# Preliminary Theory of an Electrical Computer for Mean Deviations

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### Abstract

Computation of mean deviations can be expedited by use of an electrical device, in which mean values are attained by paralleling condensers upon which the initial value is proportional to the initial datum; and deviations are arrayed in correct arithmetical sign for subsequent meaning by use of diode bridge rectifiers. Standard electrical components, available for all major functions, permit relatively easy construction and maintenance.

### Introduction

The laborious nature of the computation of a mean deviation, either "by hand," or on standard office computing machines, has inhibited the employment of this indicative statistic in many types of climatic work, as well as in other fields of investigation.

Because this computation, as normally performed, requires the rehandling of each primary datum, chances for arithmetical error are rather great. Repeated changes of operational procedure, during the computation of a single mean deviation, are confusing to all but the most skilled and alert computers, so that, even with a standard computing machine, errors tend to be numerous, requiring extensive recomputations, and raising the cost of the computation.

Within the last two decades, a part of the labor and proneness to error has been removed from this computation by the use of punch-card machines, notably those developed by International Business Machines Corp. By use of such machines, rehandling of a datum consists only of replacing the card in the machine, and, if the card is correctly punched initially, "transfer error" is virtually eliminated.

Despite this very real improvement, however, the computation of mean deviations remains a problem, for which an electrical computer offers a possible solution. This electrical computer, as here outlined, is about midway in complexity between a standard adding machine and the Bell Laboratories' relay computer. Because all fallible components are used in an "all or nothing" capacity, and are so connected that they either function or do not function, the problem of tube failure, so serious in some of the modern electrical computers, is minimized. By judicious use of suitable test problems, trouble, when it occurs, can be localized in a matter of a few seconds; and by employment of "plug in" units, trouble, when detected, can be corrected in a matter of a few minutes, usually as simply as a burned-out lamp bulb is replaced.

### Mathematical Processes

To determine a mean deviation, as the process is normally performed, the following steps are necessary:

1. Sum the initial data.
2. Divide this sum by the number of items summed. This is the arithmetical mean.
3. Determine the absolute value of the difference between the arithmetic mean and each individual datum. These differences are the absolute individual deviations.
4. Sum the absolute individual deviations.
5. Divide the sum of the absolute individual deviations by the number of deviations summed. The result is the mean deviation.

This operational breakdown shows plainly the relatively large number of operations involved in the computation of a mean deviation. As the number of errors increases at a rate approximating the square of the number of items (figures) concerned, it is obvious that an operational simplification is much to be desired.

Formulas commonly used in computation of a mean deviation are presented in Fig. 1.

## FORMULAE

Individual values = X

Number of values = N

($\Sigma$="the sum of")

$$\text{Arithmetic mean} = \bar{X} = \frac{\Sigma(X)}{N}$$

$$\text{Individual deviations} = x = |X-\bar{X}| = \left|X-\frac{\Sigma(X)}{N}\right|$$

$$\text{Mean deviation} = MD = \frac{\Sigma|x|}{N} = \frac{\Sigma|X-X|}{N} = \frac{\Sigma\left|X-\frac{\Sigma(X)}{N}\right|}{N}$$

$$\text{Variability index} = \frac{MD}{\bar{X}} = \frac{\Sigma\left|X-\frac{\Sigma(X)}{N}\right|}{\Sigma(X)}$$

Fig. 1. Formulae used in computing mean deviation by conventional methods.

## ELECTRICAL MEANING

### Introduction

Determination of mean value can be made without adding and then dividing by a number of mechanical, hydrostatic, and electrical devices, most of which are both inordinately bulky and quite slow in operation. Several electrical devices, however, show definite promise, in that they are small, inexpensive, not prone to variation, and "instantaneous" in their operation. The best of these devices is a condenser bank, which performs automatically, in microseconds, a mathematical process requiring from several seconds to several days by more familiar methods.

## Fundamental Theory

A condenser is an electrical storage device, consisting of two plates separated by a dielectric. When the potential difference (volts) between the plates of a condenser of known capacity (farads) is also known, the energy stored (coulombs) is shown by the formula:

$$Q = CE \text{ or } E = \frac{Q}{C}$$

In which:

$Q$ = charge on condenser (coulombs)
$C$ = capacity of condenser (farads)
$E$ = potential across condenser (volts).

