The Addition of Two Frequencies in Audio Apparatus

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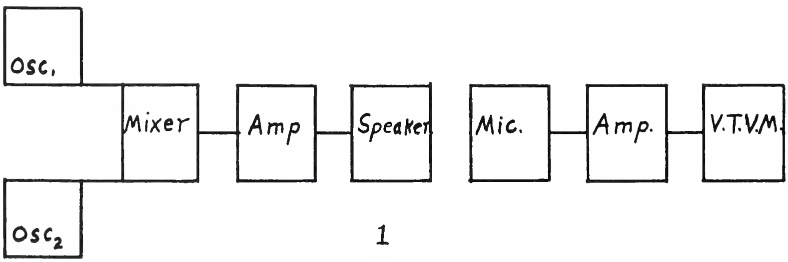
Frequency characteristics of loud speakers are always measured with a single frequency, varied over the audio range. In musical reproduction this is almost never the actual situation as there is at least a soloist with accompaniment or even an entire orchestra or chorus. It occurred to Professor R. R. Ramsey of Indiana University to measure the frequency characteristic with two frequencies impressed simultaneously on a speaker. Preliminary work by Robert Duffy, using a carbon microphone as a pick-up device, indicated that there might be some interesting results; so the problem was taken up under the direction of Professor Ramsey.

The ribbon microphone was chosen as a pick-up device because of its better stability and convenience in not requiring a polarizing voltage. The apparatus in block diagram is shown in Figure 1. It consists of two oscillators kept at different frequencies feeding into an electronic mixer, amplifier, and speaker. The sound from the speaker is picked up by the ribbon microphone feeding a preamplifier, which actuates a vacuum tube voltmeter. In order to make measurements, a mixing device was required which would prevent the output from one oscillator feeding into the other oscillator. The electronic mixer shown in Figure 2 consists of two



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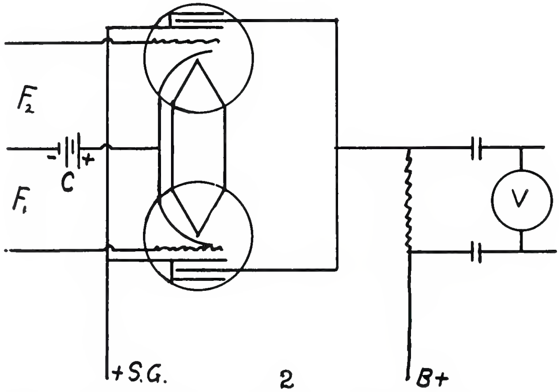

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Figs. 1, 2. Fig. 1. Block diagram of apparatus. Fig. 2. Electronic mixer.

screen grid tubes with grids driven by the two oscillators, respectively, and with the plates in parallel receiving the plate current through a common plate-coupling resistor.

The procedure was to adjust the output of one oscillator to a certain level, as measured through the system by the vacuum tube voltmeter, stop the first oscillator, adjust the second oscillator to a measured level in the same manner, then start both oscillators, and measure the resulting output.

Many measurements were made over the audio range at different levels all showing that the sum of two frequencies is the square root of the sum of the squares of the individual values. Figure 3 shows the result of adding a constant frequency of 800 cycles to a variable frequency from 100 to 6000 cycles. The straight line is the theoretical square root of the sum of the squares. The plotted points show the close agreement with the experimental result.

The theoretical result is obtained as follows:

Let $E_1 \sin \omega_1 t$ be the voltage of the first frequency, and $E_2 \sin \omega_2 t$ be the voltage of the second frequency. Then the effective value of the sum of the two waves is:

$$S = \sqrt{\frac{\int_0^{t_1} (E_1 \sin \omega_1 t + E_2 \sin \omega_2 t)^2 \, dt}{t_1}}$$

where the limit, t_1 is chosen to include an integral number of cycles of both waves.

$$S = \sqrt{\frac{E_1^2 + E_2^2}{2}} = \sqrt{\frac{E_1^2 \text{ eff.} + E_2^2 \text{ eff.}}{2}}$$

The author wishes to suggest the following equation as a general statement giving the sum of two waves of any frequency and any phase difference.

$$S = \sqrt{a^2 + b^2 + 2\,ab \cos \Theta_{\text{av.}}}$$

Where a is the effective value of the voltage of one frequency, b is the other and $\cos \Theta_{\text{av.}}$ is the average value of the cosine of the phase angle between the two frequencies.

