MATHEMATICS

Chairman: V. HLAVATY, Indiana University

E. Hopf, Indiana University, was elected chairman for 1952.

ABSTRACTS

Remarks on a theorem of Hajós. W. GUSTIN, Indiana University.—In 1942 the Hungarian mathematician Hajós proved a famous conjecture of Minkowski by means of a new theorem concerning finite abelian groups. We shall discuss this group—theoretic result of Hajós and propose some problems suggested by it.

A remark on the unified theory of special functions. M. BRENT HASLAM, Indiana University.— Truesdell's theory of special functions is based on the properties of the solutions of the differentio-difference equation

$$F(z, lc + 1) = \frac{\partial F(z, \propto)}{\partial z} \tag{1}$$

In this note we consider the solutions of (1) which also satisfy a second order differential equations

$$\frac{\partial^2 F(z, \propto)}{\partial z^2} + P(z, \propto) \frac{\partial F(z, \propto)}{\partial z} + Q(z, \propto) \underset{0}{F}(z, \propto) = 0 \tag{2}$$

Necessary and sufficient conditions that solutions of (2) satisfy (1) are found. Particular sets of equations (2) are considered and the sets in which $P(z, \propto)$ and $Q(z, \propto)$ are entire functions of $\propto$, and such that all of their solutions satisfy (1) for proper initial values are exhaustively enumerated.

Lie derivative for (V_m) in V_n. V. HLAVATY, Indiana University.—The notion of the Lie derivative in V_n is extended in such a way that it may be used for a (V_m) in V_n.

A mathematical problem of hydrodynamics. E. HOPF, Indiana University.—The problem deals with the 'integrals in the large" of the Euler equations of ideal fluid flow and it has to do with a fundamental difference between flows in two and three dimensions.

Sampling distribution of Gini's mean difference when the population is exponential. PAUL IRICK, Purdue University.—Let the variability in a sample of n independent x values be measured by the statistic

$\Delta = \Sigma \mid x_i - x_j \mid / n^2$ where x has the frequency function $\mathbf{f(x)} = e^{-x/\sigma} / \sigma$, $0 \leqq x < \infty$. Then a certain general method is used to derive the sampling distribution of Δ. The probability density function for Δ is given by the expression

$$\frac{n^2}{2\sigma(n-1)!} \sum_{j=1}^{n-1} (-1)^{n-1+j} \binom{n-1}{j} j^{n-2} e^{-n^2\Delta/2j\sigma} .$$

Although this particular example is of minor importance, the method of derivation will apply to the determination of many sampling distributions for statistics which measure dispersion.

On limit solutions of the complete equations of hydrodynamics. D. NEAD, Indiana University.—Suppose we have a sequence of solution systems of the complete equations of hydrodynamics (viscosity and thermal conductivity not zero) converging almost everywhere in a bounded open region R. Suppose further that along the sequence (1) viscosity and thermal conductivity approach zero; (2) the individual sequences composing the sequence of solution systems are (absolutely) uniformly bounded in R; (3) the density and temperature sequences are bounded uniformly away from zero in R. Then the limit system of the sequence (where sufficiently smooth) satisfies the system of equations which are obtained from the complete equations by setting viscosity and thermal conductivity equal to zero.

The probability that an arbitrary set of r positive integers be relatively prime. W. M. PEREL, Indiana University.—We define $P_r(N)$ to to be the number of ordered sets of the form $(X_1, X_2 \ldots X_r)$ such that

a) the X_v are positive integers satisfying $1 \leqslant X_v \leqslant N$, N a fixed integer

b) g.c.d. $(X_1 \ldots X_r) = 1$, for each set.

The probability that an arbitrary set of r integers, all $\leqslant$ N, be relatively prime is $\dfrac{P_r(N)}{N^r}$

The solution of the problem is obtained by passing to the limit in the above. The result is $\dfrac{1}{\zeta(r)}$.

Continuous heating of a hollow cylinder. E. A. TRABANT and W. D. WOOD, Purdue University.—Consider a hollow cylinder whose outer surface $r = b$ is thermally insulated with heat being applied continuously at the inner surface $r = a$ according to the quadratic law $q(t) = e + ft + gt^2$. If the inner surface is assumed thermally insulated after an instantaneous source of heat, then, using some results of G. Comenetz, the temperature is found to be

$$\Theta(r,t) = F_o(et + \frac{ft^2}{2} + \frac{gt^3}{3}) + P(r) + t\,Q(r) + t^2R(r)$$

$$- \frac{\pi}{\rho c} \sum_{\eta=1}^{\propto} e^{-\propto\beta^2\eta t} F\eta(r) \quad \frac{e}{\propto \beta^2\eta} - \frac{f}{\propto {}^2\beta^4\eta} + \frac{2g}{\propto {}^3\beta\eta^6}$$

where $$P(r) = \frac{\pi}{\rho c \propto} \sum_{\eta=1}^{\propto} \quad \frac{e}{\beta^2\eta} - \frac{f}{\propto \beta^4\eta} + \frac{2g}{\propto {}^2\beta^6\eta} \quad F\eta(r)$$

$$Q(r) = \frac{\pi}{\rho c \propto} \sum_{\eta=1}^{\propto} \quad \frac{f}{\beta^2\eta} - \frac{2g}{\propto \beta^4\eta} \quad F\eta(r)$$

$$R(r) = \frac{\pi}{\rho c \propto} \sum_{\eta=1}^{\propto} \frac{F\eta(r)}{\beta^2\eta}$$

with βn the successive positive roots of

$$J_1(a\beta)\,Y_1(b\beta) - Y_1(a\beta)\,J_1(b\beta) = 0$$

and where

$$Fo = \frac{2a}{\rho c(b^2 - a^2)}$$

$$F\eta(r) = \frac{\beta\eta\, J_1^2(b\beta\eta)\,[J_1(a\beta\eta)\,Y_0(r\beta\eta) - Y_1(a\beta\eta)\,J_0(r\beta\eta)]}{J_1^2(a\beta\eta) - J_1^2(b\beta\eta)}$$

For values of t sufficiently large it can be shown that the first four terms of $\Theta(r,t)$ must satisfy the heat equation with appropriate boundary conditions. Analysis of this situation reveals that P, Q and R may be written as finite expressions. Finite summation formulas for these particular series of Bessel functions are thus obtained.

Severe pure shear of an elastic body. C. TRUESDELL, Indiana University.—While the general theory of large strain of a purely elastic body was formulated a century ago, it is only in the past three years that general solutions, valid for any form of strain energy, have been obtained by Rivlin. Rivlin has shown that in order to produce a state of simple shear, shearing forces alone are insufficient: normal tractions, ultimately proportional to the square of the angle of shear, must be supplied also. In the present note a state of pure shear resulting from following one simple shear by another in a perpendicular plane is discussed. It is shown that in addition to forces of the type necessary to produce each simple shear separately, a normal force of interaction, ultimately proportional to the product of the two shear angles, must be supplied.

Anti-mathematical propaganda in textbooks. GEORGE WHAPLES, Indiana University.—Every mathematics text offers itself as a solution to the metamathematical research problem of how best to present its material under given restrictions on the assumptions and on the complications permitted in the proofs. Some high school and college texts are criticized first from this mathematical point of view and then according to the reasonableness of their demands on the pupil and their probable effectiveness as judged by educational psychology. Sections of them which are deficient on all three of these standards are called antimathematical propaganda. Examples are given; I have made an effort to confine myself to new ones.