# WHY THE LOGARITHM IN LOGARITHMIC DECREMENT?

R. R. RAMSEY, Indiana University.

When using a ballistic galvanometer it is usual to observe the first deflection and the second deflection and multiply the first deflection by the factor $1 + \frac{1}{2}\lambda$ where $\lambda$ is the logarithm of the ratio of the first deflection $d_1$ to second deflection $d_2$, in order to get the deflection which should have been obtained if the galvanometer were not damped.

If the galvanometer is not damped we have simple harmonic motion. Simple harmonic motion may be considered to be uniform circular motion projected upon a diameter of the circle. In like manner the motion of the galvanometer when damped may be considered to be the projection of the motion of a body moving with uniform angular motion but spiraling in towards the center upon a curve known as the logarithmic spiral.

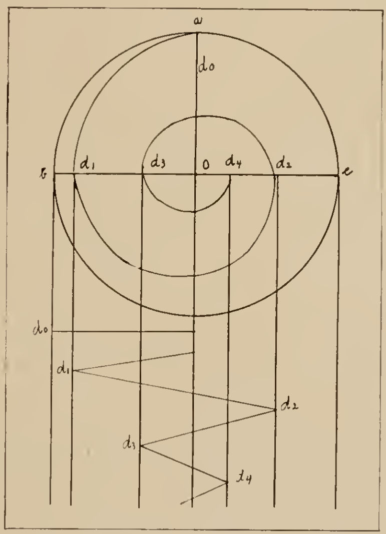


Fig. 1. Projection of logarithmic spiral.

Figure 1 represents a circular of radius $d_0$ and inside of which is the spiral whose equation is $d = d_0 e^{-\alpha\theta}$; where d is the distance from the center at any point, $d_0$ is the initial distance at the point, a — in this case $d_0$ is the radius of the circle also, $\theta$ is the angle measured from a and $\alpha$ is a constant.

The first, second, etc., deflections are the horizontal distances from the center o, when $\theta = \pi/2$, $3\pi/2$, etc.

Then since $d = d_0 e^{-\alpha\theta}$

$$d_1 = d_0 e^{-\alpha\pi/2}$$
$$d_2 = d_0 e^{-\alpha 3\pi/2}$$
$$d_3 = d_0 e^{-\alpha 5\pi/2}$$

"Proc. Ind. Acad. Sci., vol. 33, 1923 (1924)."

From the above $\frac{d_1}{d_2} = \frac{d_1 e^{-\alpha\pi/2}}{d_0 e^{-\alpha 3\pi/2}} = e^{\alpha\pi}$

$$\frac{d_2}{d_3} = e^{\alpha\pi}$$

$$\frac{d_1}{d_2} = \frac{d_2}{d_3} = \frac{d_3}{d_4} = \ldots = e^{\alpha\pi} = e^{\lambda} = K$$

Where $\lambda = \alpha\pi = \log_e \frac{d_1}{d_2}$.

From equation (1)

$$d_0 = d_1 e^{\alpha\pi/2} = d_1 e^{\lambda/2}$$

since $e^x = 1 + x + x^2/2 + \ldots$

$$e^{\lambda/2} = 1 + 1/2\lambda \text{ if } \lambda \text{ is small,}$$

and $d_0 = d_1(1 + 1/2\lambda)$.

This equation is the one usually used in making correction for decrement.

Since $e^{\lambda} = e^{\lambda/2} \times e^{\lambda/2} = K$

$$e^{\lambda/2} = \sqrt{K}$$

and $d_0 = d_1 \sqrt{K}$

why not multiply by the square root of the ratio instead of by one plus one-half the logarithm of the ratio?