On The Atomic Structure of Energy.

By A. E. Caswell.

For a number of years it has been well known that a number of the theoretical results of the classical thermodynamics are not in accord with experiment. This is especially true in domains of radiation and specific heats, and does not mean that these results are invalid, but, rather, while they contain truth they do not contain the whole truth. Eminent physicists have endeavored to formulate a theory the conclusions of which shall be true to fact. The "Quantum Hypothesis" of Planck, which involves certain assumptions relating to energy, seems to furnish the basis for such a theory. It is

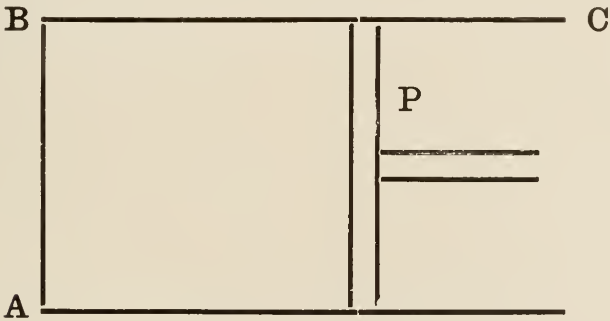

Figure 1.
A. E. Caswell—On the Atomic Structure of Energy.

the purpose of this paper to present some of the difficulties which are insurmountable on the basis of the older theory but are explainable on the basis of this hypothesis, and to indicate some of the results of the new theory and its bearing upon our conception of the nature of energy.

Two important laws of radiation which have been derived from the older theory are the Stefan-Boltzmann[1] Law and the Wien[2] Displacement Law. The former was proposed by Stefan in 1879 and was based upon the fact that Tyndall had found the ratio of the energy radiated by a platinum wire at 1200° C. to that radiated at 525° C. to be 11.6. The law states that *the energy radiated by a black body is proportional to the fourth power of the absolute*

[1]Wien. Akad. Sitz. 79, p. 391, 1879; Wied. Ann. 22, pp. 31 and 291, 1884.
[2]Ann. der Physik, 58, p. 662, 1896.

temperature, or $E = k(T^4 - T_0^4)$. The law has been amply verified for a black body by the experiments of Lummer and Pringsheim[3], but is not strictly true for other bodies. Boltzmann deduced the law theoretically in the following manner.

According to the electromagnetic theory of light, light exerts a pressure on any perfectly reflecting plane surface which is perpendicular to the direction of the light numerically equal to the energy density of the radiation. When light is incident in all directions we may assume one-third of it traveling in each of three mutually perpendicular directions, and so the pressure exerted upon the walls of a perfectly reflecting vessel filled with radiation would be equal to one-third of the energy density. Let AC (Fig. 1) be a cylinder of unit cross-section and length a, having perfectly reflecting sides and a perfectly reflecting piston P, but the end AB is a perfectly black body at a temperature T. Then the space between AB and P will be filled with radiation of energy density u corresponding to the temperature T. When equilibrium is established replace AB by a perfectly reflecting plate, and push the piston P in from a point distant x from AB to a point distant $(x - dx)$. The total amount of energy supplied, dQ, is equal to the increase of the internal energy, dU, plus the external work performed, dW. Therefore

$$dQ = dU + dW = d(xu) + p\,dx = x\,du + \frac{4}{3}u\,dx.$$

If φ is the entropy, then

$$d\varphi = \frac{dQ}{T} = \frac{x}{T}\,du + \frac{4u}{3T}\,dx = \frac{\partial\varphi}{\partial u}\,du + \frac{\partial\varphi}{\partial x}\,dx.$$

$$\therefore \frac{x}{T} = \frac{\partial\varphi}{\partial u} \text{ and } \frac{4u}{3T} = \frac{\partial\varphi}{\partial x}, \text{ or } \frac{\partial^2\varphi}{\partial u\,\partial x} = \frac{\partial}{\partial x}\left(\frac{x}{T}\right) = \frac{\partial}{\partial u}\left(\frac{4u}{3T}\right).$$

Since T is independent of x

$$\frac{1}{T} = \frac{4}{3}\left(\frac{1}{T} - \frac{u\,dT}{T^2 du}\right), \text{ or } \frac{du}{u} = \frac{4dT}{T}.$$

(1) $u = kT^4$, where k is a constant.

