

NOTE ON MULTIPLY PERFECT NUMBERS, INCLUDING A TABLE OF 204  
NEW ONES AND THE 47 OTHERS PREVIOUSLY PUBLISHED.

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BY R. D. CARMICHAEL AND T. E. MASON.

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§1. *Introduction and Historical Note.*

If the sum of all the divisors of  $N$  is  $mN$ , where  $m$  is an integer, we shall call  $N$  a multiply perfect number of multiplicity  $m$ . If  $m=2$  we shall call  $N$  a perfect number.

The study of such numbers gave rise to the principal contributions of Fermat to the higher arithmetic; and consequently they have been a means of prime importance in leading to the development of the modern theory of numbers.<sup>1</sup> As is well known their history goes back to Euclid, who proved that every number of the form  $2^{p-1}(2^p-1)$ , where  $2^p-1$  is a prime, is a perfect number. Euler and others<sup>2</sup> have shown that every even perfect number is of the Euclid type; but it remains an open question as to whether there do or do not exist odd perfect numbers. Several supposed proofs that no odd perfect number exists have been given, but none of these is rigorous. The actually known perfect numbers<sup>3</sup> are included in the Euclid formula  $2^{p-1}(2^p-1)$  for the ten values of  $p$ ,  $p=2, 3, 5, 7, 13, 17, 19, 31, 61, 89$ .

It appears that the first discovery of a multiply perfect number of multiplicity greater than 2 is due to Mersenne, who observed that 120 is one-third of the sum of all its divisors. In response to a problem proposed by Mersenne, Fermat pointed out that 672 has also the property of being equal to one-third the sum of all of its divisors. From time to time other multiply perfect numbers have been discovered.<sup>4</sup> Up to the present time

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<sup>1</sup> Cf. Lucas, *Théorie des nombres*, I, p. 376.

<sup>2</sup> A very simple proof of this theorem has recently been given by Dickson, *American Mathematical Monthly*, vol. 18 (1911), p. 109. See also a proof by Carmichael, *Annals of Mathematics*, vol. 8 (1907), p. 150.

<sup>3</sup> For reference to the literature of perfect numbers, see *Encyclopédie des sciences mathématiques*, I<sub>3</sub>, pp. 53-56.

<sup>4</sup> For a short history of these numbers, with references, see *Encyclopédie des sciences mathématiques*, I<sub>3</sub>, pp. 56-58.

there have been published altogether, so far as we have been able to find out, a total of forty-seven multiply perfect numbers. Cunningham<sup>1</sup> has announced that he has a table of eighty-five multiply perfect numbers; but he published only one of them. In the table of perfect and multiply perfect numbers in §3 we have credited to the discoverer each of the forty-seven numbers which have heretofore been published.<sup>2</sup> The remaining two hundred four numbers of the table are believed to be published here for the first time. It should be noted that numbers of multiplicity 7 occur in this table for the first time.

In §2 we have given some working rules which were found useful in obtaining new multiply perfect numbers from those already known or discovered in the process of constructing the table. Their further use would consist in the possible discovery of several new multiply perfect numbers from a single new one found by any other means whatever. It was in this way that many of the new numbers in this paper were discovered; one was obtained by direct means and others followed by use of the rules. As to the rules themselves, some of them were gotten by direct means and others by comparison of numbers in the table while the table itself was being constructed. The list of number pairs in the rules might be largely extended by a further comparison of numbers in the table. We have selected a part of those which actually proved to be of most use in the construction of the table.

## §2. Rules for Finding Multiply Perfect Numbers.

The following two theorems afford useful working rules for finding new multiply perfect numbers:

I. If  $\prod p_i^{a_i}$  and  $\prod q_i^{b_i}$  (in either order) are a pair of factor sets from the list below and if a multiply perfect number  $N$  of multiplicity  $m$  contains the factor  $\prod p_i^{a_i}$  without containing either any factor  $p_i^{a_i+1}$  or any factor  $q_i$  different from every  $p_i$ ; then the number

$$\frac{N \prod q_i^{b_i}}{\prod p_i^{a_i}}$$

is also a multiply perfect number of multiplicity  $m$ .

<sup>1</sup> *British Association Report*, 1902, pp. 528-529.

<sup>2</sup> We are indebted to Prof. Dickson for reference to the first publication of six of these numbers.

27. 17,  $2^{10}$ . 23. 89  
 27. 17,  $2^{25}$ . 19. 683. 2731. 8191  
 29. 31,  $2^{13}$ . 43. 127†  
 2<sup>11</sup>. 3<sup>5</sup>. 2<sup>20</sup>. 3<sup>3</sup>. 127. 337  
 2<sup>28</sup>. 7. 23. 233. 1103. 2089, 2<sup>36</sup>. 7<sup>5</sup>. 43. 223. 7019. 112303. 898423. 616318177  
 2<sup>33</sup>. 131071, 2<sup>37</sup>. 174763. 524289  
 2<sup>38</sup>. 53. 229. 8191. 121369, 2<sup>61</sup>. 59. 157. 43331. 3033169. 715827883. 2147483647  
 3<sup>4</sup>. 11<sup>3</sup>. 13, 3<sup>5</sup>. 11. 13<sup>2</sup>  
 3<sup>6</sup>. 137. 547. 1093, 3<sup>10</sup>. 107. 3851  
 3<sup>7</sup>. 23. 41, 3<sup>10</sup>. 23<sup>2</sup>. 79. 107. 3851  
 5<sup>2</sup>. 7<sup>2</sup>. 19. 31, 5<sup>3</sup>. 7<sup>3</sup>. 13  
 5<sup>2</sup>. 13<sup>2</sup>. 31<sup>2</sup>. 61. 83. 331, 5<sup>3</sup>. 13<sup>3</sup>. 17  
 5<sup>2</sup>. 7<sup>2</sup>. 19<sup>2</sup>. 127, 5<sup>3</sup>. 7<sup>3</sup>. 19  
 5<sup>3</sup>. 7<sup>4</sup>. 13<sup>3</sup>. 17<sup>2</sup>. 307. 467. 2801, 5<sup>4</sup>. 7<sup>3</sup>. 13. 17. 71

If  $N = r_1^{\gamma_1} r_2^{\gamma_2} \dots r_n^{\gamma_n}$ , where  $r_1, r_2, \dots, r_n$  are different primes, is a multiply perfect number of multiplicity  $m$ , then from the formula for the sum of all the divisors of  $N$  and the fact that this sum is now supposed to be  $mN$ , we have

$$m = \prod_{i=1}^n \frac{r_i^{\gamma_i+1} - r_i}{r_i(r_i - 1)}$$

Therefore in order to prove the accuracy of the rules we have only to show in each case that

$$\prod \frac{p_i^{a_i+1} - p_i^{a_i}}{p_i(p_i - 1)} = \prod \frac{q_i^{\beta_i+1} - q_i^{\beta_i}}{q_i(q_i - 1)}$$

The verification is not carried out.

II. If  $\prod p_i^{a_i} (m_1)$  and  $\prod q_i^{\beta_i} (m_2)$  (in either order) are a pair of factor sets and multiplicity from the list below and if a multiply perfect number  $N_1$  of multiplicity  $m_1$  contains the factor  $\prod p_i^{a_i}$  without containing either any factor  $p_i^{a_i+1}$  or any factor  $q_i$  different from every  $p_i$ ; then the number

$$N_2 = \frac{N_1 \prod q_i^{\beta_i}}{\prod p_i^{a_i}}$$

† This pair is due to Descartes.

is a multiply perfect number of multiplicity  $m_i$ :

$$3^3 \cdot 5 \cdot 7^3 \cdot 13 \text{ (5)}, 3^{10} \cdot 7 \cdot 23 \cdot 107 \cdot 3851 \text{ (4)}$$

$$3^4 \cdot 7 \cdot 11^2 \cdot 19 \text{ (5)}, 3^6 \cdot 23 \cdot 137 \cdot 547 \cdot 1093 \text{ (4)}$$

$$5 \cdot 7 \text{ (5)}, 5^3 \cdot 7^2 \cdot 13 \cdot 19 \text{ (6)}$$

$$5^3 \cdot 31 \text{ (5)}, 5^3 \cdot 7 \cdot 13 \text{ (6)}$$

In order to prove the theorem it is clear that we have only to show in each case that

$$\frac{1}{m_1} \prod \frac{p_i^{a_i+1} - 1}{p_i (p_i - 1)} = \frac{1}{m_2} \prod \frac{q_i^{\beta_i+1} - 1}{q_i (q_i - 1)}$$

The verification is omitted.

