# The Great Mathematics Books in the College Curriculum

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Mathematics educators are acutely sensitive to the existence of a double-aspect problem involving mathematical inadequacy. The first has to do with the inefficiency of students in the basic computational skills. The failure of large numbers of servicemen to qualify mathematically has spot-lighted a weakness abundantly evident in the results of standardized achievement tests and in daily classroom experience.

The second, more fundamental aspect, is presented by the conflicting evaluations of mathematics in current studies of education. According to one group of thinkers, mathematics lacks the "liberal" element demanded for inclusion in general education; others completely justify its liberal arts affiliation.

Edward Leen, for instance, in his recent book, *What is Education?* says of British secondary schools:

> "In our national system of education an altogether undue importance is given to the study of mathematics. Its value as a training for the mind is greatly exaggerated. . . . To impose on all students the extensive programme of mathematics that appears in our syllabuses is to lay on them a burden which hampers intellectual progress" (22).

E. K. Rand in the scholarly article "Bring Back the Liberal Arts" published in the *Atlantic Monthly*, June, 1943, unhesitatingly lists mathematics as one of the arts to be recovered. Wriston establishes its liberal arts claim on the ground of the basic disciplines of precision and reflective synthesis (41).

Sister Helen Sullivan, O.S.B., writing in the *Catholic Educational Review*, April, 1944, bases the affirmative answer to the first half of her title question, "Is Mathematics a Liberal Art or a Lost Art?" on the intrinsic values of mathematics as "a mode of thought, a system of philosophy, a means for answering some of the ultimate questions of reality," as well as on its traditional association with the fine arts as furnishing the basis of symmetry, proportion, balance, and perspective in art and architecture, and of harmony in music (39).

Oystein Ore, of Yale, points out that the correct attitude toward the problem of scientific vs. humanistic education, is a serious attempt to bring the two in contact. "Mathematics in the last century," he writes, "has experienced a brilliant growth in conjunction with the natural sciences. It should not be forgotten, however, that mathematics by its traditions and long history belongs to the liberal arts; it is evidently not to be regarded mainly as a technical tool of the sciences" (28).

Reevaluating the aims, the scope, and the content of college mathematics in accordance with an integrated objective of education, and directing learning-teaching procedures toward the attainment of that

objective, will do much to clarify the true nature of mathematics and to insure it an abiding place even in a rigorously defined liberal arts program.

Such a reevaluation was made at St. John's College, Annapolis, seven years ago. Leo Leonard Camp, instructor at St. Mary's College, California, who spent a year and a half in observation of the Restored Liberal Arts program in operation there, is convinced that students from St. John's know more and that they have better disciplined minds; they combine rigor with breadth (9). From the central idea of that program—the great books—this paper takes its orientation. The title, "The Great Mathematics Books in the College Curriculum," calls for explication. It evokes the questions:

1. What constitutes a great book?
2. Which are the great mathematics books?
3. What is their position at present in college curricula?
4. What advantages does their introduction into the curriculum offer—for students, for teachers?
5. How can they be introduced?

Books are great either in themselves or on account of their influence on other books and on the reader and the teacher (33). Six criteria, used by President Barr and Dean Buchanan of St. John's, and by President Hutchins and Mortimer Adler of Chicago, are enumerated by Adler in *How to Read a Book* (1). Except for order of arrangement, they agree with those given in St. John's catalog 1943-'44 (34). Summarized they are: 1) a great book must be a masterpiece in the liberal arts, it must direct those arts of thought and imagination to their proper ends, the understanding and exposition of truth, as the author sees it; 2) it must be immediately intelligible; 3) it must admit many possible interpretations, not ambiguities, but distinct, complete, and independent meanings, each allowing the others to stand by its side, and each supporting and complementing the others; 4) it must raise the persistent and humanly unanswerable questions about great themes in human experience—ultimate questions concerning number and measurement, form and matter, substance, tragedy, and God; 5) it is an enduring best seller; 6) it is always contemporary, intensifying the significance of other books on the same subject.

More succinctly, great books are "simply those that can most effectively induce thinking" (25). In the words of John Erskine, father of the great books idea at Columbia University, "it is the completeness of their outlook which makes great books great" (8).

There is no all-inclusive list of great books in mathematics, but the works treated below and marked with an asterisk in the bibliography at the end of this paper, do qualify according to the standards enumerated. Studied in chronological (rightly called "providential") order, they exhibit the sixth earmark. Each book is "introduced, supported, and criticized by all the other books in the list" (35).

