Direct Reading Accelerometers.

By C. R. Moore.

Every person is more or less familiar with the subject of acceleration or deceleration—changes of velocity—whether or not the laws governing the same or the mathematical expressions therefor are understood. Such everyday occurrences as passengers swaying to and fro partially suspended from street car straps, the hurry up that accompanies one's movements as he tries to reach the car door just as the motorman stops the car, are examples which prove this. Changes in the rates of motion are essential to all forms of transportation, and the more rapidly a car or train can be brought up to speed (or stopped) the shorter will be the time required between two points when a given number of stops must be made. Railway trains, street and interurban cars are therefore started and stopped as quickly as is consistent with reasonable comfort, in response to the demand of the traveling public for fast time.

It is the purpose of this paper to discuss briefly the laws of motion, and to describe a new device for measuring the rate of change of velocity, showing results of tests recently conducted in the Electrical Laboratories at Purdue University.

The author realizes at the outset that the subject of acceleration measurement is an old one and is rather reluctant to lay claim before this body of scientists that what is offered herein is new. However as far as his knowledge goes this device has not been used previous to this time. The scheme is brought to your attention for whatever consideration it may merit.

Before discussing accelerometers in detail, a brief study of just what is meant by acceleration and deceleration may be of value.

In Fig. 1 curve "D" shows distances plotted against time, the distances being taken as ordinates and the time as the abscissæ. The car is to be thought of as moving from a certain point "O," distances "d" being measured from that point at the end of the any time "td." It will be noted that during the first few time units after the car starts the distance passed through each successive unit is greater than that passed through during the preceding unit of time, i. e. the rate of motion is increasing. At the

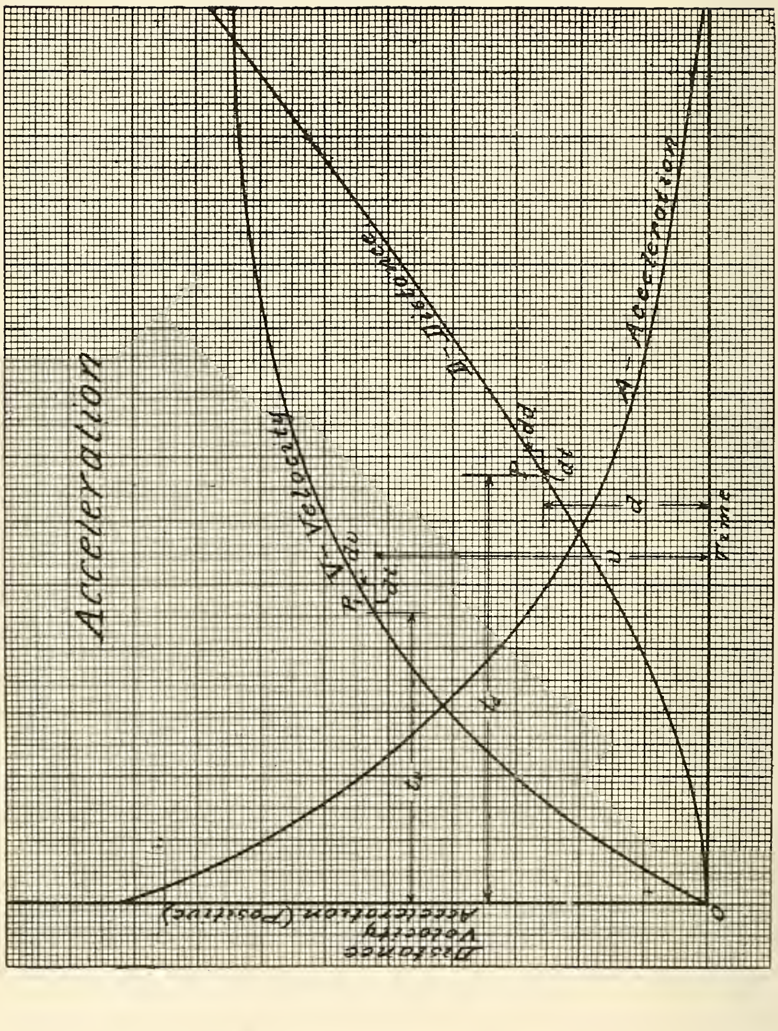
Acceleration
V - Velocity
D - Distance
A - Acceleration
Time
Distance
Velocity
Acceleration (Positive)

Fig. 1.