The following theorem, due to Descartes, is also readily proved:

III. If  $N$  is a multiply perfect number of multiplicity  $p^a$ , where  $p$  is a prime number, and if  $N$  is not divisible by  $p$ , then  $pN$  is a multiply perfect number of multiplicity  $(p+1)^a$ .

### §3. Table of Multiply Perfect Numbers.\*†

- 2) 2. 3. (Euclid, Nicomache.)
- 2) 2<sup>2</sup>. 7. (Euclid, Nicomache.)
- 4) 2<sup>2</sup>. 3<sup>2</sup>. 5. 7<sup>2</sup>. 13. 19. (Lehmer.)
- 3) 2<sup>3</sup>. 3. 5. (Mersenne.)
- 4) 2<sup>3</sup>. 3<sup>2</sup>. 5. 7. 13. (Descartes.)
- 2) 2<sup>4</sup>. 31. (Euclid, Nicomache.)
- 3) 2<sup>5</sup>. 3. 7. (Fermat.)
- 4) 2<sup>5</sup>. 3<sup>2</sup>. 5. 7. (Descartes.)
- 4) 2<sup>5</sup>. 3<sup>4</sup>. 7<sup>2</sup>. 11<sup>2</sup>. 19<sup>2</sup>. 127.
- 2) 2<sup>6</sup>. 127. (Euclid, Nicomache.)
- 4) 2<sup>7</sup>. 3<sup>2</sup>. 5<sup>2</sup>. 17. 31. (Mersenne.)
- 5) 2<sup>7</sup>. 3<sup>4</sup>. 5. 7. 11<sup>2</sup>. 17. 19. (Descartes.)
- 5) 2<sup>7</sup>. 3<sup>3</sup>. 5. 7<sup>2</sup>. 13. 17. 19. (Descartes.)
- 4) 2<sup>7</sup>. 3<sup>2</sup>. 5. 17. 23. 137. 547. 1093. (Fermat.)
- 4) 2<sup>7</sup>. 3<sup>10</sup>. 5. 17. 23. 107. 3851.
- 4) 2<sup>8</sup>. 3. 5. 7. 19. 37. 73. (Lucas.)

\* The numbers marked with a star were discovered by Mr. Mason. The remaining hitherto unpublished numbers were discovered by Mr. Carmichael.

† The multiplicity of each number is written to its left. If previously published the discoverer's name is given to the right.

- 4)  $2^3, 3^2, 7^2, 13, 19^2, 37, 73, 127$ . (Lehmer.)  
 3)  $2^3, 5, 7, 19, 37, 73$ . (Legendre.)  
 3)  $2^9, 3, 11, 31$ . (Jumeau, Fermat.)  
 4)  $2^9, 3^2, 7, 11, 13, 31$ . (Descartes.)  
 4)  $2^9, 3^3, 5, 11, 31$ . (Descartes.)  
 4)  $2^9, 3^4, 7, 11^3, 31^2, 61, 83, 331$ .  
 4)  $2^{10}, 3^3, 5^2, 23, 31, 89$ . (Mersenne.)  
 5)  $2^{10}, 3^4, 5, 7, 11^2, 19, 23, 89$ . (Fermat.)  
 5)  $2^{10}, 3^3, 5, 7^2, 13, 19, 23, 89$ . (Frenicle.)  
 5)  $2^{11}, 3^3, 5^2, 7^2, 13, 19, 31$ . (Lehmer.)  
 5)  $2^{11}, 3^3, 5, 7^2, 13^2, 19, 31, 61$ .  
 5)  $2^{11}, 3^3, 5^2, 7^3, 13^2, 31^2, 61, 83, 331$ .  
 5)  $2^{11}, 3^3, 5^3, 7^3, 13^3, 17$ .  
 5)  $2^{11}, 3^4, 5, 7^2, 13, 19, 23, 137, 547, 1093$ .  
 5)  $2^{11}, 3^{10}, 5, 7^2, 13, 19, 23, 107, 3851$ .  
 2)  $2^{12}, 8191$ . (See *Encyclopédie* I, 3, p. 55.)  
 3)  $2^{13}, 3, 11, 43, 127$ . (Descartes.)  
 4)  $2^{13}, 3^2, 7, 11, 13, 43, 127$ . (Descartes.)  
 4)  $2^{13}, 3^3, 5, 11, 43, 127$ . (Descartes.)  
 4)  $2^{14}, 3, 5, 7, 19, 31, 151$ . (Fermat.)  
 4)  $2^{14}, 3^2, 7^2, 13, 19^2, 31, 127, 151$ . (Carmichael.)  
 5)  $2^{14}, 3^2, 5^2, 7^3, 13, 19, 31^2, 83, 151, 331$ .  
 3)  $2^{14}, 5, 7, 19, 31, 151$ . (Fermat.)  
 6)  $2^{15}, 3^3, 5^2, 7^2, 11, 13, 17, 19, 31, 43, 257$ . (Carmichael.)  
<sup>\*</sup>6)  $2^{15}, 3^3, 5^3, 7^4, 11^2, 13^3, 17^2, 19, 43, 257, 307, 467, 2801$ .  
<sup>\*</sup>6)  $2^{15}, 3^3, 5^4, 7^3, 11^2, 13, 17, 19, 43, 71, 257$ .  
 5)  $2^{15}, 3^7, 5, 7, 11, 17, 41, 43, 257$ .  
 6)  $2^{15}, 3^7, 5^3, 7^2, 11, 13, 17, 19, 41, 43, 257$ .  
 2)  $2^{16}, 131071$ . (See *Encyclopédie* I, 3, p. 55.)  
 6)  $2^{17}, 3^4, 5^3, 7^3, 11^2, 13^2, 19^2, 31, 37, 61, 73, 181$ .  
 5)  $2^{17}, 3^3, 5, 7^3, 13, 19^2, 37, 73, 127$ . (Fermat.)  
 4)  $2^{17}, 3^9, 7, 19^2, 23, 37, 73, 127, 137, 547, 1093$ .  
<sup>\*</sup>6)  $2^{17}, 3^9, 5^2, 7^3, 11^2, 17, 19^4, 31^2, 37, 61, 73, 83, 101, 227, 331, 137561$ .  
 4)  $2^{17}, 3^{10}, 7, 19^2, 23, 37, 73, 107, 127, 3851$ .  
 5)  $2^{17}, 3^{11}, 7^3, 11^2, 13, 17^2, 19^4, 43, 53, 73^2, 101, 227, 307, 1801, 137561$ .  
 6)  $2^{17}, 3^{11}, 5, 7^3, 11^2, 13, 17^2, 19^4, 43, 53, 73^2, 101, 227, 307, 1801, 137561$ .  
<sup>\*</sup>6)  $2^{17}, 3^{11}, 5^3, 7^4, 11, 13^3, 17^2, 19, 53, 73^2, 307, 467, 1801, 2801$ .