Figure 4 shows two waves of the same frequency and in the same phase. Placing Θ equal to zero in the general equation, we have

$$S = a + b.$$

Figure 5 shows two waves of the same frequency but opposite phase. Placing $\Theta = 180$ degrees in the general equation we have the familiar

$$S = a - b$$

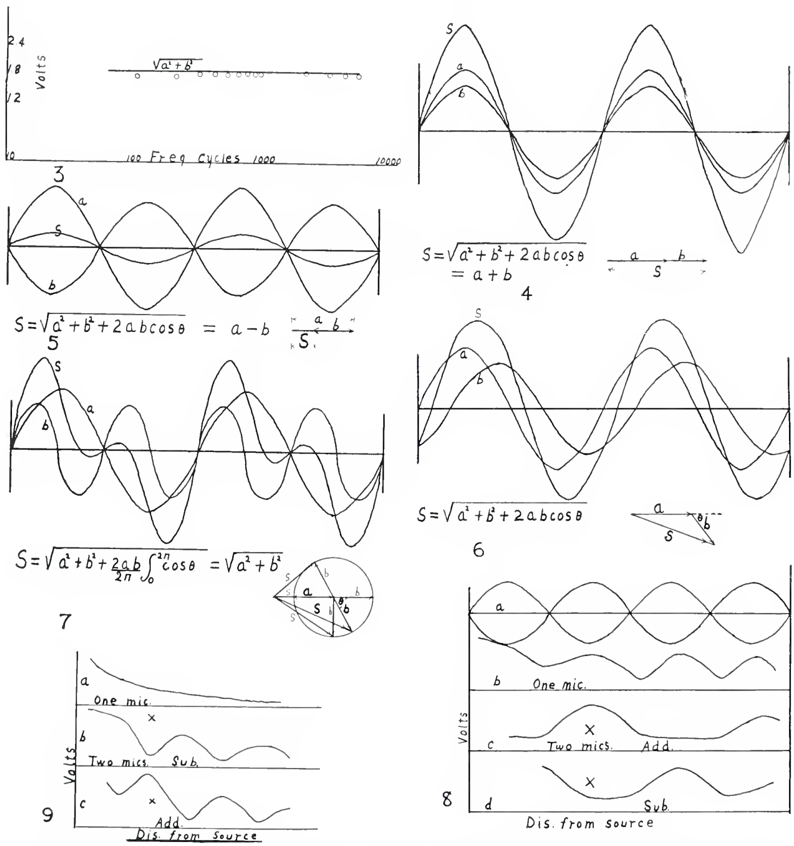
Figure 6 shows two waves of the same frequency, but b lagging a by Θ degrees. The familiar vector addition of two voltages out of phase gives

$$S = \sqrt{a^2 + b^2 + 2\,ab \cos \Theta}$$

which fits the general equation since cos Θ is constant. Figure 7 shows two waves of different frequencies added. In this case cos Θ varies from zero to unity through a period containing an integral number of cycles of both waves so the average value of cos Θ is zero and the general equation reduces to

$$S = \sqrt{a^2+b^2}$$

Each of the above cases was verified experimentally. The sum of two waves in phase and out of phase was measured by adjusting the two frequencies as nearly equal as possible and then watching the needle



Figs. 3-9. Fig. 3. The sum of the voltages from a fixed and a variable frequency. Fig. 4. The sum of two waves of the same frequency and phase. Fig. 5. The sum of two waves of the same frequency, but 180° out of phase. Fig. 6. The sum of two waves of the same frequency, but with 60° phase angle between them. Fig. 7. The sum of two frequencies, one double the other. Fig. 8. Measurements of sound intensity along the standing waves set up between the source and a wall: *a*, measurements with one microphone; *b*, with two microphones in series, one fixed, the other moved; *c*, with the microphones connected subtractive. Fig. 9. Measurements of sound intensity taken in open air without reflection: *a*, measurements with one miscrophone; *b*, with two microphones in series subtractive, one fixed at the cross, one moveable; *c*, with the microphones connected additive.

of the vacuum tube voltmeter swing to maximum and to minimum with slow beats. The maximum reading was the sum of the two waves in phase and the minimum reading was the sum of the two, 180 degrees out of phase.

It was impossible to hold the two frequencies constant enough to check their sum with a constant phase difference between them; so a method was devised to check this case which led to some interesting results. Two ribbon microphones were connected in series. The two microphones were placed in line from a sound source, and, by varying the distance between them, one could add the output from the two microphones from the same sound source, and hence of the same frequency, but with any desired phase difference.

Figure 8 illustrates the use of two microphones in series. Curve *a* is a representation of the standing waves set up between a sound source and a reflecting wall. Curve *b* is the measured intensity from a single microphone moved out from the sound source. The maxima and minima of intensity follow at each half wave length, the maxima being located at the velocity loops and minima at the nodes. Curve *c* shows the result of putting two microphones in series, keeping one fixed at the position marked X and moving the other along the direction of propagation. The maxima now follow at full wave lengths, showing that each successive half wave loop of the standing waves is out of phase by 180° with the one preceeding it. Curve *d* is the result of reversing the connections to one microphone. They are now connected subtractive and will give a maximum reading when they are situated in parts of the standing wave which are 180° out of phase.

Figure 9 shows the results of measurements in the open air where reflections are absent. Curve *a* is the intensity obtained by moving a single microphone away from a source of sound. Curve *b* was taken with two microphones in series subtractive, one at the fixed location the other moved out along the axis of propagation. This shows gradually decreasing maxima and minima at distances of a full wave length. Curve *c* is a repetition with the microphones connected additive.

It will be noted that this gives us a method of measuring the wave length of sound without the aid of standing waves. It is believed that this method of measuring wave length, and hence sound velocity, has not been used before.

Instead of using a constant frequency and moving the microphones apart to locate a wave length, the microphones may be located a fixed distance apart and the frequency varied until a maximum output reading is reached.