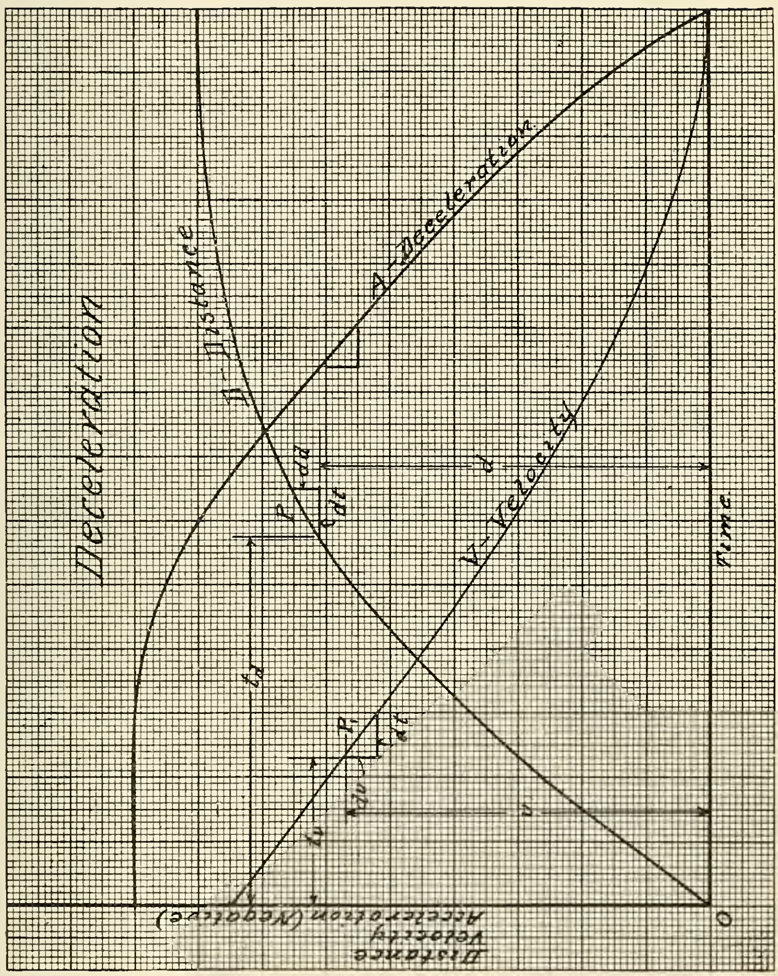
Deceleration
D-Distance
A-Deceleration
V-Velocity
time
Distance
Velocity
Acceleration (Negative)
O

Fig. 2.

end of a certain time, however, equal increments of time show equal increments of distance. The curve then becomes straight because the rate of motion has become constant.

Velocity or the average rate of motion is defined as the space passed over divided by the time required for passage. The average velocity through any point then may be found by dividing small increments of distance by the corresponding increments of time. By taking these increments sufficiently small we may make the average velocity approach the true instantaneous velocity through any given point, as closely as we please. At the limit or when the increments become zero these velocities are equal.

Near the point "P" on the distance curves shown in Figs. 1 and 2 are drawn small triangles having for their vertical components small distances "dd" and for their horizontal components the corresponding increments of time "dt." From the above definition the average velocity for the space passed over designated by the small triangle will be $v = \frac{dd}{dt}$.

By taking this triangle very small the average velocity may be made to very closely approximate the instantaneous velocity at the point "P." It is also to be noted that the ratio $\frac{dd}{dt}$ is the expression for the tangent of the angle included between the line "dt" and that portion of the curve which completes the triangle. Values proportional to "v" may therefore be found at any point on the distance curve by drawing a tangent line at that point and finding the tangent of the angle between this line and the horizontal. Plotting these values multiplied by a constant gives the velocity curves "V" (See Figs. 1 and 2). From this curve we are able to determine the velocity of the car at any time "t."

By scanning curve "V" we note that the velocities for different time values until that time is reached where the distance curve became a straight line. At this point the tangent values become constant and the velocity curve becomes horizontal.

Just as velocity may be determined by dividing space passed over by the time required, so may the acceleration be determined by dividing the velocity change by the time required to make the change. The statements relative to average and instantaneous velocity also hold for average and instantaneous values of acceleration. We may therefore write $a = \frac{dv}{dt}$

as the general expression for acceleration when derived from the velocity-time curve. As before, this expression denotes tangent values so that the acceleration curve may be obtained from the velocity curve in the same manner as the velocity curve was obtained from the distance curve. It is interesting to note that the acceleration curve reaches the X-axis at the same time the velocity curve becomes horizontal and at the same time the distance curve becomes straight. This is shown mathematically as follows:

$$v = \frac{dd}{dt} \qquad a = \frac{dv}{dt} = \frac{d^2d}{dt^2} = 0 \text{ for } v = \text{a constant.}$$

or the value of "v" can be variable only so long as the distance time curve is not straight, and unless "v" is a variable the second derivative of the distance cure will be zero.

Physicists learned early that weight could not be taken as a standard of force on account of the variation of gravity with location on the earth's surface. Knowing however that force was required to change the velocity of a body it developed that when the amount of substance—mass—in a given body was known $(m = \frac{w}{g})$ the force needed to give it a definite change in velocity in a given time was a definite function of these two quantities. The familiar expression for this is, Force = mass × acceleration.

The equation is valuable to scientists and engineers alike. Using unit mass and unit acceleration, the scientist finds thereby a unit force which is constant. (The equation of the pendulum gives him the acceleration due to gravity at any point so that mass may be easily determined.) Knowing the masses involved in a given car or machine, the engineer is able to predetermine the torque necessary at the motor shaft to bring the same up to speed in a given time. This information is valuable for purposes of design.

After the apparatus has been assembled it is sometimes necessary to determine their performance. The mass being known it remains to measure the acceleration to see if the motors meet the requirements.

This measurement of acceleration has been attempted in many ways. A few of the more important schemes will now be considered. Accelerometers employing a freely moving mass of some sort have been most used. Dr. Sheldon's device is of this type, using a suspended weight carrying a pointer at the bottom (fastened thereto by rods) which plays over a scale.

The mass being free to move is sensitive to changes of velocity and the scale may be calibrated to read acceleration directly. The calibration is fairly simple and the device is not difficult to construct.

Another device working on the same principle consists of a "U" tube partially filled with mercury so placed that its plane is parallel to the motion of the car. It is obvious that changes of velocity will cause the mercury to rise in one side of the tube and to fall in the other. The more quickly these changes occur the greater will be the difference between the heights of the mercury in the two portions of the tube. The tube may therefore be calibrated to read acceleration directly.