- 6)  $2^{17}$ ,  $3^{11}$ ,  $5^4$ ,  $7^3$ , 11, 13, 17, 19, 53, 71,  $73^2$ , 1801.
- 2)  $2^{18}$ , 524287. (See *Encyclopédie* I, 3, p. 55.)
- 6)  $2^{19}$ ,  $3^4$ ,  $5^2$ ,  $7^2$ ,  $11^3$ , 13,  $19^2$ ,  $31^3$ , 37, 41, 61, 127.
- 6)  $2^{19}$ ,  $3^4$ ,  $5^3$ ,  $7^3$ ,  $11^3$ , 13, 19,  $31^3$ , 37, 41, 61.
- 6)  $2^{19}$ ,  $3^5$ ,  $5^2$ ,  $7^2$ , 11,  $13^2$ ,  $19^2$ ,  $31^3$ , 37, 41, 61, 127.
- 6)  $2^{19}$ ,  $3^5$ ,  $5^3$ ,  $7^3$ , 11,  $13^2$ , 19,  $31^3$ , 37, 41, 61.
- 5)  $2^{19}$ ,  $3^6$ , 5, 7, 11, 23, 31, 41, 137, 547, 1093.
- 6)  $2^{19}$ ,  $3^6$ ,  $5^3$ ,  $7^2$ , 11, 13, 19, 23, 31, 41, 137, 547, 1093. (Lehmer.)
- 5)  $2^{19}$ ,  $3^7$ ,  $5^2$ , 7, 11,  $31^2$ ,  $41^2$ , 83, 331, 431, 1723.
- \*6)  $2^{19}$ ,  $3^8$ ,  $5^3$ ,  $7^2$ , 11,  $13^2$ ,  $19^2$ ,  $31^2$ , 41, 61, 83, 127, 331, 379, 757.
- 6)  $2^{19}$ ,  $3^9$ ,  $5^3$ ,  $7^4$ ,  $11^3$ ,  $13^3$ , 17, 31, 41,  $61^2$ , 97, 467, 2801.
- 5)  $2^{19}$ ,  $3^{10}$ , 5, 7, 11, 23, 31, 41, 107, 3851.
- 6)  $2^{19}$ ,  $3^{10}$ ,  $5^3$ ,  $7^2$ , 11, 13, 19, 23, 31, 41, 107, 3851.
- 5)  $2^{20}$ ,  $3^2$ ,  $5^2$ ,  $7^3$ ,  $13^3$ , 17, 31, 127, 337.
- 5)  $2^{20}$ ,  $3^3$ , 5,  $7^2$ ,  $13^2$ , 19, 31, 61, 127, 337. (Fermat.)
- 5)  $2^{20}$ ,  $3^3$ ,  $5^2$ ,  $7^3$ ,  $13^2$ ,  $31^2$ , 61, 83, 127, 331, 337.
- 5)  $2^{20}$ ,  $3^3$ ,  $5^3$ ,  $7^3$ ,  $13^3$ , 17, 127, 337.
- \*6)  $2^{20}$ ,  $3^4$ ,  $5^2$ ,  $7^4$ ,  $11^2$ ,  $13^3$ , 17, 19, 31, 127, 337, 467, 2801.
- 5)  $2^{20}$ ,  $3^7$ , 5,  $7^4$ ,  $13^3$ , 17, 41, 127, 337, 467, 2801.
- 5)  $2^{21}$ ,  $3^6$ ,  $5^2$ , 7, 19,  $23^2$ , 31, 79, 89, 137, 547, 683, 1093. (Lehmer.)
- 5)  $2^{21}$ ,  $3^7$ ,  $5^2$ , 7, 19, 23, 31, 41, 89, 683.
- 5)  $2^{21}$ ,  $3^8$ , 5, 7, 13,  $19^2$ , 23, 89, 127, 379, 683, 757.
- \*6)  $2^{21}$ ,  $3^9$ ,  $5^3$ ,  $7^3$ ,  $11^2$ ,  $13^2$ ,  $19^2$ , 23,  $61^2$ , 89, 97, 127, 683.
- 5)  $2^{21}$ ,  $3^{10}$ ,  $5^2$ , 7, 19,  $23^2$ , 31, 79, 89, 107, 683, 3851.
- \*6)  $2^{21}$ ,  $3^{11}$ ,  $5^2$ ,  $7^2$ , 13, 17,  $19^2$ , 23, 31, 37, 73, 89, 101, 227, 683, 137561.
- 5)  $2^{21}$ ,  $3^{12}$ ,  $7^2$ , 11, 13,  $17^2$ ,  $19^2$ , 23, 89, 101, 103, 227, 307, 617, 683, 137561, 398581, 797161.
- 6)  $2^{21}$ ,  $3^{12}$ , 5,  $7^2$ , 11, 13,  $17^2$ ,  $19^2$ , 23, 89, 101, 103, 227, 307, 617, 683, 137561, 398581, 797161.
- \*6)  $2^{21}$ ,  $3^{12}$ ,  $5^2$ ,  $7^4$ ,  $13^2$ , 17,  $19^2$ , 23,  $31^2$ , 61, 83, 89, 103, 181, 331, 617, 683, 398581, 797161.
- 5)  $2^{22}$ ,  $3^7$ , 5, 7, 11, 19, 41, 47, 151, 197, 178481.
- \*6)  $2^{22}$ ,  $3^8$ ,  $5^2$ ,  $7^2$ , 11,  $13^2$ ,  $19^3$ ,  $31^2$ , 47, 61, 83, 151, 181, 197, 331, 379, 757, 178481.
- \*6)  $2^{22}$ ,  $3^8$ ,  $5^3$ ,  $7^2$ , 11,  $13^3$ , 17,  $19^3$ , 47, 151, 181, 197, 379, 757, 178481.
- \*5)  $2^{22}$ ,  $3^{11}$ ,  $7^2$ , 11, 13, 17,  $19^2$ , 37, 47, 73, 101, 151, 197, 227, 137561, 178481.
- \*6)  $2^{22}$ ,  $3^{11}$ , 5,  $7^2$ , 11, 13, 17,  $19^2$ , 37, 47, 73, 101, 151, 197, 227, 137561, 178481.

