## INDUCTION AND RADIATION.

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The equation $H=\frac{I d s}{r^{2}} \cos \theta$, where $H$ is the magnetic field at $P$, (fig. 1)


Fig. 1-Illustrating a portion of a straight wire in which a current, $I$, is flowing. The field, H , at the point, P , is $\mathrm{H}=\mathrm{I}\left(\mathrm{ds} / \mathrm{r}^{2}\right) \cos 0$. When the point, P is a great distance from the wire and perpeadicular to the wire, $\mathrm{H}=\mathrm{Ih} / \mathrm{d}^{2}$. The vector potential is $A=I h / d$.

I ds is an element of current length ds, r , is the distance of the point, P , from the element l ds , and $\theta$ is the angle between the line joining P and ds and the normal to ds, may be said to be the fundamental equation giving the relation of current strength to the magnetic field about the current. From this equation we get our definition of unit current. In speaking of this equation we shall consider the point, $P$, to be on the normal to ds, and the angle $\theta=0$, so that $H=\frac{I d s}{r^{2}}$ is the form of the equation. If we apply this equation to a current flowing in a circular wire (fig. 2) we have I times length divided by $\mathrm{r}^{2}$, or $\mathrm{H}=$ $2 \pi \mathrm{r} \mathrm{I} / \mathrm{r}^{2}$ or $\mathrm{H}=2 \pi \mathrm{I} / \mathrm{r}$ for the value of H at the center of the circle.


Fig. 3-When the field is calculated for a point on the normal to the coil, $H=2 \pi r^{2} I / x^{3}$ if the distance, $x$, is great compared to the radius of the coil. Or $H=2 I A / x^{3}$.

If P (fig. 3) is on the line which is perpendicular to the circle at the center and at a distance $x$ from the center, then we have $\mathrm{H}=$ $2 \pi r\left[I /\left(r^{2}+x^{2}\right)\right]\left(r / \sqrt{r^{2}+x^{2}}\right)$ or $\mathrm{H}=2 \pi \mathrm{r}^{2} \mathrm{I} /\left(\mathrm{r}^{2}+\mathrm{x}^{2}\right)^{\frac{3}{2}}$. When the distance is great, as at $\mathrm{P}^{\prime}$, then $\mathrm{H}=2 \pi \mathrm{r}^{2} \mathrm{I} / \mathrm{x}^{3}=$ $21 \mathrm{~A} / \mathrm{x}^{3}$.
When the current is flowing in a straight wire of infinite length then $\mathrm{H}=2 \mathrm{I} / \mathrm{r}$. If the current is flowing in a wire of finite length, which is short compared to the distance x , then $\mathrm{H}=\mathrm{Ih} / \mathrm{x}^{2}$ where h is the length of the wire.

[^0]If we consider a circuit which carries a current to be equivalent to a magnetic shell (Starling, p. 225), we have for the potential at a distant point $P$, on the normal to the circuit (fig. 3) $V=I A / x^{2}=I \omega$, where $\omega$ is the solid angle at $\mathrm{P}^{\prime}$ subtended by the circuit. If the current has the same effect as the magnetic shell, then the potential at $P^{\prime}$ is $V=M / d^{2}$ where $M$ is the magnetic moment of the magnetic shell and $d$ is the distance of $P^{\prime}$ from the magnetic shell. $d$ is the distance from the north face of the shell, or d centimeters from the center of the magnet "end on'". If we solve for the potential at a point in the plane of the circuit which is the plane of the magnetic shell, (fig. 4) we get $V=I \omega=0$. This is the same as the potential at a distance $d$ from a short magnet "broad side on". (Starling, p. 13). If we differentiate the first equation with respect to the distance or with respect to $x$, we have
$H=\frac{-\mathrm{dV}}{\mathrm{dx}}=\frac{-\mathrm{d}\left(\frac{\mathrm{M}}{\mathrm{x}^{2}}\right)}{\mathrm{dx}}=\frac{2 \mathrm{M}}{\mathrm{x}^{3}}=\frac{2 \mathrm{I} \omega}{\mathrm{x}}=\frac{2 \mathrm{IA}}{\mathrm{x}^{3}}$ which is the same as was derived above.