Again the accelerometer takes the form of a slightly inclined track upon which rolls a ball. This track is made to extend in both directions and has a short level portion at the middle. Changes of velocity cause the ball to move one way or the other along the track. This device is difficult to read and is not very accurate.

All of these accelerometers are confined to horizontal motions and if the track be other than level corrections must be made therefor. This involves a great deal of labor and expense so that while the devices are simple in themselves their use is complicated. It is next to impossible to make them self-recording.

Another apparatus for reading acceleration consists of two magnetically actuated markers so arranged that dots may be made by each of them on a sheet of paper moved at a uniform rate of motion. The magnet of one of these pointers has its circuit closed through battery at regular time intervals by a clock. The other pointer has its magnet operated on a circuit which is closed through battery a definite number of times per revolution of the car wheel. From the record made by these pointers the acceleration at any time may be determined. This apparatus also involves a great deal of labor and expense and is seldom used.

The accelerometer which is the subject of this paper depends for its operation entirely upon electrical phenomena and is independent of its own location, motion or position. It will therefore read acceleration vertically or at any angle as well as in the horizontal direction. No corrections are necessary and it may easily be made self-recording. It is not difficult to calibrate and is permanent.

The circuit as originally conceived is shown in Fig. 3 in which "B" is an electric condenser, "C" an ordinary high grade direct current volt-

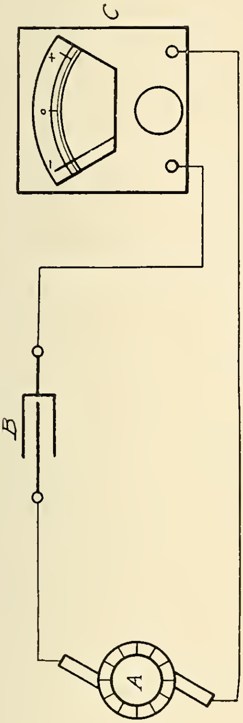

Fig 3.

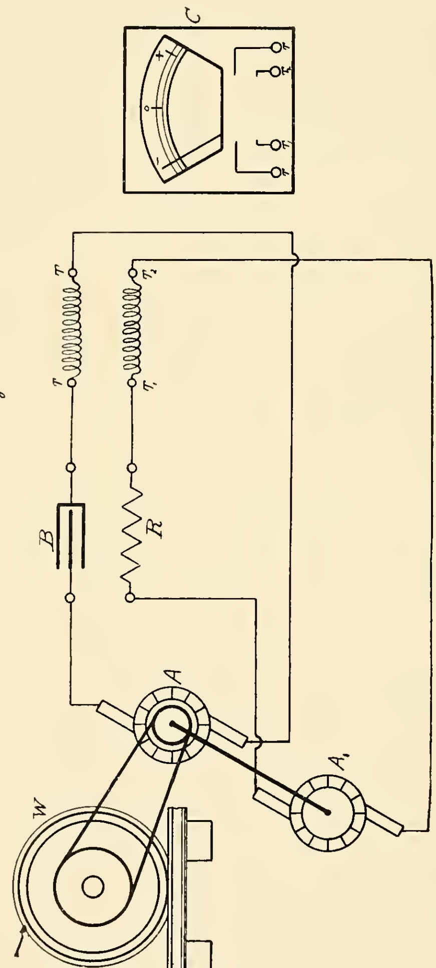

Fig. 4.

meter (with the extra resistance removed) and "A" is a direct current magneto generator having permanent magnet fields.

The equation of the condenser is Q=EC; where Q is the quantity of electricity in Coulombs (ampere seconds), E is the voltage impressed, and C is the capacity in farads of the condenser. Studying this equation we find that if E is increased uniformly the quantity of charge Q on the condenser plates will also increase uniformly. Since Q is increasing uniformly with respect to time, the inflow of current is at a constant rate. i. e., $i = \frac{dq}{dt}$. Likewise a constantly decreasing E will give a constant outflow of current. However, as soon as E reaches a fixed value all current flow in the circuit ceases since it is one property of the electric condenser to arrest the flow of direct current. (The terms "inflow" and "outflow" refer to those condenser plates that are directly connected to the instrument terminal. Of course as much current flows on to one set of plates as flows off of the other plates, the current in the line having a definite direction during an increase of voltage and the opposite direction during a decrease of voltage.) The magnitude of these currents are shown by the direct current instrument which consists merely of a coil swinging in a uniform magnetic field. So long then as the voltage is changing uniformly the instrument will read a constant value returning to zero only when E ceases changing. It follows that if E does not change uniformly the instrument will not read a constant value but that its indications will be proportional to the instantaneous rate of change of the voltage. The direct current magneto is so designed that its voltage is directly proportional to its speed, so that changes of voltage at its terminals can only occur as a result of changes in speed. Therefore the instrument reads the rate of change of speed, i. e. acceleration whether positive or negative.

In a preceding paragraph it was implied that an electric condenser allows no current to pass when the voltage E has reached a fixed value. This would be a fact if an ideal condenser could be made. but it is a well known fact that there is always some leakage even in the best condensers. This means that the dielectric has a definite value of resistance which varies with different conditions and substances, and according to Ohm's law the leakage current will be $I = \frac{E}{R}$. This state of affairs renders our ideal circuit incorrect for any speed above zero because the instrument gets a small current in a definite direction that is practically proportional to speed, and even if the voltages were constant—acceleration zero—the instrument could not return to its zero position.