Suppose that instead of the case above considered we take the case of a small radiating body at the center of a hollow sphere having perfectly reflecting walls. Then ur^2 will be constant, where r is the distance from the center of the sphere. Then $U = 4\pi\int_0^r ur^2 dr = 4\pi r^3 u$, u being in this case the energy density at the surface of the sphere. The radiation pressure on the

[3]Ann. der Physik, 63, p. 395, 1897.

whole surface is $4\ r^2u$, since all the radiation is incident normally. Then if the radius of the sphere decreases by a small amount dr

(2) $dQ = dU + dW = 4\pi[d(r^2u) + r^2u\ dr] = 4\pi r^2\ (r\ du + 4u\ dr)$.

Proceeding as before we may deduce the Stefan-Boltzmann Law from this equation, showing that the law is true for radiation from a point source.

The second law mentioned above is the Wien Displacement Law which states that *the product of the wave-length and the absolute temperature is a constant,* or λT = constant. In other words, if radiation of a particular wave-length is adiabatically altered to another wave-length the temperature changes in the inverse ratio. To prove this let us consider the sphere of the preceding paragraph. Let it expand with a constant radial velocity v, and let the velocity of the radiation be V. Then, by Doppler's principle, the wave-length will be increased at each reflection. Let λ_0 = the original wave-length, λ_n = wave-length after n reflections, and t = time elapsing between the instant when one wave is reflected and the instant when the next succeeding wave is reflected. Then

$\lambda_1 = \lambda_0 + 2vt = \lambda_0 + 2v\left(\dfrac{\lambda_0 + vt}{V}\right)$, or eliminating t

$$\lambda_1 = \lambda_0 + \frac{2v\lambda_0}{V - v} = \left(\frac{V + v}{V - v}\right)\lambda_0.$$

$$\therefore \lambda_n = \lambda_0\left(\frac{V + v}{V - v}\right)^n.$$

While the surface of the sphere moves out a distance dr the wave will travel a distance $\dfrac{V\,dr}{v}$, and since the diameter is 2r, the number of reflections which will occur is $n = \dfrac{V\,dr}{2rv}$. Consequently $\lambda_n = \lambda_0\left(\dfrac{V + v}{V - v}\right)^{\frac{Vdr}{2rv}}$, and the value of λ corresponding to an expansion dr is λ = Limit of $\lambda_0\left(\dfrac{V + v}{V - v}\right)^{\frac{Vdr}{2rv}}$, or $\lambda = \lambda_0\left(1 + \dfrac{dr}{r}\right)$, when $\dfrac{V}{2v}$ approaches infinity and the squares and higher powers of $\dfrac{v}{V}$ and $\dfrac{dr}{r}$ are neglected.

Put $d\lambda = \lambda - \lambda_0$, and $\dfrac{d\lambda}{\lambda} = \dfrac{dr}{r}$, or since $dQ = 0$, $r\ du + 4u\ dr = 0$, by equation (2), and $\dfrac{du}{u} + 4\dfrac{d\lambda}{\lambda} = 0$. On integration this gives

$$(3)\ \left(\frac{\lambda}{\lambda_0}\right)^4 = \frac{u_0}{u} = \frac{T_0^4}{T^4}, \text{ or}$$

$$(4)\ \lambda T = \lambda_0 T_0 = \text{constant}.$$

Wien's displacement law may be extended so as to give a general formula for the distribution of energy in the spectrum, viz., $E\lambda = C\lambda^{-5} f(\lambda T)$, where $E\lambda$ is the emissive power for radiation of wave-length λ and C is a constant. To prove this let r_0 change to r, then radiations of wave-lengths between λ_0 and $(\lambda_0 - d\lambda_0)$ will be changed to those of wave-lengths between λ and $(\lambda - d\lambda)$. Also $\lambda = \frac{r}{r_0}\lambda_0$, and $\lambda + d\lambda = \frac{r}{r_0}(\lambda_0 + d\lambda_0)$, whence $\frac{d\lambda}{d\lambda_0} = \frac{r}{r_0} = \frac{\lambda}{\lambda_0}$.

