On the Oscillation of Coupled Circuits

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"In 1892 Nicola Tesla captured the attention of the whole scientific world by his fascinating experiment on high frequency electric current."[1] In 1895 A. Oberbeck[2] published an article "Ueber den Verlauf der Electrichen Schwingungen bei den Tesla'schen Versuchen." Oberbeck's article is given in much detail on page 208 of Fleming's Principles. On page 200 Fleming suggests the coupled pendulum as an analogy to the coupled electric circuits.

Oberbeck set up two equations, one for the primary circuit and a second for the secondary circuit using resistance, inductance, capacity and mutual inductance. Finally, by assuming the resistance to be zero he arrived at two values for the frequency of the coupled electric oscillator. Slater and Frank[3] give practically the same mathematical reasoning, but in a shorter and neater form, in which they set up the mechanical equations for coupled pendulums and suggest the electrical circuits as an analogy. Oberbeck, and Slater and Frank use the terms coupled pendulum and coupled circuits, but do not use the terms close or loose. Apparently they think that the degree of coupling is immaterial.

Slater and Frank neglect friction and leave out the middle or damping term in their equations and say the pendulums are undamped. However, they assume that energy is transferred from one pendulum to the other. Apparently, they assume that because the frictional term is left out of the equations they are dealing with undamped motion in spite of the fact that the amplitudes are continually changing. They are in reality dealing with damped motion since the pendulums alternately deliver and receive energy, one from the other. Their conception of coupled oscillators is the same as that of Oberbeck, in 1895, who was familiar with Tesla coils of that time which were tightly coupled. This seems to be the present conception of many physicists, if we can take Slater and Frank's book as being a fair exposition of theoretical physics.

It is interesting to see how the conception of radio workers has changed since 1895. If one examines the subject of coupling in texts on wireless or radio one finds a continuous change of ideas from 1895 up to the present time. In 1915 Fleming[4] uses the term close and loose coupling. However, the distinction was one of degree. If the coupling

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[1] Fleming, J. A., 1906. The principles of electric wave telegraphy, Longmans Green and Co., London. p. 421.

[2] Oberbeck, A., 1895. Ueber den Verlauf der Electrichen Swingungen bei den Tesla'schen Versuchen, Wied. Ann. der Physik **55**:623.

[3] Slater, John C., and Frank, Nathaniel H., 1933. Introduction to theoretical physics, McGraw-Hill Co. p. 107.

[4] Fleming, J. A., 1915. The wireless telegrapher's pocket book, The Wireless Press, Marconi House, Strand, London W. C. p. 160.

coefficient, $M \div (L_1L_2)^{1/2}$, was less than 0.1 the coupling was said to be loose; if greater than 0.1 the coupling was said to be close. In 1920 George W. Pierce[5] uses the terms "sufficient" and "deficient" and "critical" coupling—critical coupling being when $M^2w^2 = R_1R_2$, deficient being when the mutual inductance is such as to make the first term less than the second, and sufficient being when the first term is greater than the second. Later, radio books use the terms critical, loose and close coupling, the definitions being the same as critical, deficient and sufficient, as defined above.

In the above it is assumed that with critical and loose coupling there is one frequency only, and with close coupling there are two frequencies. Lauer and Brown, first edition, 1920,[6] state that "There are two frequencies when the circuits are closely coupled." In the second edition, 1928,[7] they state that "The case of two closely coupled circuits is very complicated."

Zenneck and Seelig[8] in 1908 say, "Under conditions of close coupling two distinct oscillations, the so-called 'coupling oscillations' are produced in both the primary and the secondary circuits." August Hund[9] in 1935 says, "There is only one frequency in the system in the case of C.W., and that is an essential difference between tube oscillators and the spark-gap oscillator for which two frequencies exist simultaneously." We, in 1941, according to the experiments given below wish to go one step farther, and postulate that all oscillators, tube or damped wave, oscillate at one frequency at any particular instant. The above is a representative set of quotations taken from books written by radio men. Many other quotations could be given. Radio men have been close to experiment and have gradually changed their ideas of coupling and coupled oscillators. Radio engineers, being interested in results, have gradually changed their ideas but the change has been so gradual that no one has noticed or has been sufficiently interested in the change to call attention to it.