When several condensers, charged to voltages $E_1$, $E_2$, $E_3 \ldots E_n$, respectively, are connected in parallel, their capacities add directly, and the total charge is the sum of the individual charges, or:

$$\Sigma C = C_1 + C_2 + C_3 \ldots\ldots\ldots C_n$$

and:

$$\Sigma Q = Q_1 + Q_2 + Q_3 \ldots\ldots Q_n = C_1E_1 + C_2E_2 + C_3E_3 \ldots\ldots C_nE_n$$

Now, as $E = \frac{Q}{C}$, the resultant voltage, $E_r$, across the condenser bank, is:

$$\frac{\Sigma Q}{\Sigma C} = \frac{C_1E_1 + C_2E_2 + C_3E_3 \ldots\ldots C_nE_n}{C_1 + C_2 + C_3 \ldots\ldots\ldots\ldots C_n}$$

and, if all capacities are the same (and all equal to $C_1$), then:

$$Er = \frac{C_1 (E_1 + E_2 + E_3 \ldots\ldots E_n)}{NC_1} = \frac{\Sigma E}{N}$$

which is the mean of the individual voltages. Additional details and fuller derivations may be found in any standard physics text, such as Smith (3).

Thus, by charging a group of identical condensers to voltages proportional to the various items, and then connecting the condensers in parallel, the mean of the various items may be determined from the voltage across the condenser bank.

## Practical Application

One method of "putting this theory to work" is shown in Fig. 2. Here, the meaning device consists of a voltage source, a group of identical condensers, and a voltage measuring device.

In operation, the condensers are charged, one by one, from the voltage source, the applied voltage being proportional to the numerical datum concerned. When all condensers have been charged (or eliminated from the circuit by means of disconnectors—$D_1$, $D_2$, $D_3 \ldots\ldots D_{n-1}$), they are all connected in parallel, by closing of the meaning switch, and the voltage across the condenser bank is measured ("drainless" V.T.V.M.). This voltage is the arithmetical mean of the initial charging voltages.

When the meaning switch is opened, all condensers have and retain the same voltage, which is the arithmetical mean voltage determined

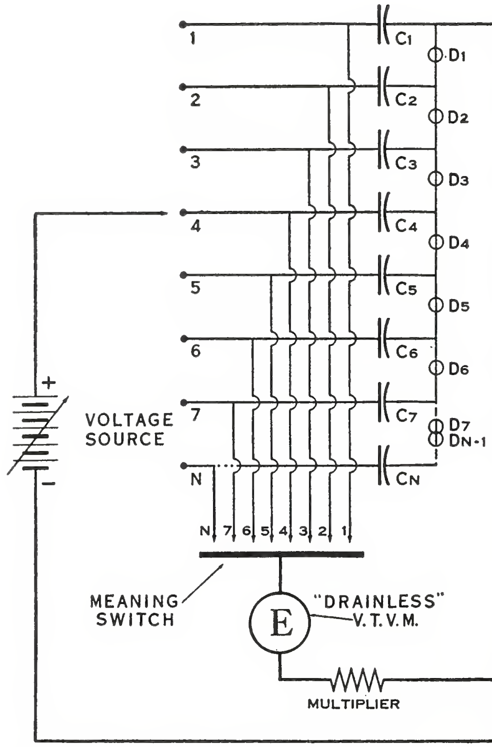


Fig. 2. Summary diagram of an electrical device for determining a mean value.

above. The meaning process is used twice in determining mean deviation: once to determine mean value, as outlined above; and a second time, with different components, to determine the mean value of the individual deviations.

## ELECTRICAL MEMORY

### Introduction

Determination of mean deviations requires that the individual data be compared with the mean of those same data, and that the absolute value of the difference, if any, be isolated and retained for further arithmetical treatment.

Various electrical and electromechanical devices can be made to retain a position, charge, or type of oscillation. Among those in common use are a ratchet-type stepping switch, a simple condenser, and a crystal-controlled mercury tank.

The simplest possible electrical memory—a condenser—is most suitable for use in this device.

## Fundamental Theory

If two identical condensers are charged to a voltage proportional to each datum; and only one of these is connected into the meaning circuit, the other meanwhile being electrically isolated; the charge on the first, or meaning, condenser, after the meaning operation, will be the mean of all the data concerned; while the charge on the second condenser will remain unchanged.