- 6)  $2^{23}$ ,  $3^7$ ,  $5^3$ ,  $7^4$ ,  $11^2$ ,  $13^3$ ,  $17^2$ , 31, 41, 61, 241, 307, 467, 2801. (Fermat.)
- \*6)  $2^{23}$ ,  $3^7$ ,  $5^4$ ,  $7^2$ ,  $11^3$ , 13, 17, 31, 41, 61, 71, 241.
- \*6)  $2^{24}$ ,  $3^7$ ,  $5^3$ ,  $7^2$ , 11, 13, 17, 31, 41, 43, 53, 601, 1801.
- \*6)  $2^{24}$ ,  $3^7$ ,  $5^4$ ,  $7^3$ ,  $11^2$ , 17, 19, 31, 41, 43, 53, 71, 601, 1801.
- 5)  $2^{24}$ ,  $3^8$ ,  $7^2$ , 11, 13, 17,  $19^2$ , 31, 43, 53, 127, 379, 601, 757, 1801.
- 6)  $2^{24}$ ,  $3^8$ , 5,  $7^2$ , 11, 13, 17,  $19^2$ , 31, 43, 53, 127, 379, 601, 757, 1801. (Lehmer.)
- 4)  $2^{25}$ ,  $3^8$ ,  $5^2$ , 19, 31, 683, 2731, 8191. (Carmichael.)
- 4)  $2^{25}$ ,  $3^4$ , 7,  $11^2$ ,  $19^2$ , 127, 683, 2731, 8191.
- 4)  $2^{25}$ ,  $3^5$ ,  $7^2$ , 13,  $19^2$ , 127, 683, 2731, 8191.
- 4)  $2^{25}$ ,  $3^6$ , 5, 19, 23, 137, 547, 683, 1093, 2731, 8791. (Carmichael.)
- 4)  $2^{25}$ ,  $3^{10}$ , 5, 19, 23, 107, 683, 2731, 3851, 8191.
- 6)  $2^{26}$ ,  $3^2$ ,  $5^6$ ,  $7^3$ ,  $11^2$ ,  $13^2$ ,  $19^3$ , 31, 37, 43, 61, 73, 181, 199, 257, 19531, 11939, 262657. .
- 6)  $2^{26}$ ,  $3^3$ ,  $5^5$ ,  $7^5$ ,  $11^2$ , 13,  $19^3$ , 31, 37, 43, 73, 181, 199, 11939, 262657.
- 6)  $2^{26}$ ,  $3^4$ ,  $5^3$ ,  $7^2$ ,  $11^3$ , 13,  $19^2$ , 31, 37, 61, 73, 127, 199, 11939, 262657.
- 6)  $2^{26}$ ,  $3^5$ ,  $5^3$ ,  $7^2$ , 11,  $13^2$ ,  $19^3$ , 31, 37, 61, 73, 127, 199, 11939, 262657.
- 6)  $2^{26}$ ,  $3^4$ ,  $5^3$ ,  $7^2$ , 11, 13,  $19^2$ , 23, 37, 73, 127, 137, 199, 547, 1093, 11939, 262657.
- 6)  $2^{26}$ ,  $3^7$ ,  $5^4$ ,  $7^4$ ,  $11^2$ , 13,  $19^2$ , 37, 41, 71, 73, 127, 199, 467, 2801, 11939, 262657.
- \*6)  $2^{26}$ ,  $3^7$ ,  $5^5$ ,  $7^4$ , 11, 13, 19, 31, 37, 41, 73, 199, 467, 2801, 11939, 262657.
- \*6)  $2^{26}$ ,  $3^7$ ,  $5^6$ ,  $7^3$ ,  $11^2$ , 17,  $19^4$ , 37, 41, 43, 73, 101, 199, 227, 257, 11939, 19531, 137561, 262657.
- 6)  $2^{26}$ ,  $3^{10}$ ,  $5^3$ ,  $7^2$ , 11, 13,  $19^2$ , 23, 37, 73, 107, 127, 199, 3851, 11939, 262657.
- 6)  $2^{27}$ ,  $3^4$ ,  $5^3$ , 7,  $11^3$ , 13, 19, 29, 31, 43, 61, 113, 127.
- 6)  $2^{27}$ ,  $3^5$ ,  $5^3$ , 7, 11,  $13^3$ , 19, 29, 31, 43, 61, 113, 127. (Fermat.)
- 6)  $2^{27}$ ,  $3^5$ ,  $5^5$ ,  $7^3$ , 11, 13, 19, 29, 31, 43, 113, 127.
- \*5)  $2^{27}$ ,  $3^6$ ,  $5^2$ , 11, 19, 23, 29, 31, 43, 113, 127, 137, 547, 1093.
- 6)  $2^{27}$ ,  $3^6$ ,  $5^3$ , 7, 11, 13, 19, 23, 29, 43, 113, 127, 137, 547, 1093.
- \*5)  $2^{27}$ ,  $3^8$ ,  $5^4$ ,  $11^4$ , 19, 29, 31, 43, 61, 71, 113, 127, 179, 3221.
- 5)  $2^{27}$ ,  $3^{10}$ ,  $5^2$ , 11, 19, 23, 29, 31, 43, 107, 113, 127, 3851.
- 6)  $2^{27}$ ,  $3^{10}$ ,  $5^3$ , 7, 11, 13, 19, 23, 29, 43, 107, 113, 127, 3851.
- \*6)  $2^{28}$ ,  $3^6$ ,  $5^2$ ,  $7^2$ , 11, 13,  $19^2$ ,  $23^2$ , 31, 79, 127, 137, 233, 547, 1093, 1103, 2089.
- 6)  $2^{28}$ ,  $3^6$ ,  $5^3$ , 7, 11,  $13^2$ , 19,  $23^2$ , 31, 61, 79, 137, 233, 547, 1093, 1103, 2089.
- 6)  $2^{28}$ ,  $3^6$ ,  $5^4$ ,  $7^3$ ,  $11^2$ , 13,  $19^2$ ,  $23^2$ , 71, 79, 127, 137, 233, 547, 1093, 1103, 2089.
- 6)  $2^{28}$ ,  $3^6$ ,  $5^5$ ,  $7^3$ , 11, 13, 19,  $23^2$ , 31, 79, 137, 233, 547, 1093, 1103, 2089.

- \*6)  $2^8, 3^7, 5^4, 7^2, 11, 13, 19^2, 23, 31, 41, 127, 233, 1103, 2089$ .
- 6)  $2^8, 3^7, 5^3, 7, 11, 13^2, 19, 23, 31, 41, 61, 233, 1103, 2089$ .
- 6)  $2^8, 3^7, 5^4, 7^3, 11^2, 13, 19^2, 23, 41, 71, 127, 233, 1103, 2089$ .
- 6)  $2^8, 3^7, 5^3, 7^3, 11, 13, 19, 23, 31, 41, 233, 1103, 2089$ .
- \*5)  $2^8, 3^8, 5, 11, 13^2, 19^2, 23, 31, 61, 127, 233, 379, 757, 1103, 2089$ .
- 6)  $2^8, 3^8, 5^2, 7, 11, 13^2, 19^2, 23, 31^2, 61, 83, 127, 233, 331, 379, 757, 1103, 2089$ .
- \*6)  $2^8, 3^8, 5^3, 7, 11, 13^3, 17, 19^2, 23, 127, 233, 379, 757, 1103, 2089$ .
- 6)  $2^8, 3^8, 5^4, 7, 11^4, 13, 19, 23, 31, 61, 71, 179, 233, 1103, 2089, 3221$ .
- \*6)  $2^8, 3^{10}, 5^2, 7^2, 11, 13, 19^2, 23^2, 31, 79, 107, 127, 233, 1103, 2089, 3851$ .
- 6)  $2^8, 3^{10}, 5^3, 7, 11, 13^2, 19, 23^2, 31, 61, 79, 107, 233, 1103, 2089, 3851$ .
- 6)  $2^8, 3^{10}, 5^4, 7^3, 11^2, 13, 19^2, 23^2, 71, 79, 107, 127, 233, 1103, 2089, 3851$ .
- 6)  $2^8, 3^{10}, 5^5, 7^3, 11, 13, 19, 23^2, 31, 79, 107, 233, 1103, 2089, 3851$ .
- \*6)  $2^8, 3^{11}, 5^3, 7^2, 11, 13^4, 19^3, 23, 37, 73, 181, 191, 233, 1103, 2089, 30941$ .
- 6)  $2^{20}, 3^4, 5^2, 7^3, 11^4, 13, 19^2, 31^3, 37, 61, 83, 127, 151, 331$ .
- 6)  $2^{20}, 3^5, 5^2, 7^3, 11, 13^2, 19^2, 31^3, 37, 61, 83, 127, 151, 331$ .
- 5)  $2^{20}, 3^6, 7^2, 11^3, 13, 19^3, 23, 31, 43, 83, 137, 151, 181, 331, 547, 1093$ .
- 6)  $2^{20}, 3^6, 5, 7^2, 11^2, 13, 19^3, 23, 31, 43, 83, 137, 151, 181, 331, 547, 1093$ .
- 6)  $2^{20}, 3^6, 5^3, 7^3, 11, 13, 19, 23, 31, 83, 137, 151, 331, 547, 1093$ .
- 5)  $2^{20}, 3^{10}, 7^2, 11^2, 13, 19^3, 23, 31, 43, 83, 107, 151, 181, 331, 3851$ .
- 6)  $2^{20}, 3^{10}, 5, 7^2, 11^2, 13, 19^3, 23, 31, 43, 83, 107, 151, 181, 331, 3851$ .
- 6)  $2^{20}, 3^{10}, 5^3, 7^3, 11, 13, 19, 23, 31, 83, 107, 151, 331, 3851$ .
- 2)  $2^{30}, 2147483647$ . (Euler.)
- \*5)  $2^{31}, 3^{13}, 7^4, 11^3, 13, 17, 31, 41, 43, 61, 83, 163, 257, 307, 331, 467, 547^2, 613, 1093, 2801, 65537$ .
- \*6)  $2^{31}, 3^{13}, 5, 7^4, 11^3, 13, 17, 31, 41, 43, 61, 83, 163, 257, 307, 331, 467, 547^2, 613, 1093, 2801, 65537$ .
- \*6)  $2^{32}, 3^8, 5^7, 7^3, 13^2, 17, 19, 23, 31, 61, 79, 89, 157, 313, 379, 757, 2141, 599479$ .
- \*7)  $2^{32}, 3^{11}, 5^3, 7^2, 11^2, 13^2, 17^2, 19^4, 23, 31, 37, 43, 61, 73, 89, 101, 227, 307, 2141, 137561, 599479$ .
- \*7)  $2^{32}, 3^{11}, 5^4, 7^3, 11^2, 13^2, 17, 19^2, 23, 31, 37, 43, 61, 71, 73, 89, 181, 2141, 599479$ .
- \*7)  $2^{32}, 3^{14}, 5^2, 7^3, 11^2, 13, 17^2, 19^4, 23, 31^2, 37, 41, 61, 83, 89, 101, 163, 227, 307, 331, 1063, 2141, 2281, 4561, 137561, 599479$ .
- 6)  $2^{32}, 3^{12}, 5^9, 7^2, 11^4, 17, 23, 29, 31, 61, 71, 89, 179, 197, 521, 1181, 2141, 3221, 599479$ .