From 1895 up to about 1915-20, all wireless communication was by means of spark or damped waves. All experiments of the early times apparently gave two frequencies at first and later when looser coupling was used it was found that only one frequency appeared. After vacuum tube oscillators came into use there was noted a sudden change or flop in frequency if the coupling was too close.

This paper deals with the mode of oscillation of tube circuits, or C.W. circuits, and gives a proposed explanation of the reason for the two

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[5] Pierce, George W., 1920. Electric oscillation and electric waves, McGraw-Hill Co., p. 167.

[6] Lauer, Henri, and Brown, Harry L., 1920. Radio engineering principles, First Edition, McGraw-Hill Co. p. 68.

[7] Lauer, Henri, and Brown, Harry L., 1928. Radio engineering principles. Second Edition, McGraw-Hill Co. p. 202.

[8] Zenneck, J. and Seelig, A. E., 1915. Wireless telegraphy, McGraw-Hill Co., p. 88.

[9] Hund, August, 1935. Phenomena in high frequency circuits, McGraw-Hill Co. p. 202.

frequencies found with spark apparatus. We wish to emphasize the fact that the coupled pendulum experiment is a case of damped motion, even if the frictional terms are omitted, as in Slater and Frank, loc. cit. The pendulum delivers or receives energy and the amplitude changes and we have a case of damped motion.

Suppose we have a simple Hartley tube oscillator, Figure 1, and connect a simple wave-meter circuit (2) by means of a two-coil link circuit (1), making the coupling between oscillator and link (1) loose and the coupling between circuit (1) and circuit (2) variable. By changing the distance between (1) and (2) and reading the radio fre-

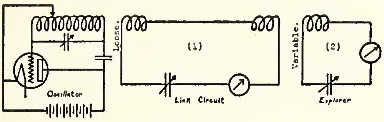


Figure 1.

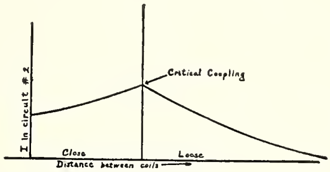


Figure 2.

Fig. 1. Diagram of tube oscillator with link circuit (1) and wave-meter circuit (2). The only way to get a smooth double humped curve with a tube oscillator is to couple loosely the oscillator to the circuit (1) and couple closely circuits (1) and (2) and then vary the frequency of the oscillator and read the current in circuit (2).

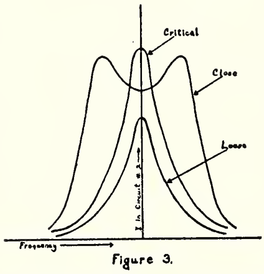
Fig. 2. Curve showing the current output in circuit (2) as the coupling between (1) and (2) is varied by changing the distance between the circuits. All three circuits of Figure 1 are loosely coupled and tuned to the same frequency at first.

quency meter in circuit (2) one gets a curve like Figure 2, the ordinate being the current I_2 and the abscissas being distance. In Figure 2 the current is small with very loose coupling and increases to a maximum at critical, then decreases with closer coupling. It is assumed that all three circuits were tuned to the same frequency when both couplings were loose.

If one sets the circuits with critical coupling, or at some point looser than critical coupling, and runs a resonance curve by changing the frequency of the oscillator one gets a single hump curve. If one sets the coupling between (1) and (2) closer than critical coupling and repeats, one gets a curve with two peaks or humps. Figure 3 shows the two resonance curves. These curves show that a loosely coupled circuit is in tune for peak response at one frequency alone. If the coupling is made close the coupled circuits (1) and (2) are tuned for peak response for two frequencies. If one takes an admittance curve by changing the admittance of circuit (2) one gets a curve with a single hump. It is impossible to get a smooth double humped curve when using a tube oscillator except by means of a procedure which is essentially the same as that given above for the double humped true resonance curve.