## Practical Applications

One method of retaining a charge electrically, or "remembering" a datum, is shown in Fig. 3, in which two identical condensers are charged simultaneously from each charging position. One of these condensers, Cm, is subsequently connected into the meaning circuit: the other, Ch, not so connected, is the "memory" element.

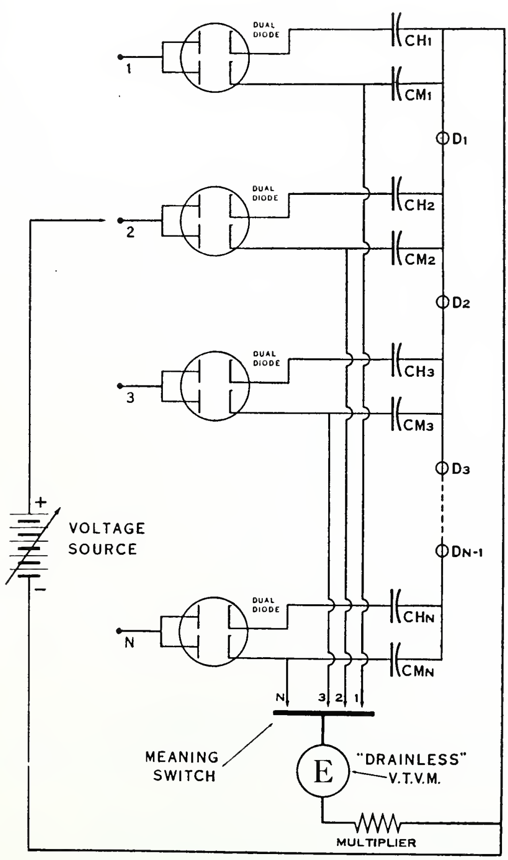


Fig. 3. Summary diagram showing one method of "memorizing" a datum while at the same time finding the mean value of a group of data.

Charging of both condensers in each pair is accomplished through a dual diode, which functions as an electrical check valve, to prevent circuit interaction. Were it not that the diode has an additional function, to be outlined later, the same operation could be performed more economically by a dual contactor on the charging switch.

## DETERMINATION OF DEVIATION

### Introduction

At this stage in the description, electrical methods of determining the mean value have been outlined, as well as a means of "remembering" the original individual data. After the meaning switch of Fig. 3 has been closed and reopened, the voltage across all the condensers designated Cm will be the same, and will be the arithmetical mean of the values to which they were initially charged. Voltages across the condensers designated Ch remain the voltages to which they were initially charged. The difference between these voltages is the deviation in each case, but, as some of the individual items are larger than the mean, and others are smaller, the absolute value of this difference, which is the desired figure, must be determined.

Because arithmetical sign of the difference between the mean value and the individual value is shown by the electrical polarity of the charge between the outer terminals (left-diode cathodes—Fig. 3) of the two condensers in each pair, determination of the mean of the absolute values of these differences, which numerically equivalent to the individual deviations from the mean, requires that they all be arranged in the same polarity, regardless of the polarity of the difference charge share by Cm and Ch.

### Fundamental Theory

When two identical charged condensers are connected "back to back" (i. e. negative terminals together) as in Fig. 3 ($Cm_1$ and $Ch_1$, for example), the voltage across the free terminals (left, Fig. 3) will be the difference between their terminal voltages, with the polarity positive at the terminal of the condenser charged to the highest value.

In ordinary machine computation, the absolute value of the deviation is determined by the operator, who subtracts the lesser value from the greater in each case, so that the result is always positive. It is, of course, possible to perform a similar process electrically, by means of a polarized relay, for example. A simpler means, however, is to transfer a portion of this charge, representing the deviation, to another condenser, of known capacity, by use of a conventional bridge rectifier (1, p. 549 or 4, 479-480) which automatically polarizes the transferred charge.