From (3) $\frac{du}{du_0} = \left(\frac{\lambda_0}{\lambda}\right)^4$. But du is proportional to $E\lambda\, d\lambda$. Therefore

$\frac{E\lambda\, d\lambda}{E\lambda_0 d\lambda_0} = \frac{du}{du_0} = \left(\frac{\lambda_0}{\lambda}\right)^4$, or $\frac{E\lambda}{E\lambda_0} = \frac{\lambda_0^5}{\lambda^5}$. Since equation (4) holds we may write

$$(5)\ E\lambda = \lambda^{-5} E\lambda_0 \lambda_0^5 = C\lambda^{-5} f(\lambda T).$$

All general distribution formulae must satisfy this equation. It remains to determine the form of the function $f(\lambda T)$. The particular form will depend upon the assumptions made. Wien found

$$f(\lambda T) = e^{-\frac{C}{\lambda T}}, \text{ or } E\lambda = C_1\lambda^{-5} e^{-\frac{C_2}{\lambda T}}.$$

For large values of λ and T this formula fails. Lord Rayleigh proposed the formula $E\lambda = C_1\lambda^{-4} Te^{-\frac{C_2}{\lambda T}}$. This formula fails for small values of λ and T. About 1901 Planck proposed the formula

$$E\lambda = \frac{C_1\lambda^{-5}}{\left(e^{\frac{C_2}{\lambda T}} - 1\right)}.$$

This formula agrees with experiment and approaches the formulæ of Wien and Rayleigh for the range of values for which each holds best. It has already supplanted Wien's to a considerable extent in commercial practice with high temperature furnaces.

This formula is based on a new and startling hypothesis which has come to be known as the "Quantum Hypothesis," to which reference has already been made. The importance of the new hypothesis is made apparent by the following quotations. Nernst[4] says, "If Newton, when he created modern mechanics, paved the way to the results of theoretical physics, if Dalton in the atomic theory gave physics and chemistry their most fruitful logical

[4] Preuss. Akad. Wiss., Berlin, Sitz. Ber. 4, pp. 65–90, 1911.

tools, then Planck in the quantum hypothesis has in his turn discovered a wholly new method of scientific calculation, and in fact this hypothesis, already so useful, is not a mere atomic theory of energy but in reality something wholly new, because the quantum can assume any chosen value from zero upward according to the motion of the atom concerned." The late Henri Poincaré wrote concerning it: "The new conception is seductive from a certain standpoint: for some time the tendency has been toward atomism. Matter appears to us as made up of indivisible atoms; electricity is no longer continuous, not infinitely divisible, it resolves itself into equally charged electrons; we have also now the magneton, the atom of magnetism. From this point of view the quanta appear as atoms of energy."[5]

The physical assumptions which Planck makes may be summarized as follows:[6]

(1) A system of many linear Hertzian oscillators, having a common period of vibration and so spaced as not to exert direct influences upon one another, are contained in a vacuum bounded by perfectly reflecting surfaces and filled with stationary black radiation.

(2) These oscillators only absorb and emit energy in the form of electrodynamic wave radiation.

(3) The vibration energy of an oscillator is given by the equation

$U = \frac{1}{2} Kf^2 + \frac{1}{2} L\left(\frac{df}{dt}\right)^2$, where f = electric moment of the oscillator, and K and L are positive constants.

