Notes on the Calibration and Use of the Ballistic Galvanometer.

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The ballistic galvanometer is an important adjunct to an electrical laboratory, inasmuch as it integrates the transient and varying currents in the case of circuits which contain inductance or capacity or both. If the time constant of the circuit is small compared with the quarter period of the suspended system of the galvanometer, the first throw is proportional to the total charge which passes, or $Q = \int i dt = G \phi$. Where Q is the charge, ϕ is the observed first throw, and G is a constant expressed in terms of coulombs or micro-coulombs per scale division. To interpret any reading the galvanometer must be calibrated by passing through it a known charge and observing the resulting first throw, the quotient giving the value of G.

This first throw is reduced somewhat by the so-called "damping," by which is meant the effect of all those resisting forces which tend to absorb the energy of a vibrating system, of any sort whatever. These forces are generally assumed to be proportional to the velocity of the moving parts, although there is no reason *a priori* why they should not depend upon other functions of the velocity, as indeed they appear to do in some cases. However, long experience has shown that the simple proportion above stated is a satisfactory generalization for slowly moving bodies, and one which introduced into the general equations of motion leads to results quite in accordance with experimental observations, for a large class of physical problems.

The earlier forms of ballistic galvanometer, now seldom seen in actual service, were designed with small, highly polished needles of the Siemens pattern, bell-shaped and slotted, and usually arranged much like the Kelvin galvanometers of the same period, astatic, and highly sensitive. An essential feature, as pointed out in the older text-books, was that the damping should be a minimum, in this type of galvanometer being due to fiber viscosity, air friction and the electro-magnetic reactions of induced currents, this latter effect however being very small. Such damping as did occur was corrected for by the use of that convenient fiction, the throw which would have occurred if there had been no damping, which is given by

$$\phi_o = \phi\left(1 + \frac{\lambda}{2}\right)$$

where ϕ is the observed throw and λ is the logarithmic decrement of Gauss, which is the natural logarithm of the ratio of successive amplitudes.

This method was known to lack precision, and indeed became unusable when the logarithmic decrement reached a value of 0.4 or 0.5. A common laboratory experiment[1] of this period was one designed to determine the resistance of a galvanometer or of an unknown coil in terms of the logarithmic decrements taken successively on open circuit, circuit closed through the galvanometer only, and circuit closed including the resistance to be measured. Satisfactory results were possible only with a needle of large magnetic leakage, and with special adjustments of the coils.

With the introduction of the suspended coil type of galvanometer and its rapid displacement of earlier types, it claimed attention also as a valuable and accurate ballistic instrument. However the normal damping is much greater in this case, first because of the increased air friction as compared with that acting on the small polished bell-shaped steel needles, and second because of the very greatly increased electro-magnetic reactions due to induced currents circulating within the coil itself.

In passing from the older to the newer type there are certain considerations which require careful attention, inasmuch as the methods applicable to the older type will usually lead to incorrect results if applied to the suspended coil type. Particularly is this true in calibrating the galvanometer. With the older type concordant results were obtained either with a standard cell and condenser, or with a mutual inductance, the logarithmic decrement being calculated in either case, and the appropriate corrections being applied. But with the suspended coil galvanometer, where the electromagnetic damping is large frequently indeed causing the motion to lose its oscillatory character entirely and become aperiodic, it is impracticable to calculate or use the logarithmic decrement in the regular way. It is then clear that the damping, and hence the discordance between the observed and fictitious throws will not only be large, but will be a function of the resistance in the external circuit, which function is not easy to determine.

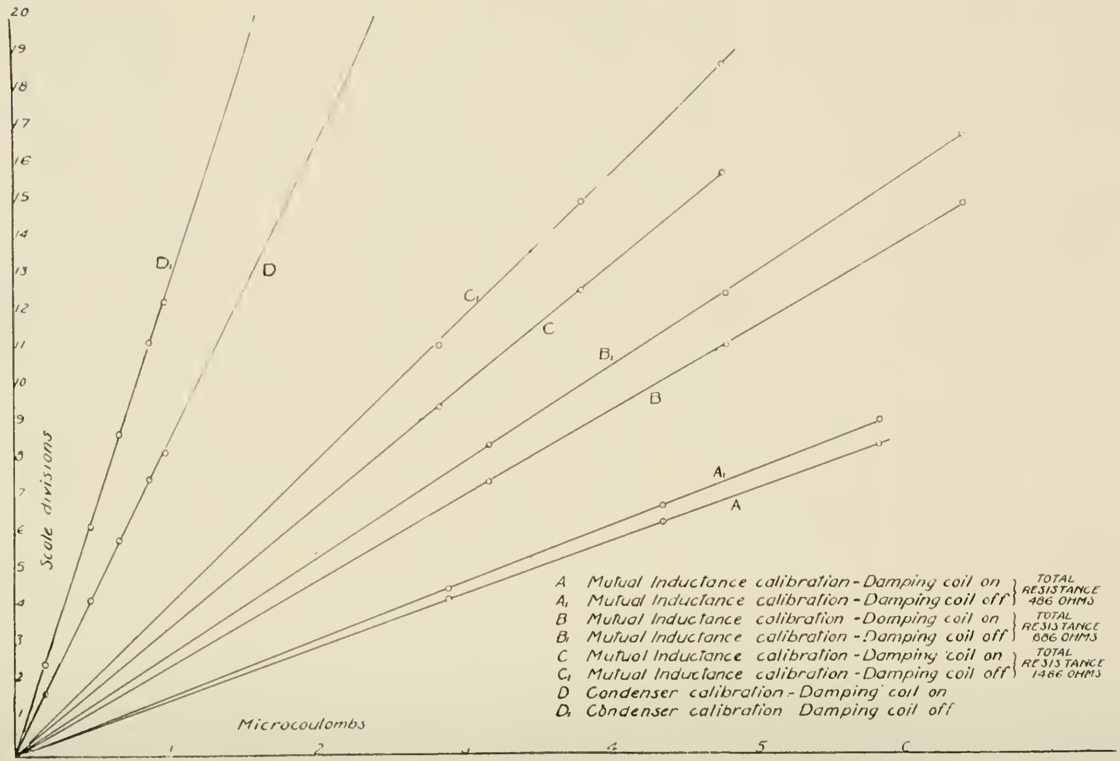
[1] Kohlrausch, Lehrbuch der Prak, Physik. 9th ed., p. 399.