If a spark oscillator is used it is possible to get double humped resonance curves, and double humped admittance curves when the coupling is close or greater than critical coupling. Due to the simplicity of the procedure in taking admittance curves, most early curves were admittance curves and were called resonance curves, since for all practical purposes spark resonance curves were the same as spark admittance curves. Apparently, closely coupled oscillator circuits produced or manufactured two frequencies when spark apparatus was used.

If one omits circuit (1) and couples circuit (2) directly to the tube oscillator and increases the coupling beyond critical coupling one gets into trouble since the oscillator suddenly changes frequency. Couple circuit (2) closely to the tube oscillator, and take an admittance curve



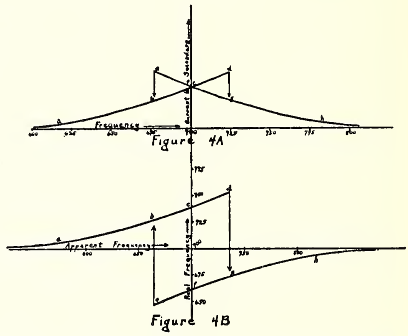


Fig. 3. Resonance curves obtained with loose, critical, and close coupling between circuits (1) and (2). The frequency of the oscillator is changed. With spark oscillators the same type of resonance curves are obtained. With spark oscillators admittance curves of the same shape are obtained when the admittance of circuit (2) is changed.

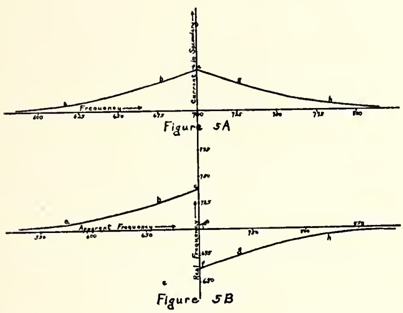
Fig. 4. 4A shows the variation of the current in a circuit which is closely coupled to a tube oscillator as the tuning of the second circuit is changed. The meter suddenly falls to a lower value. 4B shows the change of frequency as the tuning of the second circuit is changed. If the oscillator is originally tuned to some frequency, 700 kc, say, the frequency changes suddenly from a high value to a low value or vice versa. The regions, cd and fe, are metastable. It is possible for a closely coupled tube oscillator to oscillate at two different frequencies with the same circuit constants. But it does not oscillate at two frequencies at the same time.

by changing the capacity, C_2, in circuit (2). Assume that circuit (2) has been calibrated so one knows the frequency of circuit (2) in terms of the capacity of the circuit, the coupling to circuit (2) being very loose when the calibration is made. With close coupling read the current I_2 and the capacity C_2 and one gets a discontinuous curve as in Figure 4A. Reverse the order, decreasing the frequency, and one gets a sudden change of current but at a different value of the frequency.

If one plots apparent frequency as indicated from the readings of C_2, and if one determines the true frequency by means of a wavemeter or frequency meter which is coupled very loosely to the oscillator and plots this as ordinates, one gets a curve like Figure 4B.[10] The apparent

[10] This curve was worked out in our laboratory by Wilbur L. Chenault. Curve 4B is much like a part of the theoretical curve of Chaffee. Chaffee, E. Leon, 1924. Regeneration in coupled circuits, Proc. I. R. E. **12**:347.

frequency as measured in terms of C_2 is plotted as abscissas and the true frequency as measured by the loosely coupled frequency meter is plotted as ordinates. As one increases the apparent frequency one gets the broken curve abcdgh. If the apparent frequency is decreased one gets the curve hgfeba as indicated. Thus it appears that it is possible under certain conditions for the closely coupled oscillator to oscillate at two different frequencies with the same circuit constants. However, the oscillation has only one frequency at any one time. Certain conditions are metastable and others are stable. If the oscillator is oscillating in the region of cd, say, and the power is thrown off and then thrown on