When two charged condensers are connected in series opposition, or "back to back", the charge available for transfer is the difference between the charges, and, if the condensers are of like capacity, the charge is $C(E_1—E_2)$. When this capacitative pair is connected to a third condenser, by means of a bridge circuit, the condenser having the numerically greater charge will discharge into the third condenser, through the rectifier, until the terminal voltages on both sides of the rectifier are equal, or:—

$$C_1(E_1-E_2)-C_3E_3=C_1E_3$$

and, as the charge divides according to the capacities of the condensers:—

$$C_3E_3=\frac{C_1(E_1-E_2)}{C_1+C_3}$$

and:—

$$E_3=\frac{C_1(E_1-E_2)}{C_3(C_1+C_3)}$$

This permits a rather wide range of condenser values. If, for convenience in construction and computation, $C_1=C_3$, then:—

$$E_3=\frac{E_1-E_2}{2}$$

### Practical Application

One of several workable methods of isolating a charge corresponding to each individual deviation from the mean value, of arraying these deviations in correct polarity, and of preserving the charges for following operations, is shown in Fig. 4. Here, the individual condensers having been charged, and the meaning process performed, all of the condensers designated Cm are charged to the mean value, which has been measured and recorded (if desired), and the holding condensers, designated Ch, retain the charges representing the original individual data.

When the switch arms connected to the deviation condensers, designated Cd, are moved from "center" to "down" position, the bridge rectifier circuits are completed, so that a known portion of the charge difference between condensers Cm and Ch is transferred to Cd. Although this transfer is theoretically instantaneous, it actually requires a few microseconds, because the diode circuit is resistive.

When the switch arms are again centered, charges on the individual deviation condensers (Cd) are proportional to the individual deviations from the mean value, the exact proportion being a function of the various capacities involved, as previously outlined.

### Meaning of Deviations

Mean deviation is determined in the same manner as mean value, but is found by connecting all of the deviation condensers in parallel, and then measuring the voltage across the bank. This process is performed by moving all of the switch arms connected to the deviation condensers (Cd) to "up" position, and then measuring the voltage across the bank by means of the "drainless" V.T.V.M. By use of a second instrument multiplier ("Multiplier 2", Fig. 4), the mean deviation, as indicated

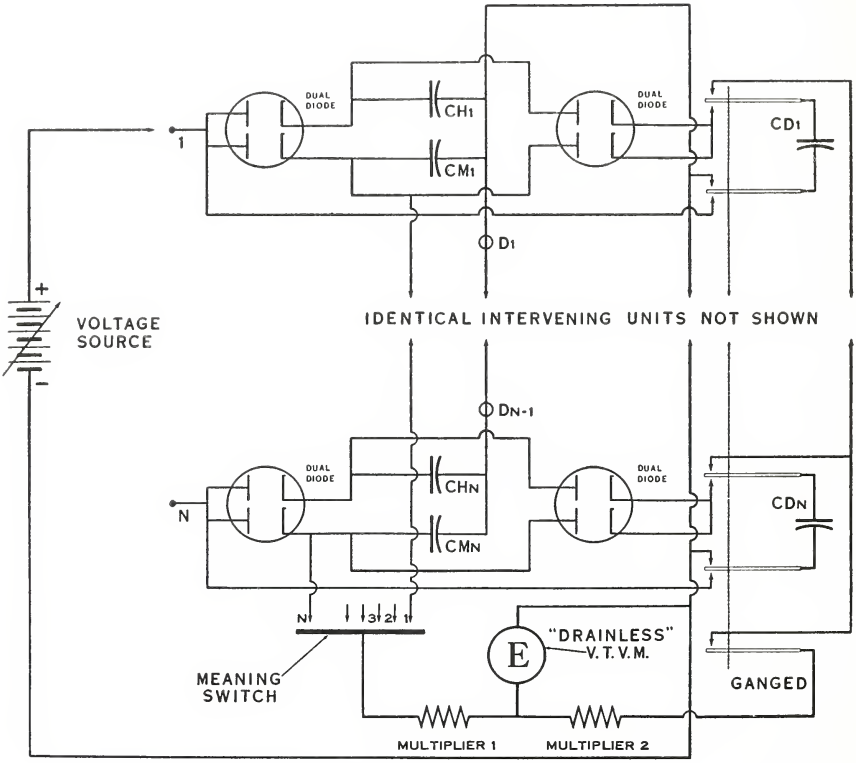


Fig. 4. Circuit for transferring deviation charges from mean and holding condensers to deviation condensers.

by the instrument, can be in the same terms as the mean value previously determined, thereby eliminating an additional computation (multiplication of the measured mean deviation voltage by a constant factor).