Euclid's *Elements of Geometry* stands at the beginning of the organized study of mathematics in western Europe, not only in time but

in relation to all subsequent developments. De Morgan, in 1848, could deliberately say:

> "There never has been, and, till we see it, we never shall believe that there can be a system of geometry worthy of the name which has any material departures (we do not speak of corrections or extensions or developments) from the plan laid down by Euclid,"

and Heath, to whom the above quotation is due, (15) adds, in 1908, that, despite the valuable recent investigations in the first principles, De Morgan would have no reason to revise that opinion.

Archimedes regularly prefaces his own works with an outline of the relevant accomplishments of his predecessors. In the letter to Dositheus which introduces the *Quadrature of the Parabola*, for instance, after stating that he has used a certain lemma to demonstrate the fact "that every segment, bounded by a straight line and a section of a right-angled cone, is four-thirds of the triangle which has the same base and equal height with the segment," he says,

> "The earlier geometers have also used this lemma; for (by it) they have shown that circles are to one another in the duplicate ratio of their diameters, and that spheres are to one another in the triplicate ratio of their diameters" (4).

The lemma referred to, states that the excess by which the greater of (two) unequal areas exceeds the less, can, by being added to itself, be made to exceed any finite area. This lemma is substantially the same as that derived by Euclid from Definition 4, Book V, and used by him to prove X, 1 and XII, 2 (3).

Apollonius, in his preface to the first of his eight books of *Conics*, speaks of Euclid's not having completely worked out the synthesis of the "three- and four-line locus," a thing impossible without some theorems proved by himself (2). It is in this same book that he makes his chief original contribution to the development of geometry, by relating the conic sections to their diameters and tangents as to the axes of a co-ordinate system.

Oresme's mid-fourteenth century invention of another form of co-ordinate system is the subject of his treatise *On the Breadths of Forms* (29).

The writings of Viete, Cavalieri, Roberval, and especially Fermat figured signally in the perfecting of analytic geometry, but Descartes' *La Geometrie* has been ranked traditionally as the cornerstone of that science. Descartes seems to be the first to have referred several curves of different orders simultaneously to the same set of coordinate axes. He distinctly does this at the beginning of his demonstration of the famous problem of Pappus. (Having given three or more lines in position, required to find a point from which an equal number of lines may be drawn, each making a given angle with one of the given lines, such that the rectangle or parallelepiped on certain of them shall equal, or, at least, bear a given ratio to the rectangle or parallelepiped on the rest) (13).

That Descartes was aware of the historic association of his work is shown by repeated references to the curves which the ancients excluded from geometry. Introducing his own solutions of the problem of Pappus, he says expressly, "neither Euclid, nor Apollonius, nor any one else has been able to solve it completely" (14).

Newton's *Principia* (26), though containing the first specimens of infinitesimal calculus, owes its fame to geometrical analysis of natural phenomena. Its astronomical portions have affinity with Aristarchus's treatise *On the Sizes and Distances of the Sun and Moon* (5), known as the "Little Astronomy," and with Ptolemy's *Almagest* (31), the "Great Astronomy" (18).

Leibniz's masterpiece, expounding the calculus (*Ueber die Analysis des Unendlichen*) does not seem to have an English translation. *The Early Manuscripts* volume (23), however, has all the rigor and classic touches of the larger work.

Lobachevski's *Theory of Parallels*, a non-Euclidean classic, is by the author's own analysis, an attempt to clarify "the obscurity in the fundamental concepts of the geometric magnitudes . . . and to fill a 'momentous gap', to fill which all efforts of mathematicians have so far been in vain." For him, writing in 1840, these "imperfections" explain why geometry, "apart from transition into analytics, can as yet make no advance from that state in which it has come to us from Euclid" (24).

Riemann's *Hypotheses of Geometry* (32), demonstrating unbounded yet finite space, and Hilbert's *Foundations of Geometry* (19) represent distinctly modern advances. Cantor and Dedekind's theory of continuity, presented in *Transfinite Numbers* (10) and *Essay on Numbers* (12), round out the theory of analytic geometry and remedy a deficiency in Euclid by establishing a one-to-one correspondence between points on a line and the real number system (11).

Nicomachus's *Introduction to Arithmetic* (27), more entertaining than scientific, is Euclid's number theory in a diluted form. Peacock's *Treatise on Algebra* (30) takes the arithmetic and algebraic foundations for its two volumes, *On Arithmetical Algebra* and *On Symbolical Algebra*, from the traditional sources. Boole's *Laws of Thought* (7) applies algebra to the laws of logic. These books, then, do have a common bond.

It would be interesting to quote from each of them to establish its liberal arts challenge to thought and imagination, its literary style, simple, yet beautiful, its concern with ultimate truths. But, to quote Descartes, "I shall not stop to explain this in more detail, because I should deprive you of the pleasure of discovering it yourself," if you have not already done so.