The circuit must therefore be modified to compensate for this small leakage current, as is shown in Fig. 4. A second direct current magneto (or another commutator on the original machine) is arranged so that it can feed current through a high resistance to another coil on the moving element of the instrument. This second coil is wound over the first and works in the same magnetic field. The current is passed through it in such a direction that the torque produced thereby opposes the torque of the original coil. By adjusting the high resistance these torques may be made equal and the instrument will read zero for any constant value of voltage within reasonable limits. This allows the charging currents to actuate the instrument entirely independent of the leakage current and condensers of reasonable cost may be employed.

In Fig. 4 the second generator is shown at A_1, the high resistance at R, and the second coil on the moving element of the instrument has its terminals shown at T_1 and T_2. These terminals are also shown in the separate sketch of the instrument C. It will be noted that the pair of magnetos are shown belted to a car axle. When this is done changes in the rate of motion of the car will produce changes in the voltages of the magnetos so that the instrument may be calibrated to read accelerations in terms of feet per second per second, as well as in terms of revolutions per second per second.

Figures 5 to 11 show the results obtained recently from tests on this type of accelerometer. Three curves (Figs. 5, 6 and 7) show positive acceleration, and three (Figs. 8, 9 and 10) show negative acceleration.

The experimental apparatus with which these results were obtained was made up as follows: the direct current machine in the condenser circuit was a separately excited generator of about 500 watts capacity having a normal speed of 1,800 R. P. M. The fields were excited from storage battery, about 140 milamperes being used. At 1,800 R. P. M. this excitation gave about 50 volts at the terminals. Since the field was constant and no appreciable current was taken from the armature the voltage remained directly proportional to the speed. The condensers had a combined capacity of about 65 micro-farads and were of the ordinary paper type. The instrument used was home made and very imperfect. Its moving element was very heavy, its frictional error large and the damping effect

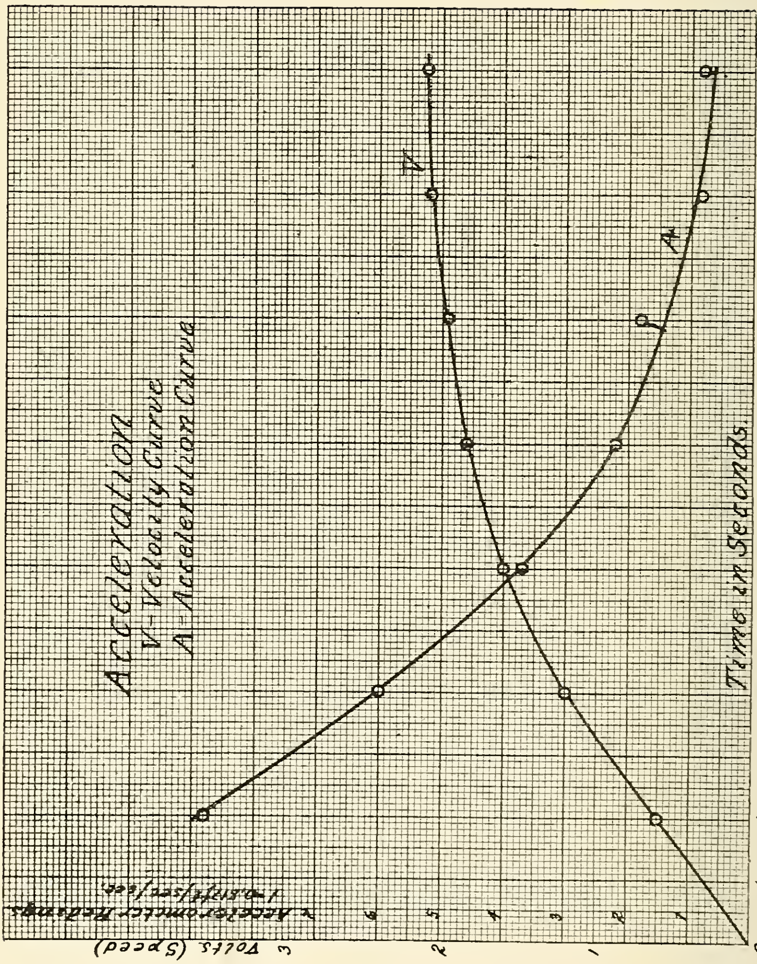
Acceleration
V-Velocity Curve
A-Acceleration Curve
V
A
Time in Seconds
Volts (Speed)
Accelerometer Readings

Fig. 5.