Regarding this matter of calibration, existing hand books, laboratory text books, and maker's catalogs are not clear, and the reader, whether he be a student, an inexperienced instructor, or a practical man can be, and to to the writer's knowledge often is, misled. In discussing the use of the ballistic galvanometer in iron testing for example, the statement in various sources which should be authoritative is not infrequently seen, that the galvanometer may be calibrated with a standard cell and condenser, and students have been known to follow these directions, without counsel from the instructor, although the condenser was introducing perhaps 4,000 megohms in series with the galvanometer, while the resistance of the secondary circuit otherwise used was less than 100 ohms. This procedure may give rise to errors of several hundred per cent. with corresponding influence on the values for the B-H curve.

Recognizing that this problem is satisfactorily treated in much of the existing literature, it must also be admitted that many of the current helps, to which one first turns for reference, are quite inadequate and misleading, and it is the purpose of this article to offer a wider discussion of the facts. A single example with calibration curves of a Leeds and Northrup type H galvanometer will serve to illustrate the principle. In figure 1, curve D gives the relation of charge to deflection for the case of calibration with a standard condenser. For the same galvanometer, A, B and C are the corresponding calibration curves when the total circuit resistances are respectively 486, 886, and 1,486 ohms. These curves show clearly the influence of diminishing total circuit resistance upon the value of the galvanometer constant. Curve D shows 8.2 scale divisions for 1 micro-coulomb, while curve A, for a circuit resistance of 486 ohms, shows 1.4 scale divisions for the same charge.

Curves A, B, C, and D were taken with the small rectangular damping coil removed. A similar set of curves, A_1, B_1, C_1, and D_1 give the cali bration values after the damping coil has been removed.

Various suggestions have been made for calculating the true value of the ballistic constant for any given condition from the known constants of the galvanometer such as period, moment of inertia, moment of torsion, strength of field, etc. These methods, entirely adequate theoretically, are nevertheless difficult to apply practically, because the values of the constants are seldom known with sufficient precision, and are themselves liable to change when the galvanometer is readjusted.


Scale divisions
Microcoulombs
D₁
D
C₁
C
B₁
B
A₁
A
A Mutual Inductance calibration - Damping coil on
A₁ Mutual Inductance calibration - Damping coil off
TOTAL RESISTANCE 486 OHMS
B Mutual Inductance calibration - Damping coil on
B₁ Mutual Inductance calibration - Damping coil off
TOTAL RESISTANCE 886 OHMS
C Mutual Inductance calibration - Damping coil on
C₁ Mutual Inductance calibration - Damping coil off
TOTAL RESISTANCE 1486 OHMS
D Condenser calibration - Damping coil on
D₁ Condenser calibration Damping coil off

To eliminate the effect of damping due to eddy currents two suggestions have been made. (1) to insert in the galvanometer circuit a special key so arranged as to break the galvanometer circuit a brief instant after the charge has passed, thus securing always the open circuit conditions of curve D; (2) to insert a special key in the galvanometer circuit so arranged that the galvanometer will always be closed through a circuit of constant resistance. Both of these methods are satisfactory, but only with perfectly operating keys, which condition is not easy to secure.

By far the safest and most convenient procedure is then to *calibrate the galvanometer for the precise conditions under which it is to be used.* This may readily be carried out by permanently including in the galvanometer circuit the secondary coil of a standard mutual inductance, and by simply reversing a known current in the primary circuit the constant can be accurately determined from the resulting throw.

For a standard of mutual inductance it has long been customary to rely on the long solenoid with a short coaxial solenoid for a secondary coil. Unless these are well made, with exceptional care and by experienced hands, they are by no means standard. The writer has measured the mutual inductance of a large number of such coils from different makers, and the subjoined table will show the discordances between measured and calculated values for a few of them.

Solenoid.	Calculated.	Measured.	Per Cent. Variation.
1	2.411	2 428	0.7%
2	1.779	1.878	5.5
3	1.010	1.053	4 0
4	0.609	0.581	4.7
5	1.027	1.068	3.8
6	0.539	0.544	0.8
7	2.1526	2.1560	0.17
8	1.0436	1.0560	1.2
9	1.056	1.073	1 6

The calculated values were all secured from the approximate formula based on 4π ni as the value of the field at the center of a solenoid, while the measured values were obtained by Maxwell's method, by comparison with a marble spool standard for which the Bureau of Standards had furnished values. These coils represent several makers, and one, No. 7, was made by the writer. It is known to be the practice of some makers to wind the coils with only approximate measurements and data, then to standardize them against a known value and subsequently to adjust certain factors in the data so that the calculated and measured values agree.

It has long the writer's belief that, except as a brief laboratory exercise, to show the student how a standard mutual inductance may be realized, the coaxial solenoids should be replaced by calibrated standards, wound preferably on white marble spools thoroughly varnished and baked hard. These when calibrated at the Bureau of Standards or elsewhere are very permanent, convenient and reliable.

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