Now, for the problem of introducing the great books into the mathematics curriculum. Is it desirable?

Wriston's comment is apropos.

> "If one seeks to stimulate ideas and to develop intellectual resourcefulness, poor books will never achieve those aims. Great minds have produced great books . . . better, by far, a struggle with Plato than easy reading about Plato and his ideas. Good as the faculty are, there are yet greater minds with which the students should make first-hand contact through books" (40).

The best current textbooks are poor when weighed in the balance of the great books. Their entire outlook is at times distorted by a professional educator's desire to publicize some particular nostrum that he has found useful. Organization of material is often the only contribution made by the textbook writer (36). What David Eugene Smith (37) says of the seventeenth century textbooks in elementary mathematics is rather generally true.

> "Their mission thenceforth was to improve the method of presenting theories already developed and to adapt the application of these theories to the needs of the world. From that time on, they ceased to be a great factor in the presentation of mathematical discoveries."

American textbooks, moreover, following the English rather than the continental type (38), tend to give a maximum amount of space to problems and a minimum to the presentation of ideas. Their dominance of college teaching results in poor assimilation and correlation of ideas.

If it is desirable from the student's point of view to substitute the "originals" for second- and third-hand books, it is doubly so from the teacher's. The great books in the college classroom would give to the teacher's knowledge greater depth and freshness. As cooperative learner with his students, he would share the new vistas of thought and inspiration which result from looking at modern scientific achievement with some of its problems unsolved. (cf. Newton's *Principia* p. 507).

There are, of course, very real difficulties involved in bringing the mathematics classics to the students. Since the great books are all of a piece, they do not fit snugly into departmentalized teaching. The teacher must be prepared to go outside the field of mathematics for their interpretation.

To illustrate. The very first definition of the first book of Euclid's thirteen books of *Elements*, variously translated "A point is that which has no part" (16), and "A point is that of which a part is nothing" (17), calls for the ability to search into Greek terminology for linguistic interpretations. It sends one to Plato and Aristotle for earlier conceptions of a point, and, through the later history of mathematics, to account for the current interpretation of that entity. It is precisely in this interlocking of the areas of learning that the liberalizing influence of the great books is felt.

But what of manipulative skill? That is the other half of the theory of using the great books in the classroom, the problem of how to use them. Reading, by the students alone and by the professors and students conjointly, is one phase. In this phase the technique of reading may need cultivation. Adler's *How to Read a Book* and Richard's *How to Read a Page* are helpful directives. Then follow discussion and weighing interpretations, application and reasoned drill in application, and, finally, correlation with the physical sciences.

At St. John's College, where the great books are the core of the entire curriculum, the system comprises, for mathematics: tutorials, seminars, occasional formal lectures, and organized laboratory periods. It is in the latter that students work out the theories expounded in the great books. There is a four-year outline of such experiments for all students.

Special exercises are planned for individuals who need additional drill or insight. Experiments in physics often serve best the purpose of mathematical applications. According to Tutor Bingley, the function of the laboratory exercises is to supplement and explicate the tutorials. The drawing board exercise assigned to clarify the only two irregularities of the moon in Ptolemy, V is typical (6).

In the fourth year, specially prepared manuals and texts in the differential and integral calculus are studied along with the works of Cantor and Lobachevski.

St. Mary's College, California, which has been experimenting with phases of the St. John's program for about five years, studies the first six books of Euclid and assigns nine geometry experiments—chiefly constructions and making models or studying models already made.

How much of the great books theory can colleges adopt under their present organization? Instructors can begin or continue to enrich their own background by setting themselves to the task of reading the books. Surely they can, on occasion, bring into class a master's solution of a problem in hand. Perhaps they can keep a reserve shelf of classics, and, by definite assignments, improve the students' acquaintance with the master minds.

Systematic study in seminars or in colloquia, of the type used at Columbia, is still more effective. Honors courses in the third and fourth years, like those at Chicago, may be feasible. At least in the case of mathematics majors, departments of mathematics can prescribe a sequence of cooperative faculty-student readings, preparatory to the senior comprehensive examination.

The great books approach to college mathematics is justified, not only by the aims of mathematics, but by the objectives of general education.

In mathematics, according to the International Commission on the Teaching of Mathematics (20),

"two ends are constantly kept in view: first, stimulation of the inventive faculty, exercise of judgment, development of logical reasoning, and the habit of concise statement; second, the association of the branches of pure mathematics with each other and with applied science, that pupils may see clearly the true relations of principles with things."

And genuine education implies the cultivation of these same habits of thought as elements of the individual's power to deal successfully with life, in its speculative as well as in its practical aspects (21).

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