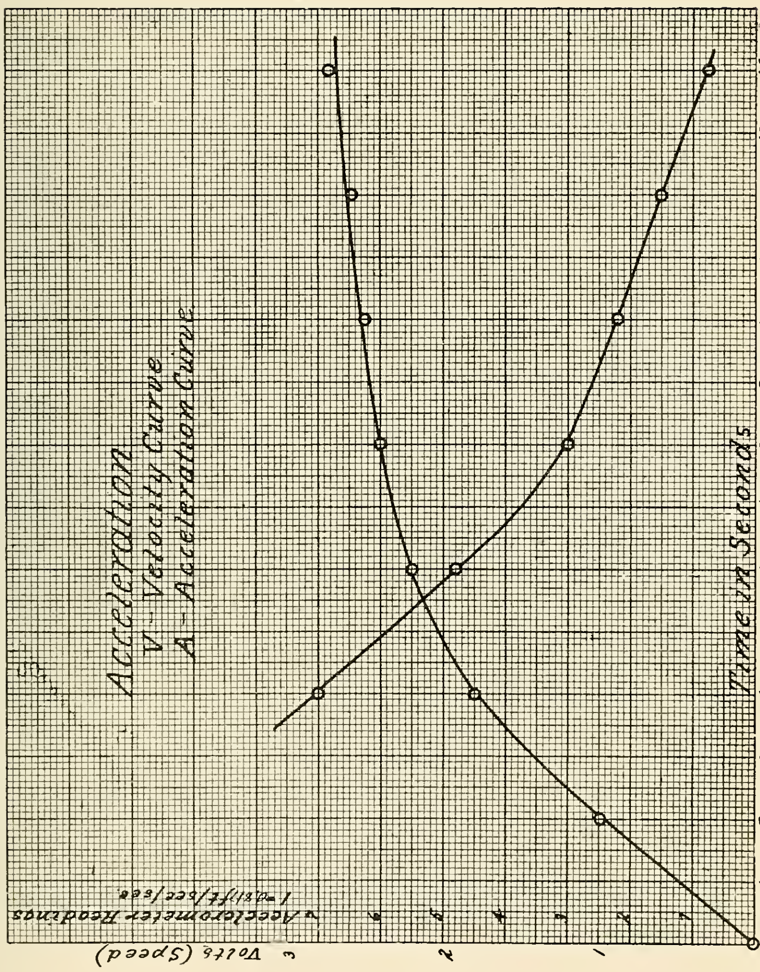
Acceleration
V - Velocity Curve
A - Acceleration Curve
Time in Seconds
Volts (Speed)
Accelerometer Readings
1 2 3 4 5 6 7 8 9 10 11 12 13 14

Fig. 6.

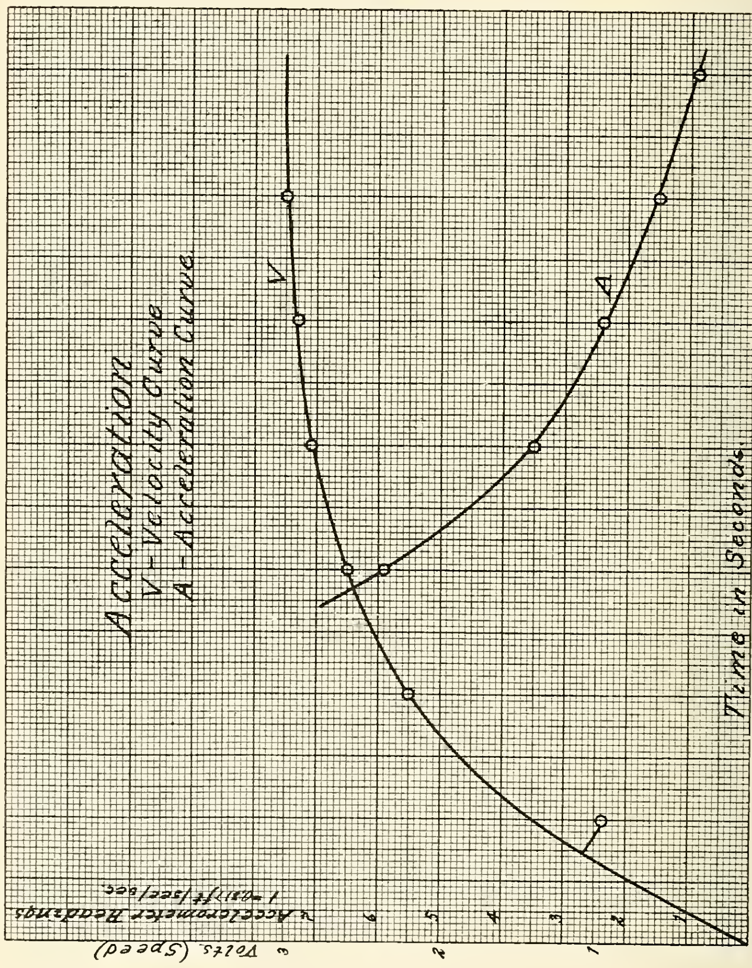
Acceleration
V - Velocity Curve
A - Acceleration Curve
V
A
Time in Seconds
Volts (Speed)
Accelerometer Readings
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14

Fig. 7.