(4) Emission only occurs when the vibration energy U is any whole number of times the energy-element, the so-called "*Elementar-quantum,*" $E = h\nu_0$, where ν_0 is the frequency of the oscillator and is equal to $\frac{1}{2\pi}\sqrt{\frac{K}{L}}$, and h is a universal constant, the so-called "*Wirkungs-quantum,*" and is equal to $6.55 \div 10^{-27}$ erg-seconds.

Whether the oscillator will actually absorb or emit energy at such times depends upon circumstances. If, however, emission occurs then the whole energy of vibration is emitted and the vibration ceases. Then through new absorption the energy again increases. Some writers on the "quanta theory" argue that the oscillators must absorb as well as emit energy in discrete amounts. It is claimed, for instance, that Einstein's formula for specific

[5]Journ. de Phys. 2, Ser. 5, pp. 5-34, 1912; *ibid* 2, Ser. 5, pp. 347-360, 1912.

[6]Ann. der Phys. 4, pp. 553-563, 1901; *ibid* 31, pp. 758-768, 1910; *ibid* 37, pp. 65-90, 1911.

heats, mentioned below, is not true unless this is so. This, apparently, is not Planck's view as he seems to consider the oscillators as absorbing energy continuously.

(5) The law of emission is: *The ratio of the probability that emission shall not occur, to the probability that emission shall occur, is proportional to the intensity of the vibration exciting the oscillator.* This intensity is defined by the equation $E_z^2 = \int_0^\infty I_\nu d\nu$, where E_z = the component of the electric intensity in the direction of the axis of the oscillator, and as before ν = frequency of the vibration. The constant of proportionality for any given period may be determined by means of Rayleigh's law of energy distribution.

By means of these assumptions the properties of the stationary state, the entropy and temperature of a system of oscillators as well as the distribution of energy in the spectrum of black-body radiation are completely determined. Planck bases his expression for absorption on electrodynamic considerations, those for emission and energy distribution upon statistical ones.

Planck's calculations will not be reproduced here, the mathematical processes being merely indicated and some of the results stated. Basing his investigation relating to the absorption of energy upon the equations given under assumptions (3) and (5) and the additional equation

$Kf + L\frac{d^2f}{dt^2} = E_z$, Planck finds that in the interval of time between two successive emissions the energy U increases uniformly according to the equation $\frac{dU}{dt} = \frac{I_0}{4L}$.

The mode of emission will obviously depend upon the theory of probability. Planck finds that, when P_n is the probability that the energy of an oscillator lies between $n\varepsilon$ and $(n + 1)\varepsilon$ and η is the probability that the energy of the oscillator shall be a whole number of times ε, the average energy of an oscillator is given by the equation

$$U = \sum_0^\infty P_n \left(n + \frac{1}{2}\right)\varepsilon = \left(\frac{1}{\eta} - \frac{1}{2}\right)\varepsilon.$$

Also $U = \left(pI + \frac{1}{2}\right)\varepsilon$, or $\frac{1}{\eta} = 1 + pI$.

The value of p is found to be $\frac{3c^2}{32\pi^2\nu^3 h}$, where c is the velocity of light.[7]

[7]Verh. Deutsch. Phys. Ges., 5, 3, Feb. 1911.

The entropy of a system of N oscillators is also shown to be

$S_n = -kN \sum_0^\infty P_n \log P_n$[8], and since $P_n = (1 - \eta)^n.\eta$,

$$\frac{1}{T} = \frac{dS}{dU} = \frac{k}{E} \log \left(\frac{\frac{U}{\varepsilon} + \frac{1}{2}}{\frac{U}{\varepsilon} - \frac{1}{2}} \right). \text{ or since } E = h\nu,\ U = \frac{\nu h}{2} \left(\frac{e^{\frac{h\nu}{kT}} + 1}{e^{\frac{h\nu}{kT}} - 1} \right)$$

Finally, the vibration intensity of black-body radiation is

$$I = \frac{1}{p} \frac{1}{\left(e^{\frac{h\nu}{kT}} - 1 \right)}.$$

When $T = 0$, $I = 0$, but $U = \frac{h\nu}{2}$. This "Energierest," or energy residue[9] is independent of the temperature, and is of importance in connection with specific heats and radio-active transformations. Planck's radiation formula readily follows from the above equation.