When the point is in the plane of the coil the potential is zero but the field $H$ is not zero. The field is the derivative of the potential with respect to the direction at right angles to the distance $d$. At a point near the plane of the coil (fig. 4) we have $V=I \omega=I A \sin \theta / y^{2}=\left(I A / y^{2}\right) x / y . \quad H=-d V / d x$ $=-\mathrm{IA} / \mathrm{y}^{3}$. The field in the plane of the coil is one-half that along the axis of the coil when the distance is the same in both cases.

The value of the field $H$ is the


Fig. 4-When the point $P$, is in the plane of the coil the field is IA/ $y^{3}, \quad y$, being the distanee of $P$, from the coil. The potential is zero but the field is the spaee derivative of the potential which gives $H=I A / y^{3}$. field due to a current, I. This current, I, is assumed to be direct current. If it is alternating current we assume that the virtual field "root mean square" field produced by the virtual current, $I$, is the same as that produced by the direct current of the same numerical value. It is this field which is called induction. Thus induction is the field we usually think of when we speak of self-induction, of mutual induction, of transformers, and of induced currents.

Due to induction, energy is stored in the field when current is increasing and is again absorbed into the circuit when the current is decreasing. In a pure inductive circuit with alternating current no energy is dissipated. The current is "wattless".

If, as is sometimes done, we try to explain the action of an aerial by assuming a perpendicular wire with a capacity such as a large ball at the top, we can picture the magnetic field as circles about the wire and the electric field as lines which start from the ball and curve downwards until they end on the ground, we get a picture (fig. 5) which answers the purpose until we try to explain the fact that the magnetic and electric fields are in phase as is shown from Maxwell's equations. The fields about the aerial as we have pictured them are out of phase. The magnetic field is a maximum when the
current is a maximum and the electric field is a maximum when the charge on the ball is a maximum, which is at a time when the current is zero and the magnetic field is zero. This is due to the fact that in the picture we have been thinking of the field called in-


Fig. 5-An elementary picture of the fields about a vertical antenna. The magnetic field is represented by the circles about the antenna and the "spray" lines represent the electric field. Our ordinary conception of magnetic field, which is induction, has led many text books to give wrong statements. duction and in the Maxwell equations we are dealing with the field called radiation.

To get the radiation we make use of the vector potential. The line integral of the vector potential is equal to the surface integral of the magnetic flux. To illustrate a line integral we will take the line integral of the electromotive force which is equal to the surface integral of the change of flux. The line integral is equal to the work done in carrying a unit quantity around the path. Suppose we have a wire bent into a rectangular loop of a single turn. Now potential difference is equal to the work required to move a unit from one point to another point. The e.m.f. in the loop is the work required to move a unit quantity of electricity around the loop, or e.m.f. is the line integral around the loop. Since $E=-d N / d t$ the e.m.f., $E$ is equal to the rate of change of flux through the coil. The total change of flux through the coil is equal to the surface integral of $\mu \mathrm{dH} / \mathrm{dt}$ taken over the surface area of the coil. In the wire if it is a closed loop there will be an induced current flowing in the wire. The rectangle does not need be a conductor. The rectangular circuit


Fig. 6-The work done by carrying a unit charge of electricity around the rectangular $p$ equal to the potential induced by the change of the magnetic field, $\beta$, through the co
might be made of a glass tube. The line integral may be thought of as being the work done by pulling a unit charge through the tube by means of a "greased" thread which had been threaded through the tube. The e.m.f. will be equal to the rate of change of flux the same as if a conducting wire were there. The rectangle may be an imaginary rectangle or path in space and the flux may not be perpendicular to the face of the imaginary circuit, and the statement that the line integral of the e.m.f. is equal to the surface integral of the charge of magnetic flux still holds.

