

# The Effect of Resistance on Dielectric Constant

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The measurement of the dielectric constant of a pure non-conducting solvent does not offer any particular difficulties and can be carried out by a number of methods. The measurement of the dielectric constant of conducting solutions is still very much a matter of controversy; even the purest solvent obtainable has some conductivity, and the reliability of the measurement decreases rapidly with increasing conductance. The accurate measurement of dielectric constant has become a matter of great importance due to its use in the determination of dipole moments and of the size and structure of a dissolved large molecule or suspended particle.

A survey of the numerical results obtained by different workers shows that considerable discrepancies exist, even in the case of pure organic liquids of low electric conductivity as has been pointed out by Davies<sup>2</sup>. The curves of figure 1<sup>3</sup> show the disagreement in the determination of the dielectric constant of KCl solutions as reported by different observers.

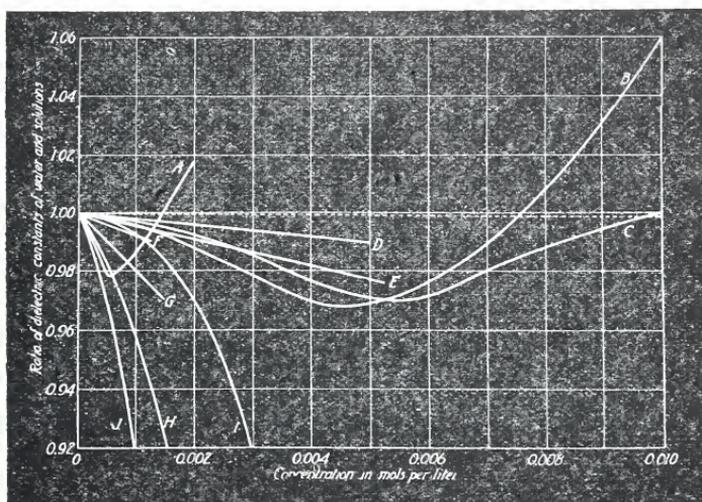


Fig. 1

The above mentioned and other existing discrepancies in the available data indicate the lack of information of the factors influencing the dielectric constant of a conducting solution. If a dielectric is a non-

<sup>1</sup> The work was done at Indiana University.

<sup>2</sup> Davies, R. M., 1936, *Phil. Mag.*, 21, ser. 7, No. 138:37.

<sup>3</sup> Austin, A., 1929, *Phys. Rev.*, 34:308.

conductor it has a zero equivalent series resistance or an infinite equivalent parallel resistance. If it is a conductor the equivalent parallel resistance has decreased, thus producing heat losses in the condenser. This pronounced heating, or loss, in a dielectric subjected to an alternating field was first observed by Siemens in 1864. The effect of the absorption is, therefore, equivalent to a resistance either in series or in parallel with the condenser.

If measurements of the capacity and resistance of a condenser are made by a bridge or a resonance method it is customary to state the results in saying that the system is equivalent to a condenser having no losses and a capacity  $C_p$  shunted by a non-inductive resistance  $R_p$ , or that it is equal to a capacity  $C_s$  in series with a non-inductive resistance  $R_s$ . Either a series resistance or a parallel resistance will change the phase angle between the electromotive force and the current. The equivalent series circuit and vector-impedance diagram for a resistance in series with a condenser is shown in figure 2. The equivalent

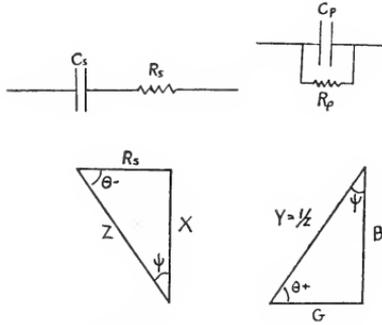


Fig. 2

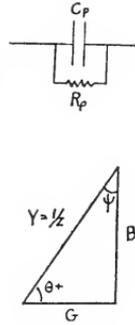


Fig. 3

parallel circuit and vector-admittance diagram for a resistance in parallel with a condenser is shown in figure 3. If the phase angle is near  $90^\circ$ —i.e., if the losses of the condenser are small—either a  $C_p$  or a  $C_s$  measurement will give approximately the same value for the capacitance measurement. However, if the phase angle deviates much from  $90^\circ$ , the values obtained by the  $C_p$  and  $C_s$  measurements will differ materially, as may be seen from the mathematical relationships which follow<sup>4</sup>.

