{\alpha T}$$

From this $\alpha T = \log.\frac{I_1}{I_2} = \log.\frac{I_2}{I_3} = \log.\frac{In}{In+1}$. This is the same as the usual logarithmic decrement used in ballistic galvanometer work, except in ballistic galvanometer work we follow the English fashion of taking the ratio of the two successive swings in the opposite direction instead of the two successive in the same direction. Or the decrement in U. S. wireless is two times the value determined by the English method. The determination of I_1, I_2, etc., or successive amplitudes of the current is impossible where the frequency is in the order of 1 million, as it is in wireless work.

In the above equation the frequency $n = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$.

If R is small or zero, this becomes $n = \frac{1}{2\pi\sqrt{LC}}$. This is the same value for n obtained from the equation of alternating current in a circuit containing resistance, inductance and capacity, with an alternating e.m.f.

$$I = \frac{E}{\sqrt{R^2 + (1/Cw - Lw)^2}}$$

The value of I is a maximum when $1/Cw - Lw = 0$, i.e. $I = E/R$. If $Lw = 1/Cw$, then $(2\pi n)^2 = 1/CL$ or $n = \frac{1}{2\pi\sqrt{LC}}$.

The above equation for I can be written $I^2 = \frac{E^2}{R^2 + (1/Cw - Lw)^2}$.

When the reactance term $1/Cw - Lw = 0$ the circuit is in resonance with the e.m.f. Then $I^2_r = \frac{E^2}{R^2 + (1/C_r w - Lw)^2} = \frac{E^2}{R^2}$ where C_r is the value of the capacity which makes the circuit in resonance with the e.m.f. Then $Lw = 1/C_r w$.

If the capacity is changed until $I^2 = 1/2\ I^2_r$, I^2_r being the resonance value, then $1/2 I_r = \frac{E^2}{R^2 + (1/Cw - 1/C_r w)^2}$ and $2R^2 = R^2 + (1/Cw - 1/C_r w)^2$ since doubling the denominator will halve the value of I^2. Then

$R^2 = 1/w^2([C_r - C]/CC_r)^2$ or $R = 1/w(C_r - C/CC_r)$

$T = 1/n = 2\pi/w$ and decrement $d = \alpha T = R/2L\ T$.

$$d = \frac{R}{2L}t = \frac{1}{w}\left(\frac{C_r - C}{CCn}\right)\frac{2}{w2L} = \pi\left(\frac{C_r - C}{C_r}\right)\frac{1}{w^2}\frac{2}{2CL}.$$

$d = \pi\frac{C_r - C}{C_r}$ where C_r is the value of the capacity at resonance and C is the value of capacity which reduces the mean square of the current to 1/2 its value. In this manner the decrement is measured by determining the resistance in terms of a capacity.

The decremeter consists of a coil, a variable condenser, and a radio frequency milliammeter or galvanometer connected in series and placed near the radiating source. The capacity is varied until the current is a maximum or the circuit is in resonance with the source. The capacity of the variable condenser is varied until the mean square of the current is reduced to 1/2 the first value. Then the decrement is calculated. This gives the sum of decrement of the source, ærial, and the decrement of the decremeter. This is exactly the same as in measuring the resistance of a 1 to 1 transformer circuit by introducing resistance in the circuit until the current is made 1/2. The value of R introduced is equal to the sum of the resistances in the two circuits. This holds if the mutual inductance is large as in a transformer.

Since $d=\alpha T=(R/2L)T$, doubling the resistance in either circuit will double the decrement of either circuit.

Thus the introduction of resistance in the decremeter circuit until the current in the decremeter is made 1/2 half, the circuit being kept in resonance all the time, will double the decrement of the decremeter. Then if $D_1=d_1+d=$ first decrement measurement and $D_2=2d_1+d=$ second decrement measurement with resistance inserted in decremeter circuit. Then $d_1=D_2-D_1$.

The decremeter is assumed to be loosely coupled to the aerial so as not to affect the aerial circuit. The method is much more simple than that usually given, as in formulae 63 and 64, page 94, Radio Instruments and and Measurements, Circular of the Bureau of Standards No. 74. This formula is:

$$\delta^1=\frac{2\delta\delta_1+\delta_1^2-\delta^2}{\delta-\delta_1}$$

Where δ^1 is the decrement of the aerial, δ the decrement of the wave meter and δ_1 the increase of δ due to the resistance added which reduces I_1^2 to 1/2 I^2.

δ^1 the decrement of the aerial seems to be given in terms of two unknown quantities. The remark is made, "It should not be forgotten that these formulae apply only when the coupling is very loose and both decrements are small". This is the condition assumed in the derivation of my formulae.

The most accurate method of getting the decrement of a decremeter is to use a continuous wave current such as is generated in the modern tube circuits or wireless telephone circuits. In these circuits the wave is continuous or the decrement is zero and the decrement measured is that of the decremeter alone.

This method can be used to determine the decrement of the decremeter and thus check the above method.

The decremeter used contained a 250 milliampere milliammeter whose D. C. resistance was 6 ohms.

When the current in the decremeter was large there was a tendency to spark over in the condenser. This brush discharge introduced a resistance in the circuit which was more or less variable. This tends to make the decrement of the decremeter greater at 200 milliamperes than at 100 milliamperes.

Due to the fact that the current is intermittant in a damped wave station. This sparking over effect is greater with damped waves than in the case of continuous waves.

The following table gives results with CW and damped waves. Decrement of wave meter at wave length indicated.

	100 mil. amp.	200 mil. amp.	
390 meters	.11	.14	
375 meters	.10	.15	
375 meters	...	.15	
348 meters	.10	.14	
310 meters	.12	.14	Average $\delta_1 = .126$

Decrement of decremeter with resistance introduced to reduce the current from 200 to 100 milliamperes.

100 milliamperes	375 meters
$2\delta_1$	.24
.26	.27

Average $\delta_1 = .127$

Decrement of decremeter with damped wave 375 meters.

$$D_2 = 2d_1 + d = .36$$
$$D_1 = d_1 + d = .23$$
$$D_2 - D_1 = d_1 = .13$$

Second:

$$D_2 = 2d_1 + d = .38$$
$$D_1 = d_1 + d = .24$$
$$D_2 - D_1 = d_1 = .14$$

Thus it is shown that the above method of determining the decremnet of a decremeter checks fairly well with the CW. method.

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