# MATHEMATICS

Chairman: W. H. CARNAHAN, Purdue University

V. Hlavaty, Indiana University, was elected chairman for 1951

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## ABSTRACTS

**On the line-space interpretation of the differential geometry of curves in Klein space.** L. K. FRAZER, Indiana University.—The Plücker coordinates of a line are six numbers which satisfy a projectively invariant quadratic equation. This yields at once the obvious mapping from the lines in ordinary space to points of a four-dimensional quadric, called the K-quadric, which lies in a five-dimensional linear space, called K-space. (Klein-space).

This paper utilizes the above mapping together with the polarity in K-space induced by the K-quadric and the polarity in three-space induced by a linear complex to find the interpretations in three space of the invariants of a K-curve. Interpretations of such projectively invariant properties as the minimal linear imbedding space and the degree of a K-curve and some examples such as K-conics are considered.

The results obtained have the advantage of reducing the differential geometry of these higher dimensional space curves from one of abstract analytic descriptions to one which can be easily visualized.

The notation involved and basic definitions of the Line-geometry follow that of the book, "Differentielle Liniengeometrie" by Dr. V. Hlavatý.

**Skepticism and the fifth axiom of Euclid.** VACLAV HLAVATY, Indiana University.—The noneuclidean geometry may be thought of as a geometry whose underlying group is a special subgroup of the general projective geometry. The corresponding representation is particularly fit to point out the failed "proofs" of the fifth axiom of Euclid, as well as the ideas of Lobachevski. The paper finishes with the Beltrami realization of the noneuclidean geometry.

**Equations with cyclic Galois groups.** RALPH HULL, Purdue University.—An outstanding unsolved problem of the Galois Theory is that of determining equations with prescribed groups. (N. Tschebotarow, Commentarii Mathematici Helvetici, Vol. 6 (1934), pp. 235-283; in particular § 2). A related problem is that of determining extension fields with prescribed groups of automorphisms relative to a given ground field. For abelian groups the latter problem falls within the scope of the class-field theory. In particular, when the ground field is the field of rational numbers, the abelian extension fields are cyclotomic fields, that is, subfields of fields of nth roots of unity. The present paper is a preliminary report of an exploratory nature on a method of attacking the first mentioned problem by way of the class-field theory for the rational ground field.

**A testing program for freshmen mathematics students in Valparaiso University.** C. O. PAULEY, Valparaiso University.—Purpose of the Testing Program: To ascertain preliminary knowledge as to the abilities of entering-freshmen mathematics students; to "screen out" the weaker ones and provide remedial measures for improving their mathematical skills. Test was administered to *Liberal-Arts* freshmen who intended to enroll in MATHEMATICS 51 (Algebra and Trigonometry). "Failure" on the test did not bar a student from enrolling in beginning mathematics course. Those making a very low score were requested to enroll in a remedial course of three hours *concurrently* with the regular four-hour course.

*Results of Test Administered in September, 1950:* 78 freshmen were given the test. It consisted of 24 items in algebra—13 items on "Computational Skills", 11 items on "Numerical Reasoning", (Multiple-Choice Type).

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| Mean score | 11.65. |
| Range of scores | 3 to 22. |
| Standard Deviation | 4.47. |

12 of the 78 students (approximately 15%) made a score of 7 or less. These were requested to enroll in the remedial class. Others may enter during the semester whenever it is thought advisable.

*Future Procedure:* The final semester grades in Mathematics 51 will be observed and compared with the grades made on the preliminary test.

A similar test will be given at the close of semester work in algebra (approximately 11 weeks devoted to algebra) and comparisons made with preceding grades.

**A note on continuity.** J. CRAWFORD POLLEY, Wabash College.—The customary definition of the continuity of a single valued function of a variable x as x varies over the interval (a, b) does not express the fundamental analytic concept of continuous variation associated with the Cartesian graph of the function. The equivalence is not so apparent as the majority of textbooks seem to indicate. Because of the importance of the concept good teaching demands that the equivalence be explicitly demonstrated. This paper presents such a demonstration. Based on the customary definition of continuity over a closed interval (a, b) and the division of the interval into sub-intervals over each of which the function is increasing, decreasing, or constant, it is shown that the conditions for continuous variation are satisfied.

**A hydrodynamical corollary of Julia's Theorem.** JAMES B. SERRIN, Indiana University.—The purpose of this note is to present a short and easy proof of an important theorem in hydrodynamics first stated in 1938 by the Russian mathematician, M. Lavrentieff.

Consider two curves $S'_1$ and $S'_2$ on the closed Riemann sphere, passing through 0 and $\infty$ and having common tangents at these points. We may suppose the tangent at $\infty$ is 0. Let $S_1$ and $S_2$ be the images of $S'_1$ and $S'_2$ in the z-plane, and let $D_1$, $D_2$ be the plane infinite regions bounded respectively by $S_1$ and $S_2$ and containing the point $-i\infty$. In addition let $D_1$ contain $D_2$. Consider two steady irrotational flows of an ideal fluid, one occupying $D_1$ and the other occupying $D_2$, and suppose that both flows have the uniform velocity U at infinity. Let $V_1$, $V_2$ be the velocities of the flow at 0 in the regions $D_1$, $D_2$, respectively.

*Theorem.* With the suppositions above,

$$V_1 \geqslant V_2,$$

and the equality holds if and only if $S_1$ is identical with $S_2$.

Numerous applications of this theorem may be given. I shall confine myself to presenting one very simple one, and to indicating certain others.

**Variant of the Theorem of Humbert.** DAVID E. VAN TIJN, Indiana University.—An answer will be provided to this question

1. For which paths defined in a parameter t by elementary functions is the arc length an elementary function of t?

**A generalization of Descartes' Rule of Signs.** EUGENE USDIN, Purdue University.—Various generalizations of Descartes' Rule of Signs have been established to determine the number of zeros of a polynomial in parts of the complex plane other than the real axis, and to treat polynomials with complex coefficients. They have in common the estimation of the number of zeros by means of the number of changes of sign or argument in the sequence of coefficients. In this paper the following generalization is established:

The number p of zeros of the polynomial $a_0+a_1 z+\dots+a_n z^n$ in the sector $|\arg z| \leqslant \gamma$ is given by the formulae

$$p=\left\{\frac{2n\gamma}{\pi}\right\}-\left\{\frac{n\gamma}{\pi}\right\}+\mu-2k \qquad \text{if } n\gamma \not\equiv \frac{\pi}{2}\ (\pi)$$

where $\mu$ is the number of changes of sign in the sequence

$$\left\{a_0, a_1 \cos\gamma, \dots, a_n \cos(n\gamma)\right\}$$

and k is a non-negative integer.

$$p=\left\{\frac{n\gamma}{\pi}\right\}+\nu-2k' \qquad \text{if } n\gamma \not\equiv 0\ (\pi)$$

where $\nu$ is the number of changes of sign in the sequence

$$\left\{a_0, a_1 \sin\gamma, \dots, a_n \sin(n\gamma)\right\}$$

and k' is a non-negative integer.

**On the impossibility of the trisection of an angle.** NELSON P. YEARDLEY, Purdue University.—By giving a counter-example, we show that it is impossible to trisect all angles.

First we establish the proposition that it is impossible to construct a line whose length is a root of a cubic equation with rational coefficients having no rational root. Then we show that to construct a 60° angle we must construct a line whose length is the root of the equation

$$x^3 - 3x - 1 = 0$$

which has no rational roots.