Imagine a rectangular path in the $\mathrm{X} Z$ plane whose area is dx dz (Fig. 6). Let the magnetic field, $H$, have the components $\alpha, \beta, \gamma$, in the $x, y$, and $z$ directions respectively. Let the electric field, H , have components $\mathrm{P}, \mathrm{Q}, \mathrm{R}$. Let the path be represented by the square $\mathrm{a} b \mathrm{c} d$ in the figure. Then the rate of change of magnetic flux through the area will be $\mu \frac{d \beta}{d t}$ and the total change of flux will be $\mu \frac{d \beta}{d t} d x d z$. Let the component of $E$ along the line ab be $P$. Then the component of E along the line dc will be that along ab plus the change in moving a distance $d z$ in the $Z$ direction. This change is equal to the space rate of change, $\frac{d P}{d z}$, times the distance. Then the value of E along de will be $\mathrm{P}+(\mathrm{dP} / \mathrm{dz}) \mathrm{dz}$.

The component of E along $a d$ is R , and that along be is $\mathrm{R}+(\mathrm{dR} / \mathrm{dx}) \mathrm{dx}$. Then the work done in taking our unit quantity along the path abcd is Pdx $[R+(d R / d x) d x] d z+[P+(d P / d z) d z] d x-R$ dz which when added and simplified is $(d P / d z-d R / d x) d x d z$. Since $E=-d N / d t$ this is equal to the negative of the rate of change of flux, which is $-\mu \frac{d \beta}{d t} d x d z$, or $-\mu \frac{d \beta}{d t} d x d z$ $=(d P / d z-d P / d x) d x d z . d x d z$ can be cancelled out. Going through similar operations in the other two planes we get the two similar equations. The three equations are:

$$
\begin{aligned}
& -\mu \frac{d \alpha}{d t}=\frac{d R}{d y}-\frac{d Q}{d z} \\
& -\mu \frac{d \beta}{d t}=\frac{d P}{d z}-\frac{d R}{d x} \\
& -\mu \frac{d \gamma}{d t}=\frac{d Q}{d x}-\frac{d P}{d y}
\end{aligned}
$$

The shorthand method of writing these equations is $-\mu \frac{d H}{d t}=$ Curl of $E$.
Now for the line integral of a vector potential. Let $A$ be the vector potential with components $A_{x} A_{y} A_{z}$. So far we have not said what $A$ is. If the line integral of A is equal to the surface integral of the magnetic field we can from the analogy of the line integral of the e.m.f. write the equation in "shorthand",

$$
\begin{aligned}
& H=\text { Curl of } A \text {, or writing in full } \\
& \alpha=\frac{d A_{z}}{d y}-\frac{d A_{y}}{d z}
\end{aligned}
$$

$$
\begin{aligned}
& \beta=\frac{d A_{x}}{d z}-\frac{d A_{z}}{d x} \\
& \gamma=\frac{d A_{y}}{d x} \quad \frac{d A_{x}}{d y}
\end{aligned}
$$

If the magnetic field at P (fig. 7) is due to current in the vertical aerial at the origin, then $\alpha=0, \gamma=0$, and $\beta$, the component parallel to the Y axis, is the only component.

Then $\beta=\frac{d A_{x}}{d z}-\frac{d A_{z}}{d x}$.


Fig. 7-The magnetic field, radiation, at $P$ due to alternating current in a distant aerial, OZ , is the Y component, $\beta, \quad \beta$ is in phase with the electric field which is parallel to the antenna OZ . The two fields are in time, phase; in space quadrature.

A must in some manner depend on the current in the aerial and since this is a long vertical current A will not change as we move up in the Z direction above P . Then $\frac{\mathrm{dA}_{\mathrm{x}}}{\mathrm{dz}}=0$.