Planck finds the following values for k and h. $k = 1.35 \div 10^{-16}$, and $h = 6.55 \div 10^{-27}$ in C. G. S. units. Using these values together with $\frac{e}{m} = 1.77 \div 10^6 c$, $e = 4.69 \div 10^{-7}$, c, $e = 4.68 \times 18^{-10}$, and $c = 3 \times 10^{10}$, the mean number of undisturbed vibrations is found to be $1.37\ 10^{11}.\lambda e^{\frac{1.46}{\lambda T}}$, or $1.37 \times 10^7 . e^{\frac{14600}{\lambda T}}$, if λ is measured in μ. In the same units the emission number of an oscillator per second is $\frac{2.18 \times 10^7 e^{\frac{14600}{\lambda T}}}{\lambda_2}$, and the mean accumulation time is $4.58 \times 10^{-8} . \lambda^2 e^{\frac{14600}{\lambda T}}$. The equations used to obtain the three preceding results are, respectively:

$$\text{Number of vibrations} = \frac{3c^3 L}{8\pi^2 \nu} e^{\frac{h\nu}{kT}},$$

$$\text{Emission number} = \frac{8\pi^2 \nu^2}{3c^3 L} e^{-\frac{h\nu}{kT}} N,$$

[8]Berliner Ber., 5, 13, July, 1911.

[9]Ann. der Physik, 26, p. 30, 1908.

Time between two successive emissions $= \frac{3c^3L}{8\pi^2\nu^2} \; e^{\frac{h\nu}{kT}}$.

Larmor[10] in an expansion and generalization of ideas implied in Planck's theory divides a system which is a seat of energy into elementary receptacles of energy called "cells." The "element of disturbance" possessing the element of energy under consideration is as likely in its travels to occupy any one of these cells as any other. Instead of the relation $\varepsilon = h\nu_0$, which Planck obtains, Larmor finds that the ratio of the energy-element to the extent of his standard cell is an absolute physical quantity. Larmor claims that his theory evades an atomic constitution of energy although this seems to be open to argument. Planck believes that his constant *h* provides for Larmor's "elements of disturbance." Larmor's radiation formula reduces to that of Planck.

Jeans[11] has worked out a rather complete and satisfactory electron theory of metals, but when applied to radiation his results, expressed in terms of a single universal constant, are in conflict with experiment. Planck considers that Jeans' formula requires a second universal constant which he identifies with *h*, the "wirkungs-quantum," Jeans' formula being a special case where h = 0.

Let us now turn to some of the experimental facts which can be accounted for on the basis of the quantum hypothesis. The agreement with the experimental facts of radiation has already been mentioned.

The experimentally determined specific heats of crystalline substances, especially at low temperatures, do not agree with the older theories, but Einstein[12] by applying the quantum hypothesis to this case has deduced the formula $c = 3R\,\Sigma \frac{e^{\frac{\beta\nu}{T}}\left(\frac{\beta\nu}{T}\right)^2}{\left(e^{\frac{\beta\nu}{T}} - 1\right)^2}$, where R is the gas constant, β a positive constant, and ν and T as before, are the vibration frequency and the absolute temperature. Nernst and Magnus have found that this formulais only approximate and have added a term $bT^{\frac{3}{2}}$[13]. This formula agrees with the results

[10] Roy. Soc. Proc., Ser. A, 83, pp. 82-95, 1909.

[11] Phil. Mag., 17, pp. 773-791, 1909; *ibid* 18, pp. 209-226, 1909.

[12] Ann. der Physik, 22, 1, pp. 180-190, 1906.