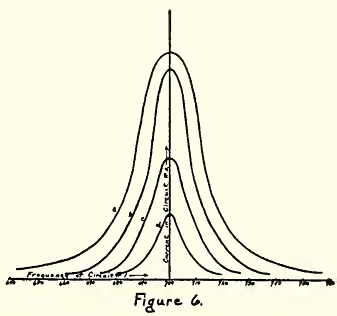


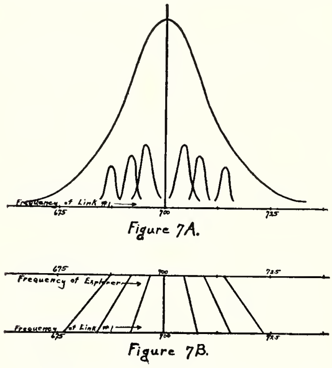
Fig. 5. This is the same as Figure 4 except that the tube has ac potential on the plate. The tube starts and stops oscillating 60 times per second. There is no metastable oscillation.

Fig. 6. Admittance curves with link circuit tuned and detuned. Curve a, link circuit tuned to resonance 700 kc. Curve b, link detuned to 690 or 710 kc. Curve c, link detuned to 680 or 720 kc. Curve d, link detuned to 670 or 730 kc. Maximum response is obtained at the frequency of the tube oscillator which is 700 kc.

again the oscillations will be in the stable region near gf. If oscillating near e and the power is interrupted, the oscillator will be near b. To have the oscillator oscillating in the metastable regions it is necessary to start in the stable region and gradually change to the metastable by changing the condenser C_2. If one has a variable frequency oscillator made by rapidly changing the variable condenser in the oscillator circuit from one value to a second, periodically, then peak response in circuit (2) can be found at two different settings. Of if one has closely coupled circuits as (1) and (2) and exposes them to a rapidly recurring band of frequencies the closely coupled circuits will pick out and emphasize the two frequencies to which the closely coupled circuits are tuned.

In the above experiments it is assumed that the plate potential on the tube is a continuous direct current potential. If the potential on the oscillating tube is that of 60 cycle alternating current the curves obtained are as in Figure 5A and Figure 5B. Since the oscillating tube is stopping and starting 60 times per second there is no metastable condition.

As stated before, the frequency of the current found in the coupled circuits depends on the oscillator and not on the coupling of the circuits. This is shown by the following: if in Figure 1 the link circuit (1) is detuned the frequency as measured by circuit (2) is the same as when (1) is tuned to the oscillator. The intensity of the current I_2 is diminished by the detuning but the frequency is constant. Figure 6 shows admittance curves taken by changing the capacity C_2 in circuit (2) when the link circuit was tuned to the frequencies indicated in Figure 6. The frequency is that of the oscillator which is constant at 700 KC.



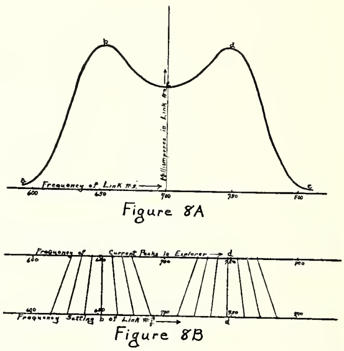


Fig. 7. Figure 7A is obtained with the same general set up as that of Figure 6, except that the oscillator is a spark oscillator. The position of peak response is given by the small peaked curves. Figure 7B, lower line, gives the frequency to which the link was tuned while the upper line gives the frequency at which maximum response was obtained. The oscillator gives a band of frequencies whose maximum electromotive force is near 700 kc. The impedance of the link is a minimum at the frequency to which the link is tuned. The maximum current is obtained at some intermediate frequency.