Then $\beta=H=-\frac{d A_{z}}{d x}$
From the above we have $d A_{z}=-H d x \quad A=-\int H d x$.
But we have assumed for constant current that $H=\frac{I d s}{r^{2}}=\frac{I h}{x^{2}}$ where $h$ is the height of the aerial $x=r$ the distance between $P$ and the aerial.

Then $A z=-\operatorname{Ih} \int \frac{d x}{x^{2}}=\frac{I h}{x}$.
From this we have $H=\frac{d A}{d x}=\frac{d\left(\frac{I h}{x}\right)}{d x}$.

Of course, if we take the derivative we will get our old value for H , but we have heretofore assumed that the current in the aerial is direct current or the equivalent to direct current. This is not true. The current is alternating current, or I $=I-\sin \omega t$. Also, it takes some time for the field to reach the point, $P$. The field at $P$ is due to the current which was in the aerial a fraction of a second before the time $t$.

Then $I=I_{0} \sin \omega\left(t-t^{\prime}\right)$. Since space equals velocity times time, $t^{\prime}=x / v$ where $v$ is the velocity of light, then the current is $I=I_{0} \sin \omega(t-x / v)$
and

$$
H=\frac{-d}{d x}\left(\frac{h I_{0} \sin \omega(t-x / v)}{x}\right)
$$

differentiating,

$$
H=+\frac{h I_{0}}{x^{2}} \sin \omega\left(t-\frac{x}{v}\right)+\frac{h I_{0} \omega}{v x} \cos \omega\left(t-\frac{x}{v}\right) .
$$

Thus we see that the field, $H$, consists of two parts. The first is the field we get by considering the current to be constant, or if alternating current, by considering the field to be independent of the sine of the angle. This virtual field is numerically the same as the field due to a D.C. current.

The second part is that in which we consider the angle to depend on the distance $x$. The two parts are out of phase by 90 degrees. We remember we had trouble with the ordinary field in our elementary picture because it was out of phase with the electric field. This second part is in phase with the electric field.

The first part is induction. The second part is radiation. The first part the induction diminishes as the square of the distance while the second, the radiation, diminishes as the distance.

We can write the virtual values of the magnetic field by considering the sine and cosine to be unity, and writing I for the virtual current, then

Induction, $\mathrm{H}=\mathrm{hI} / \mathrm{x}^{2}$
Radiation, $\mathrm{H}=\mathrm{h} \mathrm{I} \omega / \mathrm{vx}$.
If I is measured in amperes, $I / 10$ will give the value of I to make the field in lines per square centimeters.
Since $\quad \frac{\omega}{\mathrm{v}}=\frac{2 \pi}{\lambda}$

$$
\text { Radiation, } H=\frac{h I 2 \pi}{10 \lambda x} \text {. }
$$

Equating the two values and solving for x we find that the two components of $H$ are numerically equal when $x=\lambda / 2 \pi$. At a distance equal to $1 / 6.28$ of a wave length the two values are numerically equal. Since they are in time quadrature the measured value will be 1.414 times the calculated value of one. Closer to the aerial the value of H is nearly all induction and diminishes as the square of the distance. Beyond this point the field is mostly all radiation and varies inversely as the distance.

For practical purposes when the distance is less than $1 / 20$ of a wave length the radiation can be neglected, and when the distance is greater than $1 / 2$ wave length the induction can be neglected.

If instead of an antenna aerial we have a coil aerial the induction can be calculated as is done in the first part of the paper. Induction is the ordinary
field due to direct current. It is found to diminish as the cube of the distance from a coil. It is $21 \mathrm{~A} / \mathrm{d}^{3}$ perpendicular to the plane of the coil and $1 \mathrm{~A} / \mathrm{d}^{3}$ in the plane of the coil.