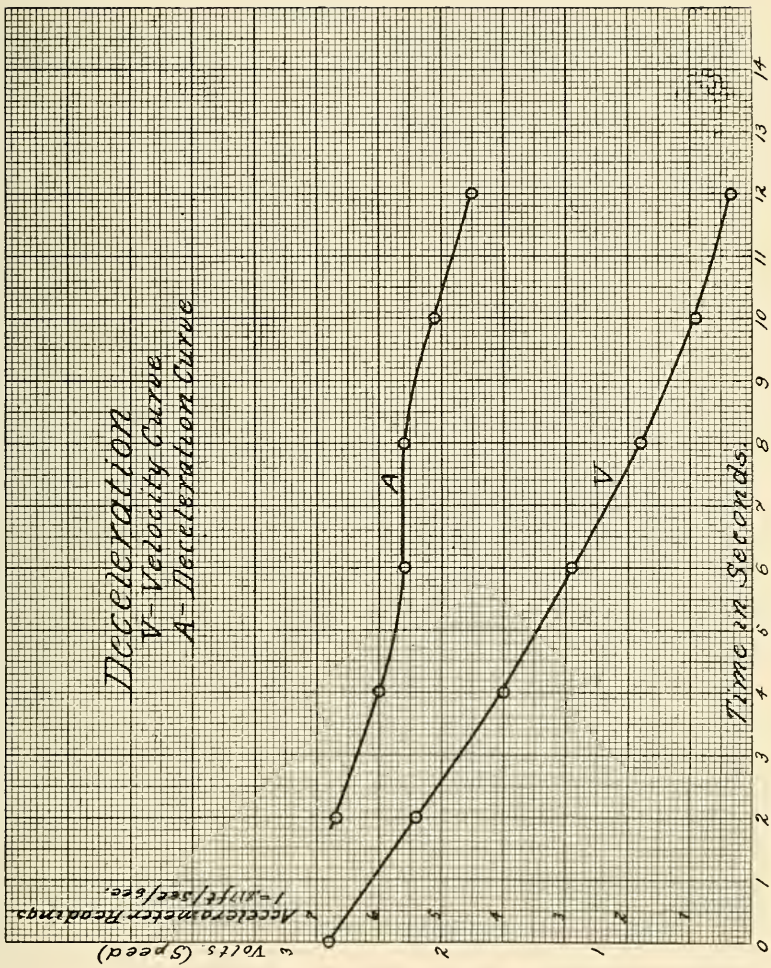
Deceleration
V-Velocity Curve
A-Deceleration Curve
A
V
Time in Seconds
Volts (Speed)
Accelerometer Readings
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

Fig. 8.

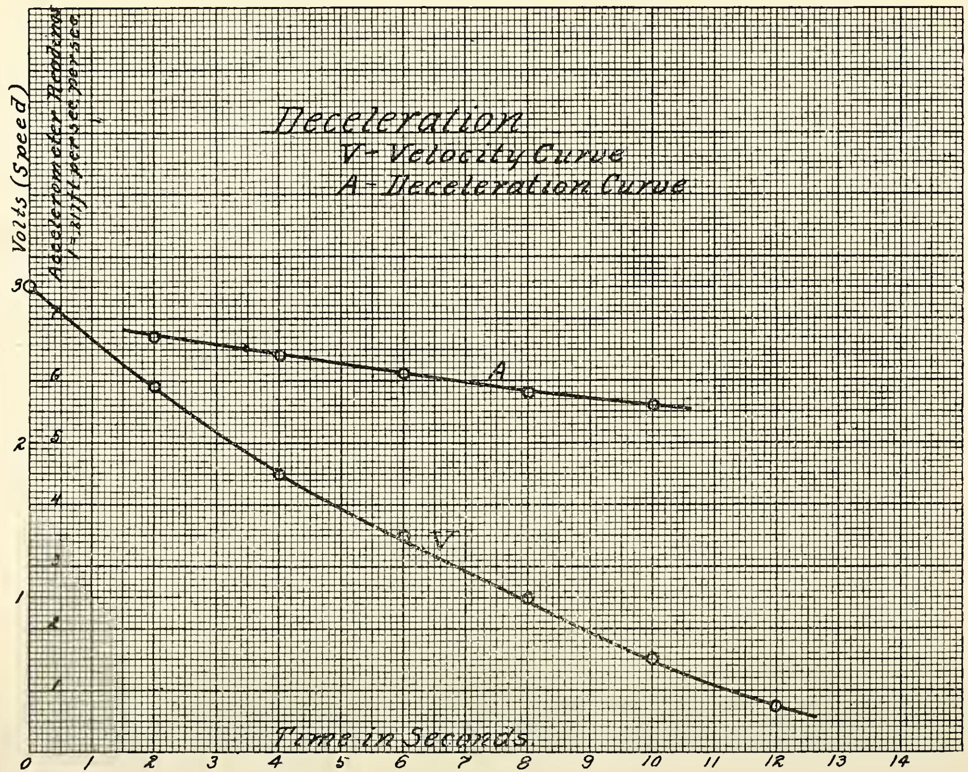
Deceleration
V - Velocity Curve
A - Deceleration Curve
Volts (Speed)
Accelerometer Readings
1 = .8117 ft. per sec. per sec.
A
V
Time in Seconds
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14

Fig. 9.

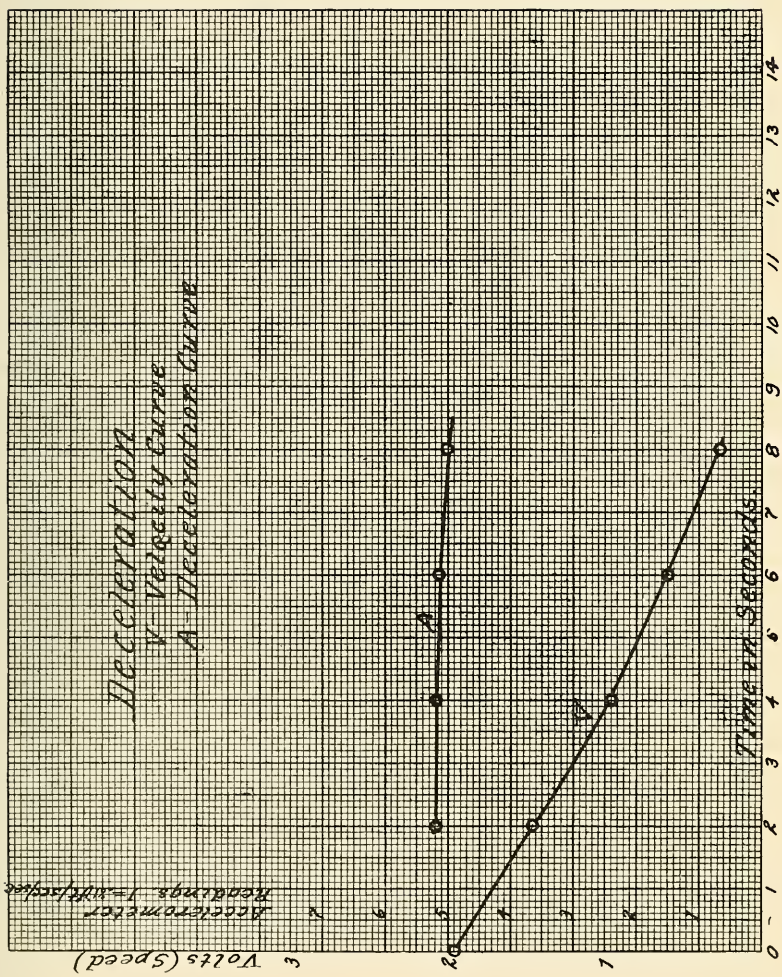
Deceleration
V - Velocity Curve
A - Deceleration Curve
Time in Seconds
Volts (Speed)
Accelerometer