- 4)  $2^{33}$ ,  $3^4$ , 7,  $11^3$ , 31, 61, 83, 331, 43691, 131071.
- 5)  $2^{33}$ ,  $3^5$ , 5,  $7^3$ , 11, 13, 83, 331, 43691, 131071.
- 4)  $2^{23}$ ,  $3^0$ , 7, 11, 23, 83, 137, 331, 547, 1093, 43691, 131071.
- 5)  $2^{23}$ ,  $3^7$ , 5,  $7^2$ , 11, 19, 41, 83, 331, 43691, 131071.
- 4)  $2^{33}$ ,  $3^{10}$ , 7, 11, 23, 83, 107, 331, 3851, 43691, 131071.
- \*6)  $2^{35}$ ,  $3^{14}$ ,  $5^2$ ,  $7^4$ ,  $11^4$ ,  $13^2$ , 17,  $31^2$ , 41, 61, 163, 179,  $331^2$ , 467, 2281, 2617, 2801, 3221, 4561, 5233, 43691, 131071.
- \*6)  $2^{34}$ ,  $3^8$ ,  $5^4$ , 7, 11, 13, 17,  $19^2$ , 31,  $71^2$ ,  $127^2$ , 271, 379, 683, 757, 1279, 2557, 5113, 5419, 6829, 122921.
- 5)  $2^{34}$ ,  $3^{12}$ ,  $7^2$ , 11, 13,  $17^2$ ,  $19^4$ , 31, 71, 101, 103, 127, 227, 307, 617, 683, 6829, 122921, 137561, 398581, 797161.
- 6)  $2^{34}$ ,  $3^{12}$ , 5,  $7^2$ , 11, 13,  $17^2$ ,  $19^4$ , 31, 71, 101, 103, 127, 227, 307, 617, 683, 6829, 122921, 137561, 398581, 797161.
- \*6)  $2^{34}$ ,  $3^{13}$ ,  $5^3$ ,  $7^4$ , 11, 13, 19,  $31^2$ , 41, 71, 83, 127, 163, 307, 331, 467,  $547^2$ , 613, 683, 1093, 2801, 6829, 122921.
- \*6)  $2^{34}$ ,  $3^{15}$ ,  $5^3$ ,  $7^4$ ,  $13^2$ , 17, 19,  $31^2$ , 41, 61, 71, 83, 97, 127, 193, 331, 467, 683, 2801, 6829, 122921.
- \*6)  $2^{34}$ ,  $3^{19}$ ,  $5^3$ , 7,  $11^4$ , 17,  $19^2$ ,  $31^2$ , 43, 61, 71, 83,  $127^2$ , 179, 197, 257, 271, 331, 683, 1181, 3221, 5419, 6829, 19531, 122921.
- 7)  $2^{35}$ ,  $3^{13}$ ,  $5^2$ ,  $7^5$ ,  $11^2$ , 13, 17,  $19^2$ ,  $31^2$ ,  $37^2$ , 41, 43, 61, 67, 73, 83, 109, 127, 163, 307, 331,  $547^2$ , 613, 1093.
- \*7)  $2^{35}$ ,  $3^{13}$ ,  $5^3$ ,  $7^4$ ,  $11^2$ ,  $13^3$ ,  $17^2$ ,  $19^2$ , 23,  $37^2$ , 41, 43, 67, 73, 109, 127, 163,  $307^2$ , 367, 467,  $547^2$ , 613, 733, 1093, 2801.
- \*7)  $2^{35}$ ,  $3^{14}$ ,  $5^3$ ,  $7^3$ ,  $11^4$ ,  $13^2$ , 17,  $19^2$ , 31,  $37^2$ , 41, 43, 61, 67, 73, 109, 127, 163, 179, 257, 2281, 3221, 4561, 19531.
- 7)  $2^{35}$ ,  $3^{15}$ ,  $5^3$ ,  $7^7$ ,  $11^4$ , 13,  $17^2$ , 19, 29,  $37^2$ , 41, 43, 67, 71, 73, 97, 109, 179, 193, 307, 521, 601, 1201, 3221.
- 6)  $2^{38}$ ,  $3^5$ ,  $5^4$ ,  $7^7$ ,  $11^2$ ,  $13^2$ , 19, 31, 43, 61, 71, 223, 601, 1201, 7019, 112303, 898423, 616318177.
- \*6)  $2^{36}$ ,  $3^6$ ,  $5^2$ ,  $7^3$ , 11, 13, 19, 23, 31, 43, 137, 223, 547, 1093, 7019, 112303, 898423, 616318177.
- 6)  $2^{38}$ ,  $3^0$ ,  $5^4$ ,  $7^7$ ,  $11^2$ , 13, 19, 23, 43, 71, 137, 223, 547, 601, 1093, 1201, 7019, 112303, 898423, 616318177.
- \*6)  $2^{36}$ ,  $3^7$ ,  $5^3$ ,  $7^5$ , 11,  $13^2$ , 19, 31, 41, 43, 61, 223, 7019, 112303, 898423, 616318177.
- \*6)  $2^{36}$ ,  $3^8$ ,  $5^2$ ,  $7^5$ , 11,  $13^2$ ,  $19^2$ ,  $31^2$ , 43, 61, 83, 127, 223, 331, 379, 757, 7019, 112303, 898423, 616318177.