[13] Journ. de Phys., 9, pp. 721-749, 1910; Zeitschr. Electrochem., 17, pp. 265-275, 1911; Ann. der Physik, 36, 2, pp. 395-439, 1911.

obtained by Nernst, Lindemann and others for the specific heats of a large number of substances including such anomalous substances as diamond. Moreover, by means of Einstein's formula it is possible to calculate the frequency of the radiation emitted from the specific heat of a substance and the results so obtained agree very well with the frequency as determined by optical methods. It may be noted that the classical thermodynamics would lead one to expect the specific heat of a substance to become zero at the absolute zero of temperature. This is not the case and Planck's "energierest" would lead one to suspect the truth.

Stark[14] has found that when secondary kathode rays are generated by X-rays that the electrons in the secondary rays possess energy of the same order of magnitude as those in the primary kathode rays which produce the X-rays and that this does not depend upon the intensity of the X-rays. The number of electrons in the secondary stream, however, does vary with the intensity of the X-rays. This can be accounted for by saying that the quantum possessed by an electron in the primary stream is handed on by the X-rays to an electron in the secondary stream. Thus when secondary rays are produced the velocity of the individual electrons will not depend upon the intensity of the X-rays but upon the size of the quantum. With more intense X-rays more quanta are transmitted and more electrons set free.

Paschen found that when canal rays were examined for the Doppler effect that, instead of having the original spectral line displaced to one side or broadened on one side, he had a "rest," or undisplaced, and *two* displaced lines. Stark[15] has pointed out that according to Planck's theory a positive ion will only radiate when its kinetic energy of translation is some multiple of the "elementar-quantum." Consequently the ions which are radiating have perfectly definite velocities depending upon the number of quanta they possess, and so we should expect to find a displaced line corresponding to each of these velocities. Stark has also been able to calculate the velocity of the radiating ions and finds that the results tend to confirm Planck's theory.

Haber[16] has applied the quantum hypothesis to the absorption spectra of solids and obtains an equation relating the wave-length in the infra-red, λ_r, the wave-length in the ultra-violet, λ_v, and the molecular weight, viz., $\lambda_r = \lambda_v \times 42.81 \sqrt{M}$. This formula holds for regular crystalline substances such as NaCl, KCl, and fluorspar.

[14]Phys. Zeitschr., 10, pp. 902–913, 1909.
[15]Verh. Deutsch. Phys. Ges., 10, 20, pp. 713–715, 1908.
[16]Verh. Deutsch. Phys. Ges., 13, 24, pp. 1117–1136, 1911.

One of the most useful theorems in thermodynamics is a sort of supplement to the Second Law, and is due to Nernst. It states that *the entropy of a substance at the absolute zero of temperature is zero.* This theorem is amply justified by experiments on specific heats, thermo-neutral chemical reactions, and so forth. Boltzmann has shown that if the energy be subdivided into a large number of equal parts a quantity can be calculated, by means of the theory of probability, which is proportional to the entropy as deduced with the aid of Nernst's theorem, the proportionality factor depending upon the magnitude of the elementary amounts of energy. Some value of this amount should give the value of the entropy exactly. This value, according to Planck, would be the "elementar-quantum." Nernst's theorem may then be considered as another ground for belief in the basic truth of Planck's theory.[17] Additional evidence in favor of the theory is to be found in the phenomena of fluorescence, the photo-electric effect, and others.