## Clearing of Machine

After a computation is completed, it is necessary to clear the machine before a new sequence is started. This process is analogous to erasing a blackboard, and is done by "dumping" all of the charges residual upon the various condensers. Most convenient method of doing this is to short-circuit all of the condensers, which can be done, with the circuit of Fig. 4, by grounding the common of the meaning switch to battery—for the condensers Cm; commoning and grounding the upper cathodes of the left diodes for the condensers Ch; and, with the switch arms in "up" position, connecting switch arm at "multiplier 2" to battery— for the condensers Cd.

Several other charge dumping methods are possible. Choice of the exact method is unimportant, and depends largely upon the type of switching mechanism used.

### Voltage Supply

Sources of potential for this computer, and for others of similar design, must be very stable, and not subject to either slow drift, or "poltergeist" variations in output potential. At the time of this writing (spring, 1949), several supplies having an output of 250 volts, with an accuracy of plus or minus one millivolt, are available (2, p. 258). Greater accuracy is attainable, but is quite costly at the time of this writing. The simplest way of converting output of a stabilized supply into the various voltages needed to represent individual data is by use of a decade voltage divider connection, which is ideally suited for key-punch operation.

### Voltage Measurement

Measurement of the voltage across the condenser banks after the various meaning processes is best performed by use of a vacuum tube voltmeter, which can be designed to remove an entirely negligible portion of the charge during the measuring process. Various commercial types, of high accuracy, are available; some being designed to make a printed record of the measurements, on familiar "adding machine tape".

With some instruments, the power supply for the V.T.V.M. can be taken from the charging supply in such a manner that long-term voltage drifts are "cancelled out". When this is done, the exact output voltage of the supply need no longer be known, and the inserted and measured charges will be in approximate volts, rather than in standard volts.

It is also possible, by means of a vacuum tube bridge device, to determine the voltage across the condenser bank in terms of the supply voltage (or any known fraction of it); and several printing voltage comparators, of high accuracy and dependability, are now (1949) commercially available.

### Adjunct Equipment

Various other items of equipment called for by the theoretical design here presented are all available commercially. The condensers can be standard low-leakage elements. Some now available have a loss of less than 0.1 percent of charge per hour, at full rating. As the time required for a single mean deviation computation is usually less than two seconds per unit, losses due to condenser leakage are negligible in most work.

Switching equipment of phenomenal dependability has been made for the communications industry for several decades, and is ideally suited for use in computing mechanism. Stepping switches and similar devices of this manufacture will commonly operate from 100,000 to more than 5,000,000 times without a failure. This equipment is very costly, compared to the items which perform similar functions in pinball machines and juke boxes, but, on a per operation basis, it is quite inexpensive.

Rectifiers used in the bridge circuit are standard dual diodes, of which the 6H6 is a familiar example. These have a nominal life of about 1,000 hours; and an actual life, in some instances, of more than 30,000 hours.

## Economic Factors

A device of this general type, designed with 32 positions and an accuracy of five significant figures, such as is needed for meteorological and climatic computations, costs somewhere between $5,000 and $10,000, initially. Maintenance cost per operation, at least theoretically, is about the same as for a standard mechanical computing machine. Operator time required for computing each mean deviation is, or can be made, about one third that required for standard methods, and chances for operator error are about quartered.

In consequence, construction of a device of this type will be worth considering only when a very large number of computations must be made each year; or where speed of computation is essential.

Although use of "plug-in" units expedites repairs and maintenance, an electrical computer is usually not very satisfactory in locations where a competent maintainer is not available. Thus, although this electrical computer will theoretically expedite the computation of mean deviations, its construction is inadvisable unless and until it can be used on a full time basis in some location where dependable maintenance (and unfailing power supply) is available.

## Summary and Conclusions

The foregoing theoretical outline shows that many troublesome computations can be expedited by use of electrical computing mechanisms. Determination of arithmetical means, and of mean deviations, can be performed electrically by straightforward application of well-known principles, employing standard mass-produced components.

Economically, however, use of electrical computers, even though they shorten work time and reduce chances of error, will probably and should be restricted to large computing centers, where the machines are operated nearly full time, and where adequate stable power supplies and skilled maintenance are available at all times.

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