- \*6)  $2^{30}$ ,  $3^8$ ,  $5^3$ ,  $7^3$ , 11,  $13^3$ , 17,  $19^2$ , 43, 127, 223, 379, 757, 7019, 112303, 898423, 616318177.
- 6)  $2^{30}$ ,  $3^8$ ,  $5^3$ ,  $7^3$ , 11,  $13^2$ , 19,  $31^2$ , 43, 61, 83, 223, 331, 379, 601, 757, 1201, 7019, 112303, 898423, 616318177. (Gérardin.)
- \*6)  $2^{30}$ ,  $3^9$ ,  $5^4$ ,  $7^3$ ,  $11^4$ , 13, 19, 31, 43, 61, 71, 179, 223, 3221, 7019, 112303, 898423, 616318177.
- 6)  $2^{36}$ ,  $3^9$ ,  $5^3$ ,  $7^6$ ,  $11^3$ ,  $13^2$ , 19, 29, 31,  $61^3$ , 223, 263, 1861, 4733, 7019, 112303, 898423, 616318177.
- \*6)  $2^{30}$ ,  $3^{10}$ ,  $5^2$ ,  $7^3$ , 11, 13, 19, 23, 31, 43, 107, 223, 3851, 7019, 112303, 898423, 616318177.
- 6)  $2^{30}$ ,  $3^{10}$ ,  $5^4$ , 7,  $11^2$ , 13, 19, 23, 43, 71, 107, 223, 601, 1201, 3851, 7019, 112303, 898423, 616318177.
- \*6)  $2^{30}$ ,  $3^{11}$ ,  $5^2$ ,  $7^8$ ,  $13^2$ , 17,  $19^2$ ,  $31^2$ ,  $37^2$ , 61, 67, 73, 83, 127, 223, 331, 1063, 7019, 112303, 898423, 616318177.
- \*5)  $2^{30}$ ,  $3^{13}$ ,  $7^8$ , 11, 13, 17,  $19^4$ , 37, 41, 101, 163, 223, 227, 307,  $547^2$ , 613, 1063, 1003, 7019, 112303, 137561, 898423, 616318177.
- \*6)  $2^{30}$ ,  $3^{13}$ , 5,  $7^8$ , 11, 13, 17,  $19^4$ , 37, 41, 101, 163, 223, 227, 307,  $547^2$ , 613, 1063, 1003, 7019, 112303, 137561, 898423, 616318177.
- \*6)  $2^{30}$ ,  $3^{14}$ ,  $5^4$ ,  $7^6$ ,  $11^4$ ,  $13^3$ , 29, 31, 41, 61, 71, 163, 179, 223, 263, 2281, 3221, 4561, 4733, 7019, 112303, 898423, 616318177.
- 4)  $2^{37}$ ,  $3^4$ , 7,  $11^3$ , 31, 61, 83, 331, 43691, 174763, 524287.
- 5)  $2^{37}$ ,  $3^5$ , 5,  $7^3$ , 11, 13, 83, 331, 43691, 174763, 524287.
- 4)  $2^{37}$ ,  $3^6$ , 7, 11, 23, 83, 137, 331, 547, 1063, 43691, 174763, 524287.
- 5)  $2^{37}$ ,  $3^7$ , 5,  $7^2$ , 11, 19, 41, 83, 331, 43691, 174763, 524287.
- 4)  $2^{37}$ ,  $3^{10}$ , 7, 11, 23, 83, 107, 331, 3851, 43691, 174763, 524287.
- \*6)  $2^{37}$ ,  $3^{14}$ ,  $5^2$ ,  $7^4$ ,  $11^4$ ,  $13^2$ , 17,  $31^2$ , 41, 61, 163, 179,  $331^2$ , 467, 2281, 2617, 2801, 3221, 4561, 5233, 43691, 174763, 524287.
- 6)  $2^{38}$ ,  $3^5$ ,  $5^6$ ,  $7^3$ , 11, 13, 19, 23, 43, 53, 79, 229, 257, 8191, 19531, 121369.
- \*5)  $2^{38}$ ,  $3^7$ ,  $5^5$ ,  $7^3$ , 23, 31, 41, 53, 79, 229, 8191, 121369.
- \*6)  $2^{38}$ ,  $3^8$ ,  $5^4$ ,  $7^2$ , 11, 13,  $19^2$ , 23, 53, 71, 79, 127, 229, 379, 757, 8191, 121369.
- \*6)  $2^{38}$ ,  $3^9$ ,  $5^2$ ,  $7^5$ ,  $11^2$ ,  $19^2$ , 23,  $31^2$ ,  $43^2$ , 53, 61,  $79^2$ , 83, 127, 229, 331, 631, 8191, 121369.
- \*6)  $2^{38}$ ,  $3^9$ ,  $5^4$ ,  $7^3$ ,  $11^3$ , 13, 23,  $43^2$ , 53,  $61^2$ , 71,  $79^2$ , 97, 229, 601, 631, 1201, 8191, 121369.
- \*6)  $2^{38}$ ,  $3^{11}$ ,  $5^5$ ,  $7^4$ ,  $13^2$ , 19, 23,  $31^2$ , 37, 53, 61, 73, 79, 83, 229, 331, 467, 2801, 8191, 121369.

- \*7)  $2^{39}$ ,  $3^{11}$ ,  $5^7$ ,  $7^3$ , 11,  $13^2$ , 17,  $19^2$ , 29,  $31^2$ , 37, 41, 61, 73, 79, 83, 127, 157, 313, 331, 2203, 30841, 61681.
- \*7)  $2^{39}$ ,  $3^{12}$ ,  $5^5$ ,  $7^7$ ,  $11^3$ ,  $13^2$ ,  $17^2$ ,  $19^2$ , 29,  $31^2$ , 41, 43,  $61^2$ , 83, 97, 103, 127, 307, 331, 601, 617, 1201, 2203, 30841, 61681, 398581, 797161.
- \*7)  $2^{39}$ ,  $3^{13}$ ,  $5^3$ ,  $7^2$ ,  $11^2$ , 13,  $17^2$ ,  $19^4$ , 23, 29, 31,  $41^2$ , 43, 101, 163, 227,  $307^2$ , 367, 431,  $547^2$ , 613, 733, 1093, 1723, 2203, 30841, 61681, 137561.
- \*7)  $2^{39}$ ,  $3^{15}$ ,  $5^4$ ,  $7^4$ ,  $11^2$ ,  $13^2$ ,  $17^3$ ,  $19^2$ ,  $29^2$ ,  $31^2$ ,  $41^2$ , 61, 67, 71, 83, 97, 127, 193, 331, 431, 467, 1723, 2203, 2801, 30841, 61681.
- \*5)  $2^{40}$ ,  $3^{12}$ ,  $7^8$ ,  $11^2$ ,  $17^2$ ,  $19^5$ , 81,  $37^2$ , 43, 47, 67,  $103^2$ , 127, 257, 307, 557, 617, 1063, 3571, 7621, 13367, 15241, 398581, 797161, 164511353.
- \*6)  $2^{40}$ ,  $3^2$ ,  $5 \cdot 7^8$ ,  $11^2$ ,  $17^2$ ,  $19^5$ , 31,  $37^2$ , 43, 47, 67,  $103^2$ , 127, 257, 307, 557, 617, 1063, 3571, 7621, 13367, 15241, 398581, 797161, 164511353.
- \*7)  $2^{40}$ ,  $3^{15}$ ,  $5^2$ ,  $7^6$ ,  $11^2$ , 13, 17,  $19^2$ , 29,  $31^2$ , 37, 41, 43, 83, 97, 103, 127, 193, 257, 263, 331, 557, 4733, 7621, 13367, 15241, 164511353.
- \*6)  $2^{41}$ ,  $3^5$ ,  $5^3$ ,  $7^7$ , 11,  $13^3$ ,  $17^2$ , 31,  $43^2$ , 79, 127, 271, 307, 337, 601, 631, 1201, 5419.
- \*7)  $2^{41}$ ,  $3^{11}$ ,  $5^5$ ,  $7^6$ ,  $11^3$ ,  $13^2$ ,  $17^2$ , 19, 29,  $31^2$ , 37, 43, 61, 73, 83, 127, 263, 271, 307, 331, 337, 4733, 5419.
- \*5)  $2^{41}$ ,  $3^{14}$ ,  $7^4$ ,  $11^3$ ,  $13^4$ , 17, 31, 41, 43, 61, 127, 163, 191, 271, 337, 467, 2281, 2801, 4561, 5419, 30941.
- \*6)  $2^{41}$ ,  $3^{14}$ , 5,  $7^4$ ,  $11^3$ ,  $13^4$ , 17, 31, 41, 43, 61, 127, 163, 191, 271, 337, 467, 2281, 2801, 4561, 5419, 30941.
- 6)  $2^{42}$ ,  $3^{11}$ ,  $5^7$ ,  $7^3$ ,  $13^3$ , 17, 19, 31, 37, 61, 73, 79, 83, 157, 313, 331, 431, 9719, 2099863.
- 5)  $2^{42}$ ,  $3^{13}$ ,  $7^5$ ,  $11^2$ , 13,  $19^2$ , 31, 41, 43, 61, 83, 127, 163, 307, 331, 431,  $547^2$ , 613, 1093, 9719, 2099863.
- 6)  $2^{42}$ ,  $3^{12}$ , 5,  $7^3$ ,  $11^2$ , 13,  $19^2$ , 31, 41, 43, 61, 83, 127, 163, 307, 331, 431,  $547^2$ , 613, 1093, 9719, 2099863.
- \*6)  $2^{43}$ ,  $3^{13}$ ,  $5^5$ ,  $7^8$ , 11,  $19^5$ , 23, 31, 37, 41, 89, 127, 151, 163, 199, 307, 397,  $547^2$ , 613, 683, 1063, 1093, 2113.
- \*7)  $2^{43}$ ,  $3^{14}$ ,  $5^6$ ,  $7^9$ ,  $11^3$ , 13, 17,  $19^4$ , 23, 31, 41, 43, 61, 89, 101, 151, 163, 199, 227, 257, 397, 683, 2113, 2281, 4561, 19531, 137561.
- \*7)  $2^{43}$ ,  $3^{15}$ ,  $5^8$ ,  $7^6$ , 11, 13, 17,  $19^3$ , 23, 29, 31, 41, 83, 89, 97, 151, 181, 193, 199, 263, 397, 683, 829, 2113, 4733.
- \*6)  $2^{43}$ ,  $3^{10}$ ,  $5^3$ , 7,  $11^4$ ,  $19^2$ , 23, 29, 31, 61, 71, 89, 127, 151, 179, 197, 199, 397, 521, 683, 1181, 2113, 3221.

