Motion of N Bodies.

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The relative motion of n bodies, in any order of space, and subject to any law of mutual action, is given by

$$(1) \quad \ddot{\phi} = \phi\pi$$

where φ is a matrix which transforms n determining points of a reference space of order $n - 1$ into the positions of the n bodies, and π is a self-conjugate matrix, depending solely upon the ratios of the mutual reactions to the corresponding mutual distances.

The matrix ϕ is of order $n - 1$, if the motion of the bodies is within the reference space, and ϕ', the conjugate of ϕ, annuls every direction of the reference space exterior to the space of the moving bodies. If the space which contains the moving bodies be greater than $n - 1'$st order the matrix ϕ must be of the same order, but must annul all directions outside of the reference space.

The reduced equations of motion are,

$$(2) \quad (\dot{\psi} + W)\, \psi^{-1}\, (\dot{\psi} - W) = 2\, (\ddot{\psi} - \psi\pi - \pi\psi),$$

$$(3) \quad \dot{W} = \pi\psi - \psi\pi,$$

where $\psi = \phi'\phi$, a function of the mutual distances, and W is a skew conjugate matrix, whose elements are to be found from the quadratic equations between them in (2), and thence substituted in the remaining equations of (2) and in (3), giving a certain number of reduced equations of second and third order.

Another equation which is linear in the elements of W enables us to find the reduced equations in third and fourth orders,

$$(4) \quad D_t\, (\ddot{\psi} - \psi\pi - \pi\psi) = \pi\dot{\psi} + \dot{\psi}\pi + W\pi - \pi W.$$

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