Fig. 10.

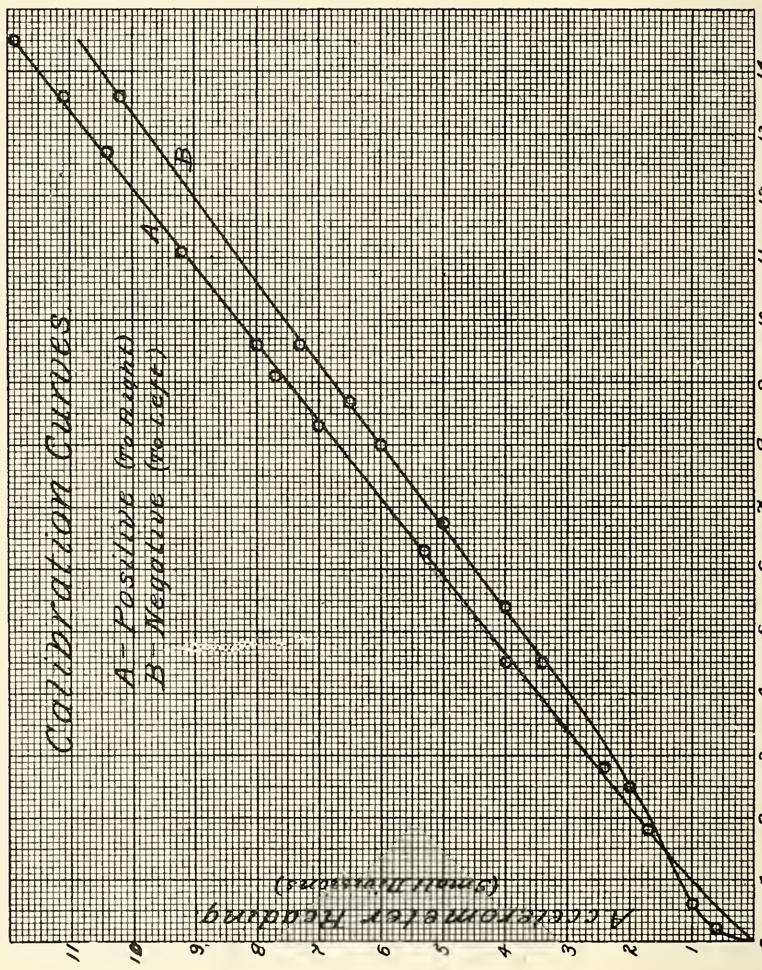
Calibration Curves.
A- Positive (To Right)
B- Negative (To Left)
A
B
Accelerometer Reading
(Small Divisions)
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14

Fig. 11.

poor. Its calibration curves are shown in Fig. 11. These imperfections account for the variation in its calibration constant as will be stated later. The resistance circuit contained a three-volt, 1,800 R. P. M. magneto (permanent fields) directly connected to the motor shaft, as was the generator in the condenser circuit. The resistance employed was of the ordinary box type.

Acceleration was obtained by impressing suddenly a fixed voltage on the driving motor and reading values of speed and the accelerometer every two seconds. Deceleration was obtained by opening the motor switch and reading speed and the accelerometer every two seconds. The speed readings were secured by attaching a voltmeter to the three-volt magneto. Some of the readings thus taken are shown in Figs. 5 to 10 which are self-explanatory.

Scanning these curves brings out their similarity to the mathematical curves on Figs. 1 and 2.

Calibration is effected by drawing tangents at various points on the speed time curve and dividing the accelerometer reading at this point by the value of the tangent of the angle between this line and the horizontal. This quotient should be constant. Now by noting actual voltage and the corresponding speed the number of volts per revolution may be obtained. Our tangent value indicates volts change in a given time "t" which may now be reduced to revolutions change in the same time. If the generators be belted to a car axle the wheels of which have a known diameter this revolution change may be reduced to the corresponding change of linear velocity in the given time "t."

For the tests herein described, however, the instrument scale was arbitrarily drawn and, with the particular circuit set up, each small division corresponds to an acceleration of 0.33 revolutions per second per second. If it had been used on an interurban car having 24″ wheels its scale would indicate 0.817 feet per second per second per small division. This value could be reduced to a workable figure by using a larger condenser, a higher voltage and a more sensitive voltmeter.