Fig. 8-A square coil carrying radio frequency current produces two fields at the point, $P$. The induction diminishes as the cube of the distance while the radiation diminishes as the first power of the distance.

For radiation we follow Dellinger. Consider a square coil in the XZ plane of height h , and length 1. (Fig. 8.) The horizontal parts will not contribute to field at a point $P$ in the horizontal plane. Then the radiation at P consists of two components one each from the two vertical wires. These two will be equal but slightly out of phase because the distance of one is greater than the other by $\lambda$ centimeters. The resultant field at P is the vector difference of the two equal vectors which differ in direction by a small angle, $\theta . \theta / 2 \pi=1 / \lambda$ or $\theta=2 \pi \mathrm{l} / \mathrm{x}$. In the diagram (fig. 9) oa $=2 \mathrm{H}_{1} \sin$ $\theta / 2$. Since $\theta$ is small, $\sin \theta / 2=\theta / 2$, then


Fig. 9-The two vertical sides of length, $h$ of the square coil produces two equal radiation fields but due the distance, 1 , they are slightly out of phase. The vector difference is proportional to the vector oa.

$$
\mathrm{oa}=\mathrm{H}=2\left(\frac{\mathrm{hI} \omega}{\mathrm{vx}}\right) \frac{2 \pi \mathrm{~h}}{2}=4 \pi^{2} \mathrm{~h} \ln \mathrm{I} / 10 \lambda^{2} \mathrm{~d} .
$$

Thus the radiation from a coil varies inversely as the distance while the induction varies inversely as the cube of the distance.

The radiation from a coil varies inversely as the square of the wave length while from an antenna inversely as the wave length.

The induction from a coil is $1 \mathrm{~A} / \mathrm{d}^{3}$ and the radiation from the same coil is $4 \pi^{2} I A / \lambda^{2} d$. Equating the two values we get $\frac{1}{\mathrm{~d}^{2}}=4 \pi^{2} / \lambda^{2}$ or $\mathrm{d}=\lambda / 2 \pi$. Thus the two components are equal at a distance $\lambda / 6.28$, the same being true for an antenna aerial. Figure 10 gives the relative distribution of radiation and induction about a coil at a distance, $\mathrm{d}=\lambda / 2 \pi$. Close to the coil or antenna aerial the field is primarily induction. Figure 11 gives the distribution when $d=\lambda / 20$. The energy represented by induction does not leave the aerial. It is stored in the medium during the first fourth of a cycle and then returns to the acrial during the second fourth of the cycle in the same manner as the field of an ordinary transformer or choke coil. The induction is the ficld which stays at home. The energy of the field of the radiation does not return to the aerial but passes out to infinity unless it is absorbed by intervening objects. The energy is radiated into space.

Of course it is possible to absorb a part of the energy of induction if the absorber is in the field of the induction, that is, near the aerial. This is the same
as in a transformer; part of the energy may be absorbed by the secondary coil, in which case it can not return to the primary.

It will be noted that radiation from a given aerial depends upon the frequency or wave length. Induction is independent of frequency. The virtual value of induction field for 60 cycle, 300 meters or 41 meters is numerically the same as that produced by D.C. current.


Fig. 10-In the plane of a coil the radiation field and the induction fields are numerically equal when the distance from the coil is equal to the wave length divided by 6.2832 . The figure shows the relative values in all directions about the coil. Perpendicular to the coil radiation is zero and induction is two times that in the plane of the coil.


Fig. 11-Shows the relative values of the two fields when the distance is one twentieth of a wave length. The induction is many times the radiation.