In the equivalent series circuit:

$$\text{Impedance} = Z = R - iX = R_s - i/C_s \omega \tag{1}$$

$$\text{P.F.} = \cos \theta = \cos[\tan^{-1}(1/\tan \Psi)] = \cos[\tan^{-1}(1/R_s C_s \omega)]. \tag{2}$$

In the equivalent parallel circuit:

$$\text{Admittance} = 1/Z = Y = G + iB = 1/R_p + iC_p \omega. \tag{3}$$

$$\text{P.F.} = \cos \theta = \cos[\tan^{-1}(1/\tan \Psi)] = \cos[\tan^{-1}(R_p C_p \omega)]. \tag{4}$$

<sup>4</sup> Reference to any advanced textbook on Alternating Currents as, Hund, August, 1933, *High Frequency Measurements*. McGraw-Hill Book Co., Inc., N. Y., 4-5.

The general equations involving R, G, X, and B relating equivalent parallel to equivalent series circuits are:

$$G = R/(R^2 + X^2) \quad (5)$$

$$B = X/(R^2 + X^2) \quad (6)$$

and relating equivalent series to equivalent parallel circuits are:

$$R = G/(G^2 + B^2) \quad (7)$$

$$X = B/(G^2 + B^2) \quad (8)$$

From the above considerations the following equations may readily be derived:

$$R_p = R_s(1 + 1/R_s^2 C_s^2 \omega^2) \quad (9)$$

$$C_p = C_s/(1 + R_s^2 C_s^2 \omega^2) \quad (10)$$

$$R_s = R_p/(1 + R_p^2 C_p^2 \omega^2) \quad (11)$$

$$C_s = C_p(1 + 1/R_p^2 C_p^2 \omega^2) \quad (12)$$

It is seen from figures 2 and 3 and from equations 2 and 4 that as the power factor, P.F., varies from zero to .707, the resistance component increases in magnitude until the resistance and capacitance components are equal when the phase angle is  $45^\circ$ . As the phase angle varies from  $45^\circ$  to zero the power factor varies from .707 to 1. For a power factor equal to unity, the power loss equals EI or  $I^2R$ . In any measurement when the power factor is greater than .707 the resistance component is larger than the capacitance component, and as the power factor increases toward unity the measurement resolves into the measurement of varying resistance with the capacitative effect becoming negligible. It thus appears that a capacitance measurement holds little meaning in this region and the determination of the dielectric constant from this measurement would hardly seem reliable since the value of the capacity could change considerably without appreciably affecting the measurement.

It is also significant that, from equations 9, 10, 11, and 12, as the power factor varies from zero to .707 the values of  $C_s$  and  $C_p$  vary from equality to values where  $C_s = 2C_p$ , and the values of  $R_s$  and  $R_p$  vary from zero and infinity respectively to values where  $R_s = R_p/2$ . As the phase angle varies from  $45^\circ$  to zero the values of  $C_s$  and  $C_p$  approach infinity and zero respectively. Computations show that a 28% power factor denotes a variation of approximately 10% between the equivalent series and equivalent parallel capacitance. If the dielectric constant of the dielectric of such a condenser were to be computed as it is for a non-conducting solution from these measurements by setting it equal to the ratio of the capacitance of the condenser with the material as dielectric to the capacitance of the same condenser with air as dielectric it would make a 10% variation in the value of the dielectric constant depending upon whether  $C_s$  or  $C_p$  values were used. In the cases of certain conducting solutions the value of  $C_s$  is  $10^3$  to  $10^6$  times larger than

that of  $C_p$ . However, Debye's<sup>5</sup> generalized dielectric should be used when the phase angle deviates much from  $90^\circ$ .

It may be noted that there are cases in the literature where the dielectric constant for conducting solutions have been computed from  $C_s$  or  $C_p$  measurements as above set forth and not from the Debye equation, consequently the above mentioned discrepancies in dielectric constant are present. Also, when measurements are made for solutions where the power factor approaches unity the measurements become largely one of resistance.

Examples of data in which the power factor is greater than .707 are found in the literature. A typical set of data which seems to show this to good advantage, taken by Gore,<sup>6</sup> is shown in Table 1. This data, obtained by the bridge method at 1000 cycles per second and at  $25^\circ \text{C}$ ., is the mean of several measurements of ortho chloro-benzoic acid solutions. This is one of several sets of similar data for solutions of organic acids in ethyl alcohol and benzene. Data taken by Lunt<sup>7</sup> and Rau on the variation of power factor and frequency shows power factors exceeding .707. Similar data reported by Hamburger<sup>8</sup> shows the same type of variation.