We shall now examine Planck's assumptions and attempt to interpret them physically. We know that Hertzian waves only differ in wave-length from the luminous waves emitted by an incandescent body. What is then more natural than to assume that the atoms of bodies contain tiny Hertzian resonators, or oscillators? We say "atoms" because the spectral lines of an element appear in the spectra of its compounds. The "perfectly reflecting walls" may be nothing more than a useful mathematical fiction, or may represent true physical boundaries corresponding to the cell walls of Larmor's elementary receptacles of energy. The assumption of a mechanical model is of vastly less interest than those implying at least an atomic structure of energy. If there are atoms of energy do they preserve their identity? Are they invariable? Planck assumes a discontinuous emission but a more or less continuous absorption of energy. May we not ask the question: "Are these discontinuities due to the oscillator or to the energy itself"? If the energy exists in discrete quantities, why is it not absorbed as well as emitted in multiples of the "elementar-quantum"? If absorbed in discrete amounts are these identical with those emitted? Apparently not, for in the phenomena of fluorescence the fluorescent light is nearly always of lower frequency than the light which causes the fluorescence, or, speaking in terms of Planck's hypothesis, the emergent quanta are smaller than the incident quanta, for the so-called *atoms of energy* are larger in proportion to the frequency of the light, so that "atoms" corresponding to blue light are larger than those corresponding to red light. If, then, we are to account for fluorescence by means

[17]Preuss. Akad. Wiss. Berlin, Sitz. Ber., 4. pp. 65–90, 1911.

of the quantum hypothesis we must conclude that energy is absorbed in amounts differing from those in which it is emitted or may even be absorbed continuously. If energy exists in the form of atoms these must undergo some change in the oscillator analogous to the chemical transformation of ozygen into ozone, or *vice versa*. Furthermore, it would appear that there would need to be as many different sorts of atoms of energy as there are wave-lengths of electro-dynamic wave radiation.

An alternative which has occurred to me is that the discontinuities in the radiation are due to the mechanical form of the oscillator, that is, they are not due to an atomic structure of the energy itself but rather to the oscillator. Imagine, if you will, a circuit consisting of a condenser, inductive resistance and spark gap in series, and in addition assume that the system can absorb energy falling upon it in the form of radiation. For some potential difference, V, the dielectric in the spark gap will break down and a discharge will occur, all of the electric energy, $\frac{1}{2}CV^2$, where C = capacity of system, being transformed into radiant energy of the wave-train produced, or into heat. The discharge will be oscillatory when $CR^2 < 4L$, R being resistance and L inductance, and its frequency will be $\nu = \frac{1}{2\pi}\sqrt{\frac{1}{CL} - \frac{R^2}{4L^2}}$. If we further assume that R is negligible when compared with the other quantities, we have $\nu = \frac{1}{2\pi}\sqrt{\frac{1}{CL}}$. Let E = energy of system at the instant discharge occurred, then $E = \frac{1}{2} CV^2 = \frac{1}{2} CV^2.2\pi\sqrt{CL}.\nu$, or $E = \pi C^{\frac{3}{2}} V^2 L^{\frac{1}{2}} \nu$.

This equation is identical with Planck's equation, $E = h\nu$, if we put $h = C^{\frac{3}{2}} V^2 L^{\frac{1}{2}} =$ constant. Thus the energy is proportional to the frequency. Some such oscillator may provide a means of escape from the conclusion that energy is atomic.

The emission law follows at once from the conception that the oscillator will not be as apt to radiate into a space where the radiant energy is dense as into one where it is rare.

Whatever our conception as to the nature of energy itself there is abundant reason for believing that it is emitted by bodies in discrete amounts. Experimental verification is to be found in widely-separated fields of research. The quantum idea has gained such a foothold that Einstein and Stark would abandon the electromagnetic theory of light for a corpuscular energy theory although Planck himself considers this unnecessary. Concerning the hypothesis the late Henri Poincaré has said, "The present state of the question is as follows: the old theories, which hitherto seemed to account for all known phenomena, have met with an unexpected obstacle. An hypothesis has presented itself to M. Planck's mind, but so strange a one that one is tempted to seek every means of escaping it; these means, however, have been sought vainly. The new theory, however, raises a host of difficulties, many of which are real and not simply illusions due to the inertia of our minds unwilling to change their modes of thought."

NOTE—Since this paper was read Professor Millikan has given a masterly presentation of the various atomic theories of radiation. See *Science*, Jan. 24, 1913.