- \*7)  $2^{13}$ ,  $3^{21}$ ,  $5^5$ ,  $7^6$ , 11, 13, 17,  $19^3$ ,  $23^2$ , 29,  $43^2$ , 67,  $79^2$ , 83, 89, 107, 151, 181, 199, 257, 263, 331, 397, 631, 661, 683, 2113, 3851, 4733, 19531.
- \*6)  $2^{14}$ ,  $3^8$ ,  $5^3$ ,  $7^3$ , 13,  $19^2$ , 23,  $31^2$ , 37, 47, 73, 79, 83, 127, 137, 151, 331, 547, 631, 1093, 23311.
- 6)  $2^{14}$ ,  $3^7$ ,  $5^4$ ,  $7^3$ , 11,  $19^2$ ,  $31^2$ , 37, 41, 47, 71, 73, 79, 83, 127, 151, 331, 631, 23311.
- \*6)  $2^{14}$ ,  $3^{10}$ ,  $5^3$ ,  $7^3$ , 13,  $19^2$ , 23,  $31^2$ , 37, 47, 73, 79, 83, 107, 127, 151, 331, 631, 3851, 23311.
- \*7)  $2^{14}$ ,  $3^{11}$ ,  $5^3$ ,  $7^4$ ,  $11^2$ ,  $13^3$ ,  $17^2$ ,  $19^3$ , 29,  $31^3$ , 37, 47, 53,  $73^2$ , 79, 151, 181, 263, 307, 631, 1801, 4733, 23311.
- \*5)  $2^{15}$ ,  $3^{13}$ ,  $7^3$ ,  $11^3$ , 17,  $19^2$ , 31, 41, 43, 47, 61, 127, 151, 163, 197, 271, 307, 547<sup>2</sup>, 613, 1093, 5419, 178481, 2796203.
- \*6)  $2^{15}$ ,  $3^{13}$ , 5,  $7^2$ ,  $11^3$ , 17,  $19^2$ , 31, 41, 43, 47, 61, 127, 151, 163, 197, 271, 307, 547<sup>2</sup>, 613, 1093, 5419, 178481, 2796203.
- \*7)  $2^{15}$ ,  $3^{14}$ ,  $5^3$ ,  $7^3$ ,  $11^5$ ,  $13^2$ ,  $17^2$ ,  $19^4$ , 31, 37, 41, 43, 47, 61, 101, 151, 163, 197, 227, 271, 307, 2281, 4561, 5419, 137561, 178481, 2796203.
- \*7)  $2^{15}$ ,  $3^{14}$ ,  $5^4$ ,  $7^3$ ,  $11^5$ ,  $13^2$ , 17,  $19^3$ , 31, 37, 41, 43, 47, 61, 71, 151, 163, 181, 197, 271, 2281, 4561, 5419, 178481, 2796203.
- \*7)  $2^{15}$ ,  $3^{15}$ ,  $5^4$ ,  $7^3$ ,  $11^5$ , 13,  $17^2$ ,  $19^3$ , 29, 37, 41, 43, 47, 71, 97, 151, 181, 193, 197, 263, 271, 307, 4733, 5419, 178481, 2796203.
- \*7)  $2^{15}$ ,  $3^{21}$ ,  $5^3$ ,  $7^3$ ,  $11^2$ , 13,  $17^3$ ,  $19^4$ , 23, 29, 43, 47, 67, 83, 101, 107, 151, 197, 227, 271, 331, 661, 3851, 5419, 137561, 178481, 2796203.
- \*7)  $2^{15}$ ,  $3^{15}$ ,  $5^3$ ,  $7^8$ , 11, 13,  $17^2$ ,  $19^2$ , 23, 31,  $37^2$ , 41, 61, 89, 97, 127, 193, 307, 1003, 2351, 4513, 442151, 13264529.
- \*7)  $2^{16}$ ,  $3^{17}$ ,  $5^3$ ,  $7^7$ , 11, 13, 17,  $19^2$ , 23,  $31^2$ ,  $37^2$ , 43, 61, 67, 83, 89, 127, 331, 379, 601, 757, 1201, 2351, 4513, 442151, 13264529.
- \*7)  $2^{16}$ ,  $3^{21}$ ,  $5^4$ ,  $7^4$ , 11, 13, 17, 19,  $23^2$ , 31, 37, 61, 67, 71, 79, 83, 89, 107, 331, 467, 661, 2351, 2801, 3851, 4513, 442151, 13264529.
- \*6)  $2^{20}$ ,  $3^4$ ,  $5^3$ ,  $7^4$ ,  $11^3$ ,  $13^3$ ,  $17^2$ , 31, 61, 67, 103, 139, 307, 467, 2143, 2801, 11119, 131071.
- 6)  $2^{20}$ ,  $3^4$ ,  $5^4$ ,  $7^3$ ,  $11^3$ , 13, 17, 31, 61, 67, 71, 103, 139, 2143, 11119, 131071.
- 6)  $2^{20}$ ,  $3^5$ ,  $5^4$ ,  $7^3$ , 11,  $13^2$ , 17, 31, 61, 67, 71, 103, 139, 2143, 11119, 131071.
- \*6)  $2^{20}$ ,  $3^6$ ,  $5^3$ ,  $7^2$ ,  $13^2$ , 17, 19, 23, 31, 61, 67, 103, 137, 139, 547, 1093, 2143, 11119, 131071.
- \*6)  $2^{20}$ ,  $3^6$ ,  $5^3$ ,  $7^4$ , 11,  $13^3$ ,  $17^2$ , 23, 67, 103, 137, 139, 307, 467, 547, 1093, 2143, 2801, 11119, 131071.