TABLE I

Conc. mg.m/1	Data Given			Calculated From Given Data			
	$k_p$	$C_p$ m.f.	$R_p$ ohms	$C_s$ m.f.	$R_s$ ohms	P.F.	$k$ .
0	24.183	.000632	41,696	.0237	40,600	.9865	912.
1	24.384	.000637	9,080	.482	9,068	.9993	185.x10 <sup>2</sup>
5	25.287	.000661	6,151	1.010	6,147	.9997	389.x10 <sup>2</sup>
10	24.740	.000647	6,613	.904	6,609	.9996	348.x10 <sup>2</sup>
15	25.214	.000660	5,144	1.442	5,142	.9998	555.x10 <sup>2</sup>
20	25.970	.000679	4,125	2.184	4,124	.9998	840.x10 <sup>2</sup>
100	34.704	.000907	1,574	11.220	1,574	1.000	4,320.x10 <sup>2</sup>

The capacity of the condenser in air was given as 26.15 m.m.f.

When a dielectric is introduced between the plates of a condenser, it is possible to account for the capacitance increase by considering that the dielectric effectively diminishes the distance between the plates. One resorts to this idea when measuring the dielectric constant of a non-conducting material, as a plate of paraffin, which does not completely fill the space between the condenser plates, in which case the equation for the capacitance is written

<sup>5</sup> Debye, Paul, 1929, *Polar Molecules*, The Chem. Catalog Co., Inc., N. Y., N. Y. 97.

<sup>6</sup> Gore, R. C., 1933. Thesis, Ind. Univ.; Published, 1936, *J. Phy. Chem.*, 40:619-625.

<sup>7</sup> Lunt, R. W., and Rau, M. A. C., 1929-1930, *Proc. Royal Soc., Ser. A.*, 126:226.

<sup>8</sup> Hamburger, F., 1930. *Phys. Rev.*, 35:1121.

$$C = \frac{A}{4\pi \left[ \frac{d-d_1}{k} + \frac{d_1}{k_1} \right]}$$

If a plate of poor-conducting material, such as carbon, of the same thickness,  $d_1$ , is inserted between the plates of the condenser, the capacitance is increased due to an actual decrease of  $d_1$  in the distance between the plates. It would seem, then, that the dielectric constant of a conductor equals infinity. Here

$$C_1 = \frac{A}{4\pi \left[ \frac{d-d_1}{k} \right]}$$

and the resistance of the carbon is a series resistance in the circuit.

Let us consider a condenser filled with a liquid containing dipoles which turn end for end as the potential on the plates is changed, such as when the condenser is subjected to an alternating potential. The heat effect produced by a dipole turning is essentially the same as if the current flowed through the dipole resistance a distance equal to the length of the dipole. The total effective distance would depend upon the number of dipoles present. This would cause an effective decrease of the  $d$  in the same manner as was produced by the carbon plate with the series resistance. The effective distance between the plates would depend upon the length of the dipoles and the number of dipoles present. The resistance would also depend upon the length of the dipole and the number of dipoles present. Then the resistance would be increased in proportion to the concentration of dipoles, or in proportion to the effective length of the dipoles,  $d_2$ , and the capacity of the condenser would be

$$C_2 = \frac{A}{4\pi \left[ \frac{d-d_2}{k_2} \right]}$$

where  $k_2$  remains constant and is the dielectric constant of the solution in the no-loss condition. But  $(d-d_2)$  equals a certain percentage of  $d$ , the percentage depending upon the concentration of dipoles in the conducting solution, which, in turn, is also proportional to the resistance of the solution. Therefore,  $(d-d_2) = d(1-F)$  where  $F$  is the percentage concentration of dipoles. Then

$$C_2 = \frac{A}{4\pi \left[ \frac{d(1-F)}{k_2} \right]}$$

In a condition as just described it is logical to conclude that the resistance is in series with the capacitance. It is suggested, then, that the

dielectric constant of a conducting solution remains constant and equal to its value in the no-loss condition, and that the change in capacitance is due to the diminution of the effective distance between the plates, which, in turn, is proportional to the resistance of the solution.