We have spoken of the magnetic field only. Maxwell's equations show that the encrgy of the magnetic field is exactly equal to the energy of the electric field. Thus if we know the magnetic field we know the electric field. The electric field and magnetic fields are always associated together. They
are two aspects of the same thing. They really are the same thing in a certain sense. For an illustration take a simple D.C. circuit. The current through a coil is equal to the total e.m.f. divided by the total resistance. The current is also equal to the Pd . at the terminals of the coil divided by the resistance of the coil. These two values are equal. The current in the coil is not 2I. In the electro-magnetic system of units the ratio between E and H is $3 \times 10^{10}$. E in absolute units of potential per centimeter is equal to H in Gilberts times $\mathrm{v},\left(3 \times 10^{10}\right)$.

$$
\mathrm{E}=\mathrm{Hv}=3 \times 10^{10} \mathrm{H}
$$

If we divide by $10^{\S}$ we have E in volts per centimeter. Multiply this by 100 and we have E in volts per meter. If we reduce this to micro-volts by multiplying by $10^{6}$ we again have
$\mathrm{E}=3 \times 10^{10} \mathrm{H}$ micro-volts per meter; thus $\mathrm{H} \times \mathrm{v}$ is either absolute units of potential per centimeter or micro-volts per meter.

E is usually expressed in micro-volts per meter written $\mu \mathrm{V} / \mathrm{M}$

$$
\mathrm{E}=3 \times 10^{10} \mathrm{H} \frac{\mu \mathrm{~V}}{\mathrm{M}}
$$

The e.m.f. induced in a coil by the magnetic field is $\mathrm{e}=\mathrm{AH} 2 \pi \mathrm{n} / 10^{8}$ volts where $A$ is the area of coil and $n$ is the frequency.

The e.m.f. induced in a vertical antennae of height $h$, is the number of lines cut per second, $\mathrm{e}=\mathrm{hvH}$ abs, or $\mathrm{e}=$ Eh abs, or instead of absolute units the e.m.f. may be expressed in micro-volts, if $h$, height, is expressed in meters instead of centimeters.

The received current can be determined by dividing the e.m.f. in volts by the resistance of coil or antenna.

From these fundamental equations Dellinger's four equations ${ }^{1}$ for received current can be obtained. They are:

From antenna to antenna

$$
\mathrm{I}_{\mathrm{r}}=188 \mathrm{~h}_{\mathrm{s}} \mathrm{~h}_{\mathrm{r}} \mathrm{I}_{\mathrm{s}} / \mathrm{R} \lambda \mathrm{~d}
$$

Antenna to coil

$$
\mathrm{I}_{\mathrm{r}}=1184 \mathrm{~h}_{\mathrm{s}} \mathrm{~h}_{\mathrm{r}} \mathrm{l}_{\mathrm{r}} \mathrm{~N}_{\mathrm{r}} \mathrm{I}_{\mathrm{s}} / R \lambda^{2} \mathrm{~d}
$$

Coil to antenna

$$
\mathrm{I}_{\mathrm{r}}=1184 \mathrm{~h}_{\mathrm{s}} \mathrm{l}_{\mathrm{s}} \mathrm{~h}_{\mathrm{r}} \mathrm{~N}_{\mathrm{s}} \mathrm{I}_{\mathrm{s}} / \mathrm{R} \lambda^{2} \mathrm{~d}
$$

Coil to coil

$$
\mathrm{I}_{\mathrm{r}}=7450 \mathrm{~h}_{\mathrm{s}} \mathrm{l}_{\mathrm{s}} \mathrm{~h}_{\mathrm{r}} \mathrm{l}_{\mathrm{r}}^{1} \mathrm{~N}_{\mathrm{s}} \mathrm{~N}_{\mathrm{r}} \mathrm{I}_{\mathrm{s}} / R \lambda^{3} \mathrm{~d}
$$

where h is the height, l is the length, n is the number of turns of coil, I is current in amperes, $R$ is resistance in ohms, $\lambda$ is wave length. The subscripts $s$ and $r$, refer to the sending and receiving stations respectively. The lengths may be in centimeters, meters, feet or miles, provided all lengths are measured in the same unit. These formulae are for radiation. Induction must be calculated from other formulae.