- 6)  $2^{50}$ ,  $3^9$ ,  $5^3$ ,  $7^3$ , 11, 13, 17, 23, 67, 71, 103, 137, 139, 547, 1093, 2143, 11119, 131071.
- \*6)  $2^{50}$ ,  $3^{10}$ ,  $5^3$ ,  $7^2$ ,  $13^2$ , 17, 19, 23, 31, 61, 67, 103, 107, 139, 2143, 3851, 11119, 131071.
- \*6)  $2^{50}$ ,  $3^{10}$ ,  $5^3$ ,  $7^4$ , 11,  $13^3$ ,  $17^2$ , 23, 67, 103, 107, 139, 307, 467, 2143, 2801, 3851, 11119, 131071.
- 6)  $2^{50}$ ,  $3^{10}$ ,  $5^3$ ,  $7^3$ , 11, 13, 17, 23, 67, 71, 103, 107, 139, 2143, 3851, 11119, 131071.
- \*6)  $2^{53}$ ,  $3^{15}$ ,  $5^6$ ,  $7^3$ , 11, 17,  $19^2$ , 41, 43, 53, 79, 97, 127, 157, 193, 257, 269, 683, 1613, 2731, 8191, 19531.
- \*6)  $2^{53}$ ,  $3^{17}$ ,  $5^5$ ,  $7^3$ , 13, 17,  $19^2$ , 31,  $37^2$ , 53, 67, 79, 127, 157, 269, 379, 683, 757, 1063, 1613, 2731, 8191.
- \*5)  $2^{53}$ ,  $3^{19}$ ,  $5^6$ ,  $11^4$ ,  $19^2$ , 31, 43, 53, 61, 79, 127, 157, 179, 197, 257, 269, 683, 1181, 1613, 2731, 3221, 8191, 19531.
- \*6)  $2^{53}$ ,  $3^{21}$ ,  $5^3$ ,  $7^2$ , 13, 17,  $19^2$ , 23, 53, 67, 79, 83, 107, 127, 157, 269, 331, 661, 683, 1613, 2731, 3851, 8191.
- \*6)  $2^{53}$ ,  $3^{21}$ ,  $5^4$ , 7, 11, 17, 19, 23, 53, 67, 71, 79, 83, 107, 157, 269, 331, 661, 683, 1613, 2731, 3851, 8191.
- \*7)  $2^{53}$ ,  $3^{23}$ ,  $5^4$ ,  $7^5$ , 11, 13, 17,  $19^4$ , 29, 37, 41,  $43^2$ , 53, 71, 73,  $79^2$ , 101, 157, 227, 269, 463, 631, 683, 1613, 2731, 6481, 8191, 137561.
- \*7)  $2^{52}$ ,  $3^{15}$ ,  $5^3$ ,  $7^2$ ,  $11^3$ , 13, 17,  $19^2$ , 31, 37, 41, 43, 53, 61, 79,  $97^2$ , 127, 157, 193, 263, 313, 317, 2503, 3169, 3181, 6361, 69431, 485581, 20394401.
- \*6)  $2^{56}$ ,  $3^{11}$ ,  $5^7$ ,  $7^8$ ,  $13^3$ ,  $17^3$ ,  $19^2$ , 29,  $37^2$ , 67, 73, 79, 127, 157, 313, 1063, 1619, 16189, 32377, 524287, 1212847.
- \*6)  $2^{56}$ ,  $3^{14}$ ,  $5^5$ ,  $7^9$ ,  $11^4$ ,  $13^3$ ,  $17^2$ , 31, 41, 163, 179, 191, 307, 467, 1619, 2281, 2801, 3221, 4561, 16189, 32377, 524287, 1212847.
- \*7)  $2^{59}$ ,  $3^{23}$ ,  $5^7$ ,  $7^6$ ,  $11^2$ ,  $13^4$ ,  $19^3$ ,  $29^3$ ,  $31^2$ , 37,  $41^2$ , 53, 61, 73, 79, 83, 151, 157, 181, 191, 211, 263, 313,  $331^3$ , 421, 431, 463, 661, 1321, 1723, 1889, 4733, 6481, 30941.
- 2)  $2^{59}$ , 2305843009213693951. (Seelhoff, Pervusin.)
- 6)  $2^{63}$ ,  $3^5$ ,  $5^6$ ,  $7^3$ , 11, 13, 19, 23, 43, 59, 79, 157, 257, 19531, 43331, 3033169, 715827883, 2147483647.
- \*5)  $2^{63}$ ,  $3^7$ ,  $5^5$ ,  $7^3$ , 23, 31, 41, 59, 79, 157, 43331, 3033169, 715827883, 2147483647.
- 6)  $2^{63}$ ,  $3^8$ ,  $5^4$ ,  $7^2$ , 11, 13,  $19^2$ , 23, 59, 71, 79, 127, 157, 379, 757, 43331, 3033169, 715827883, 2147483647. (Cunningham.)

- \*6)  $2^{61}$ ,  $3^9$ ,  $5^2$ ,  $7^5$ ,  $11^2$ ,  $19^2$ ,  $23$ ,  $31^2$ ,  $43^2$ ,  $59$ ,  $61$ ,  $79^2$ ,  $83$ ,  $127$ ,  $157$ ,  $331$ ,  $631$ ,  
 43331, 3033169, 715827883, 2147483647.
- \*6)  $2^{61}$ ,  $3^9$ ,  $5^3$ ,  $7^7$ ,  $11^3$ ,  $13$ ,  $23$ ,  $43^2$ ,  $59$ ,  $61^2$ ,  $71$ ,  $79^2$ ,  $97$ ,  $157$ ,  $601$ ,  $631$ ,  $1201$ ,  
 43331, 3033169, 715827883, 2147483647.
- \*6)  $2^{61}$ ,  $3^9$ ,  $5^3$ ,  $7^4$ ,  $13^2$ ,  $19$ ,  $23$ ,  $31^2$ ,  $37$ ,  $59$ ,  $61$ ,  $73$ ,  $79$ ,  $83$ ,  $157$ ,  $331$ ,  $467$ ,  $2801$ ,  
 43331, 3033169, 715827883, 2147483647.
- \*7)  $2^{67}$ ,  $3^{15}$ ,  $5^7$ ,  $7^7$ ,  $11^2$ ,  $13$ ,  $17$ ,  $19$ ,  $23$ ,  $41$ ,  $43$ ,  $47$ ,  $53$ ,  $79$ ,  $83$ ,  $97$ ,  $137$ ,  $157$ ,  $193$ ,  
 313,  $331$ ,  $601$ ,  $953$ ,  $1201$ ,  $13159$ ,  $26317$ ,  $43691$ ,  $131071$ .
- \*7)  $2^{75}$ ,  $3^{21}$ ,  $5^6$ ,  $7^9$ ,  $11^3$ ,  $13$ ,  $17^2$ ,  $19^3$ ,  $23$ ,  $37$ ,  $43^2$ ,  $67$ ,  $79^2$ ,  $97$ ,  $107$ ,  $139$ ,  $181^2$ ,  
 191,  $199$ ,  $229^2$ ,  $257$ ,  $307$ ,  $331^2$ ,  $457$ ,  $467$ ,  $631$ ,  $661$ ,  $2617$ ,  $2801$ ,  $3851$ ,  
 5233,  $11939$ ,  $19531$ ,  $43691$ ,  $174763$ ,  $262657$ ,  $524287$ ,  $525313$ .
- 2)  $2^{83}$ , 618970019642690137449562111. (Powers.)

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