A liquid containing dipoles may be likened to oil containing particles of conducting material, such as suspended carbon particles. A condition similar to the one just described is illustrated by certain measurements and assumptions made with a condenser filled with oil. Measurements of  $R_s$  and  $C_s$  employing the resonance method at 405,000 cycles per second were made on new S.A.E. number 30 crank case oil which showed very slight conductivity, then on the same oil containing suspended carbon and possibly other conducting particles. The temperature of the oil was 23° C. From these measurements it was found that by adding enough carbon to cause an increase of 1.3 ohms equivalent series resistance a 1% increase of capacity was obtained or,  $k$  remaining constant, a 1% decrease in  $d$  was produced in the constant area plate condenser. These measurements were taken in the low loss condition of the solution, therefore, the equivalent parallel resistance would be high and the values of  $C_s$  and  $C_p$  would be equal, as seen from equations 10 and 12. Then, assuming the relationship between the equivalent series resistance and the decrease in  $d$  to hold, arbitrary values of  $R_s$  were assumed and the values of  $C_s$  computed by dividing the value of  $C$  in the no-loss condition by the corresponding decreasing values of  $d$ . The values of  $R_p$  and  $C_p$  were then computed from equations 9 and 10 respectively. The values of phase angle were computed from the relationship, as seen from figure 2,  $\tan \theta = 1/R_s C_s \omega$ , and the corresponding power factors determined. The partial set of data is listed in Table II. Figure 4 shows the variation of  $R_s$  with  $R_p$ , of  $C_s$  with  $R_p$ , and of  $C_p$  with  $R_p$  in Table II.

TABLE II

$R_s$ Ohms	For % Decrease		$C_s$ (m.f.)	Phase Angle	$C_p$ (m.f.)	$R_p$ Ohms	P.F.
	in $d$	$d_s$					
0.0	0.0	1.00d	.000276	90°	.000276	∞	0.000
1.3	1.0	.99d	.000279	89°57'	.000276	1,570,000	.0009
13.0	10.0	.90d	.000307	89°25'	.000307	125,500	.0102
26.0	20.0	.80d	.000353	88°42'	.000344	50,200	.0227
59.0	30.0	.70d	.000394	87°46'	.000383	25,740	.0390
78.0	60.0	.40d	.000690	82°14'	.000678	4,258	.135
104.0	80.0	.20d	.00138	70°1'	.00103	886	.342
117.0	90.0	.10d	.00276	50°39'	.00165	291	.634
119.0	91.6	.084d	.00329	45°00'	.00165	238	.707
121.0	93.0	.07d	.00394	39°34'	.00160	204	.771
124.9	96.0	.04d	.00690	24°35'	.00119	151	.910
127.5	98.0	.02d	.0138	12°38'	.000660	133	.976
128.8	99.0	.01d	.0276	6°20'	.000338	130	.994
130.0	100.0	.00d	∞	0°00'	0	130	1.000

As may be seen from Table II or figure 4, the value of  $R_s = R_p/2$  and the value of  $C_s = 2C_p$  under the conditions when the phase angle equals  $45^\circ$ , or the power factor equals .707.

The above consideration shows how the dielectric constant of a conducting solution may be considered to remain constant and the variation in capacity all accounted for by the variation of the effective distance between the plates due to the variation in the concentration of dipole molecules or of small conducting particles, which, in turn, is proportional to the variation of resistance of the conducting solution.

The author wishes to express his appreciation to the members of the Physics Department of Indiana University for their kind cooperation and assistance and especially to Dr. R. R. Ramsey under whose guidance the problem was completed.

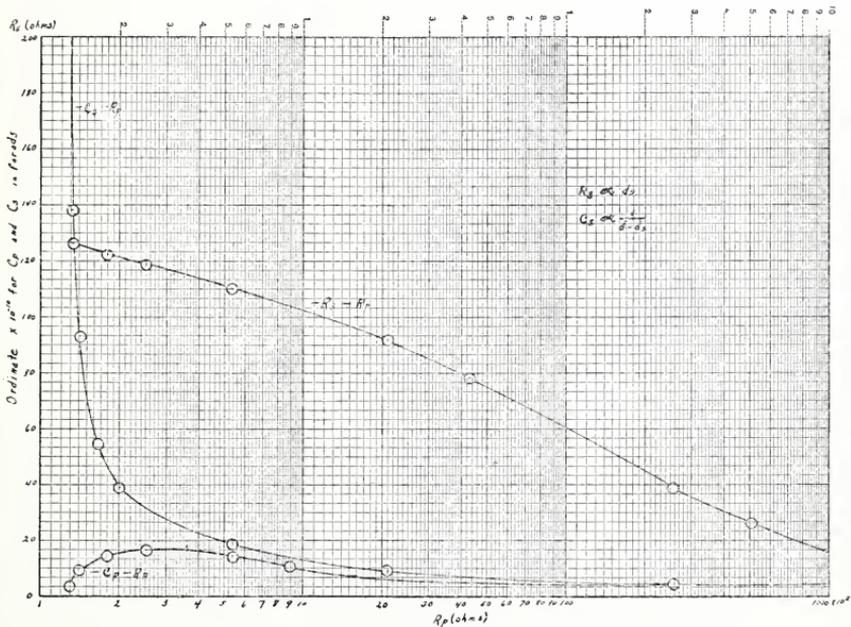


Fig. 4