The height, $h$, is the effective height of the antenna aerial. In the original equation, Ids $\cos \theta / \mathrm{r}^{2}$, we assumed that the value of $\cos \theta$ was unity, and again we assumed that all the current flowed to the top of the aerial. Since we know the capacity at the top is distributed and not bunched, we know that the height will in general be less than the measured height. The effective height is the height of a theoretical aerial with all the capacity at the top and one in which all the alternating current flows ifrom bottom to top wand which will

[^1]produce at a distant point the same field as the aerial in question. However, the distant point must not be too far removed, since there is always a certain amount of absorption by intervening objects which diminishes the intensity at a great distance. The "distant" point should be a few wave lengths removed from the aerial, one wave at least. With small power the distance may of necessity be a fraction of a wave and may be so close that the field is mostly induction field, in which case the exact distance as measured is at best an approximation. There seems to be some confusion in the definition of h . Some use $h$ as the distance from the ground to the "top", others use $h$ as twice this distance, arguing that the earth being a good conductor will reflect and give the same effect as an aerial in free space with the center of the aerial being the point of connection to the ground. Practically this confusion does not make much difference since the height, $h$, must be determined experimentally.

The height, h , is determined by winding a coil or loop of rather large dimensions or diameter, and connecting a tuning condenser and a radio frequency milliammeter in the circuit and measuring the received current when the coil is placed at a distance, $d$, from the aerial. The area is calculated from the formula $\mathrm{A}=\mathrm{n} \pi \mathrm{r}^{2}$ if circular, or nhl if rectangular, n being the number of turns, $r$ the radius of the coil, $h, l$, being the dimension of the rectangle. The resistance can be measured by the resistance variation method if a radio frequency resistance box is available, or by the reactance variation method if the capacity of the condenser is known.

From the received current when the coil is in resonance, and resistance of the circuit, the e.m.f. is calculated. From the e.m.f. and the area the field is calculated. From the field and the distance from the aerial and the current


Fig. 12-When the distance is as great as one-hal! a wave the induction is relatively small and can be neglected. in the sending aerial, the height, $h$, is calculated, assuming the equation which gives the value of the field at the given distance. If the distance is less than $1 / 2$ wave length it will be necessary to take the induction into account. Figure 12 gives the distribution of radiation and induction when $\mathrm{d}=\lambda / 2$. If the distance is less than one-sixth of a wave, the field is mostly induction. It will be well to assume an approximate value of the height, $h$, and calculate both the radiation and the induction for the point. From the relative values one can determine which is the larger, and the error which will be made if one is neglected. Of course, use the larger of the two if one is discarded.

If the values are of the same order, then since they are in time quadrature the effective field is the square root of the sum of the squares. If the ratio of one to the other is one to two, then the received field is $\sqrt{1^{2}+2^{2}}=2.24$. If the smaller value is neglected the error is more than $10 \%$. If the relative values are one to four, then $\sqrt{1^{2}+4^{2}}=4.13$. The error is 3 per cent. If the ratio is one to ten, then $\sqrt{1^{2}+10^{2}}=10.05$. The error is $1 / 2$ per cent. When the ratio is one to seven the error is 1 per cent. Taking other errors into account, one can neglect the smaller without appreciable error when the smaller is not greater than one-fifth of the larger.

If it is necessary to place the coil near the aerial to get enough received current to read accurately, there is a question when the distance, $d$, is measured. The induction from an antenna varies inversely as $\mathrm{d}^{2}$. The center of "gravity" of the field is not at the center of the coil. Unless the distance, $d$, is rather great, there will be an error in placing the coil. If the radiating aerial is a coil, then the induction varies inversely as the cube of the distance and the error will be greater if the distance is measured from the center of the coils. If the distance, $d$, is great compared to the dimensions of the coil, the error will become negligible. Then the question arises, why not make the coil of small dimensions with a large number of turns?