Fig. 8. This is much the same as Figure 7 except the spark oscillator is closely coupled and the curve has two peaks. The closely coupled circuits emphasize or pick out two frequencies of the band of frequencies which the spark oscillator produces. 8B shows the frequency at which the link circuit is tuned and also the frequency at which the frequency is obtained. 8B corresponds to 7B.

If instead of a tube (C.W.) oscillator a spark or damped wave oscillator is used the admittance curves are as in Figure 7. In this there is a shift with the detuning of the link circuit. However, the peak of the curve does not appear at the frequency to which the link is tuned. Figure 7B, lower line, shows the frequency to which the link was tuned and the upper line shows the frequency at which maximum current was obtained. Figures 8A and B show the corresponding results for a close coupled spark oscillator. The shifts are always toward the peaks.

Explanation of the results. It is seen that a tube oscillator, whether loosely or closely coupled, oscillates at one frequency at any one time. When the tube oscillator is closely coupled to another circuit it is possible under certain conditions to oscillate at two frequencies with the same circuit constants. However, if it is oscillating at one of these frequencies, the metastable frequency, say, it is not oscillating at the stable frequency and vice versa. Since closely coupled circuits are in tune for peak response at two frequencies at the same time, these two frequencies are found provided an oscillator or oscillators in the neighborhood are oscillating at these two frequencies. It is well known that when an oscillator is modulated with a sine wave frequency we have amplitude modulation in which the amplitude of the carrier wave varies as a sine wave with the frequency of the modulation or "voice" frequency. This, as is well known, produces two new frequencies—side bands or combination frequencies.[11] If the modulation consists of two sine waves there are four side bands, two for each frequency. Thus, applying this reasoning to the limit, whenever the amplitude varies new frequencies are introduced or manufactured. In a spark oscillator or damped wave oscillator the amplitude varies logarithmically. This can be thought of as being a number of sine waves and a number of new frequencies are introduced. In other words, the frequency of a damped wave is not single valued but consists of a band of frequencies.[12] A wave meter not having great resolving power or selectivity gives a response at some average frequency where the intensity is a maximum. The detuned link circuit increases the resolving power of the measuring circuit for the frequency to which the link is tuned. The result with a spark oscillator is a response between the frequency of the link and the frequency whose intensity is the greatest. If two link circuits are introduced between the spark oscillators the response of the wave-meter, although being very feeble, is very near the frequency of the link circuits. The curves for this are not shown.

With a closely coupled spark oscillator, since the closely coupled circuit is in tune for two frequencies at the same time, the closely coupled circuit picks out and emphasizes the two frequencies and we have a spark "resonance" curve with two humps. Most of these so-called resonance curves are really admittance curves, since the resulting curves are very much alike and the process of making admittance curves is more simple than that of making resonance curves.

With single frequency tube oscillators it is impossible to make double humped admittance curves, when the circuits are closely coupled. With spark oscillators double humped admittance curves are found, not because the closely coupled circuits manufactured two frequencies but because the damped waves contain a band of frequencies.

Thus we feel justified in making the assertion that all radio oscillators oscillate at one frequency at any one instant. That the apparent two frequencies found in closely coupled spark oscillators are due to the rapid change of frequency due to the flopping from metastable to stable states and also due to the fact that new frequencies or side bands are produced whenever the intensity or amplitude of the vibration

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[11] Bragg, Sir William, 1939, Combination tones in sound and light, Proc. Roy. Inst. **30**, 3:424-433; Nature, **143**:542.

[12] Hazel, Herbert C., 1935. Beat notes, combination tones and side bands, Phil. Mag. **19**:103.

changes. Damped waves are those in which the amplitude is changing rapidly and the effect of a band of frequencies is shown in our apparatus which integrates the effect over a time which is long compared to the instantaneous time over which the frequency is constant.

The mathematics which has been interpreted as showing that closely coupled oscillators manufacture two frequencies applies to closely coupled circuits showing that they are tuned for peak response at two frequencies.