These calibration values varied from 15 to 25 revolutions per second per second per small scale division on account of imperfections in the instruments and the small readings made necessary by having insufficient capacity.

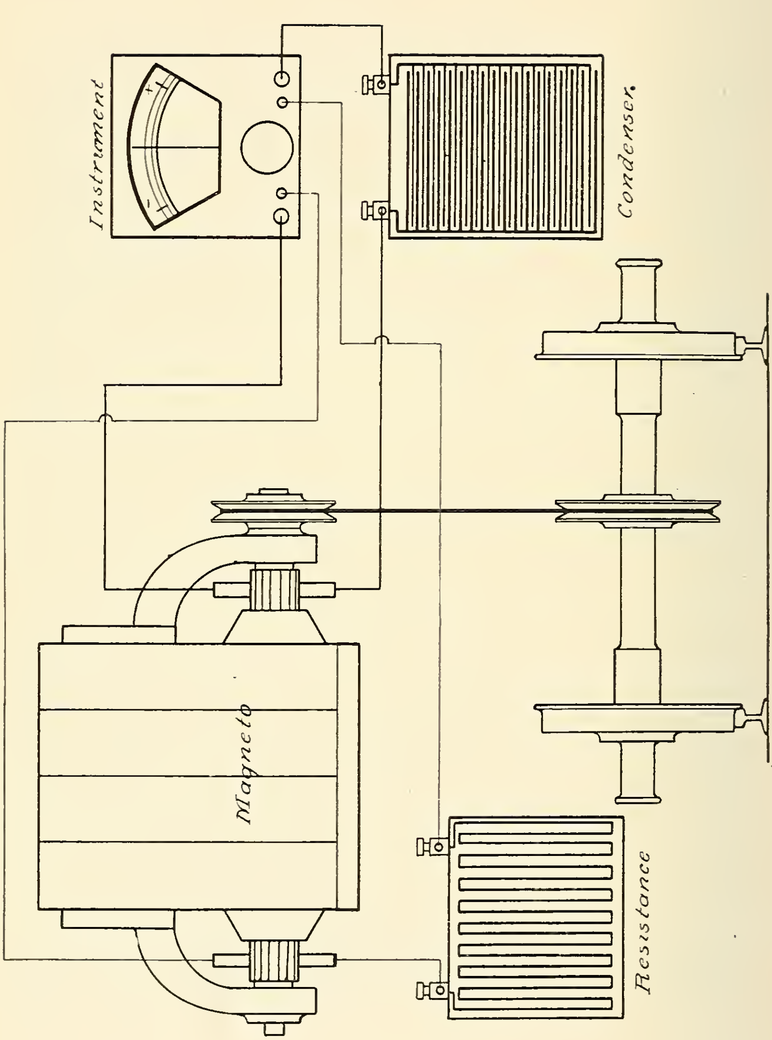

Fig. 12.

Almost any condenser when suddenly discharged if allowed to stand a few minutes will experience a rise in potential at its terminals. This rise is due to what is known as the residual charge. This phenomenon is explained as follows: When a condenser is charged its dielectric is strained and being non-homogeneous the strains are unequal. (By strain is meant the actual compression of the plates.) When discharged these strains are relieved but they do not decrease at the same rate, so that some parts of the dielectric become strained in the opposite sense and balance those parts which are slower in acting. The condenseer is then apparently discharged, but after standing a while these strains tend to diminish and usually there is a resultant strain set up. This resultant strain is due to the fact that while the forces were originally balanced at the end of the first discharge, yet the distances are unequal and in nonhomogeneous materials stress is seldom proportional to strain.

The condenser may now be discharged again and after a time may show still another rise of potential. In the apparatus herein described this effect is entirely negligible, for the reason that the condenser is never charged or discharged suddenly, some few seconds being required to complete the action.

In all condensers there is also some absorption, but with good condensers used at the voltages proposed for this apparatus this effect is also quite negligible, and we may with certainty say that for a given voltage change at any part of the potential range equal quantities of electricity pass through the instrument.

With an instrument giving a uniform scale therefore we have an apparatus which will show equal increments of readings for equal rates of change of velocity, i. e. a direct reading accelerometer.

Fig. 12 shows the apparatus as assembled for use in railway work. The double commutator magneto is here shown belted directly to the car axle. It is obvious that the readings of the instrument are unaffected by grades or side tiltings of the car.

The apparatus may be made self-recording by employing a recording instrument instead of an indicating one, as shown in Fig. 13. These recorders may be obtained in the market and are very sensitive and reliable. The record is made by placing a pen on the end of the voltmeter pointer, the whole being pulled down upon a sheet of paper moving at a uniform rate of motion by means of a small magnet whose circuit is closed through battery by a clock. The record is thus made automatically and needs no correction.

The accelerometer may be made self-contained and is easily transferred from one car to another.

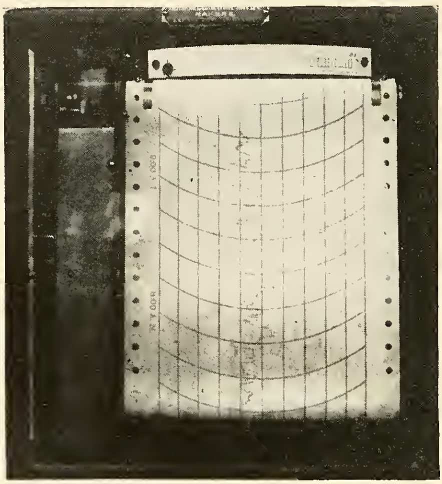

Fig. 13.

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