We are trying to get as large value of the received current as possible. Since we are working with a particular frequency there is a maximum value of inductance which the coil can not exceed.

The received current depends upon the area, $\mathrm{A}, \mathrm{n}$ times the area of one turn, and inversely upon the resistance of the coil. A little practical experience with receiving coils will convince one that with a given inductance $A / R$ is a maximum when the coil is made with a small number of turns and large diameter-one turn if practical to handle.

Assuming we have readings which are practically correct, then the height, $h$, of the transmitting aerial can be calculated.

After knowing the effective height of the transmitter the field at other points can be calculated. A receiving antenna can be erected and the received current in the antenna measured as well as the resistance of the antenna. Knowing the current, resistance and field, the effective height of the receiver can be calculated from the formula $\mathrm{E}=\mathrm{I} / \mathrm{R}=\mathrm{Hvh}, \mathrm{h}=\mathrm{I} / \mathrm{RHv}$. The height $h$, can be assumed to be constant for frequencies which do not differ greatly from the frequency used in the above determination. If the determination is made at 300 meters the same value of $h$ can not be used at 40 meters.

If the antenna is a directive aerial such as an $\Gamma$ aerial, the field will be different in different directions. The current should be measured at several points distributed around the aerial and the mean value of h used.

Effective height of a Coil. The effective height of a coil is the theoretical height of an antenna in which the received e.m.f. will be the same as that received in the coil. One in which the received current is the same assuming the resistance of the coil and antenna to be the same. One can get an idea of the effective height of coils from the following table:

Effective Height of Colls ${ }^{2}$

| Turns | Diameter | Frequency | Effective Height |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
| 4 | 44.2 | $4.9 \times 10^{6}$ | 6.3 cm. |
| 4 | 44.2 | 5.4 | 6.9 |
| 4 | 44.2 | 7.8 | 10.0 |
| 2 | 35.0 | 8.6 | 3.5 |
| 2 | 35.0 | 16.1 | 6.5 |
| 2 | 35.0 | 17.0 | 6.8 |

Instead of a milliammeter in the circuit a tube voltmeter can be used to measure the Pd across the condenser.
$\mathrm{Pd}=\mathrm{ILw}=\mathrm{I} / \mathrm{Cw}$, since $\mathrm{I}=\mathrm{E} / \mathrm{R}, \mathrm{E}=\mathrm{PdR} / \mathrm{Lw}=\mathrm{PdRCw}$.
With a tube, voltmeter readings can be made at a greater distance. However, the resistance of the circuit is increased when the voltmeter is used. It is necessary to measure the resistance with the voltmeter connected.

Table Showing Change of Effective Height with Wave Length

| Turns | Area | Frequency | Effective <br> Height | Wave <br> Length |
| :--- | :---: | :---: | :---: | :---: |
|  | $88 \times 88$ |  |  |  |
| 8 | $88 \times 88$ | $.6 \times 10^{6}$ | 7.8 cm. | 500 meters |
| 8 | $88 \times 88$ | 1.0 | 13.0 | 300 |
| 8 | $56 \times 56$ | 10.0 | 19.4 | 200 |
| 2 | $56 \times 56$ | 7.5 | 13.0 | 30 |
| 2 | $56 \times 56$ | 3.0 | 9.9 | 40 |
| 2 |  | 4.96 | 100 |  |

In the above tables it will be seen that the height of a coil is very small. This is partly overcome by the fact that as a general thing it is much easier to get a coil of small resistance than it is to construct an antenna of low resistance. A coil is also usually more portable than an antenna.

[^2]
[^0]:    "Proc. Ind. Acad. Sci., vol. 37, 1927 (1928)."

[^1]:    ${ }^{1}$ J. H. Dellinger. Scientific Papers, Bureau of Standards, No. 354, p. 463.
    $13-43033$

[^2]:    ${ }^{3}$ Friis and Bruce. Inst. Rad. Engs. Proc. 14. 518, 1926.

