

MATHEMATICS

Chairman: ARTHUR ROSENTHAL, Purdue University
RALPH HULL, Purdue University, was elected chairman for 1954

ABSTRACTS

Quadratic forms in vector spaces. EBERHARD HOPF, Indiana University.—A quadratic form in a vector space is defined by a bilinear form in that space. Every quadratic form satisfies a certain functional equation—the parallelogram identity. There is the converse question whether every solution of this functional equation is a quadratic form or not. The answer is no, but under very mild additional conditions of boundedness or measurability the answer is shown to be yes.

A property of integrals of the product of the Lagrange interpolating polynomials and the exponential function. DONALD B. OWEN, Purdue University.—A simple set of integral rational fractions with integral coefficients which estimate the exponential function may be obtained from the integrals indicated in the title. By combining certain of these estimates additional estimates may be obtained such that the polynomials in the numerator and denominator differ only in that the signs of the terms alternate in the denominator whereas they are all positive in the numerator. One such estimate is given by

$$\exp(x) = (12 + 6x + x^2)/(12 - 6x + x^2),$$

which is accurate to seven decimal places for $-0.1 \leq x \leq +0.1$. Six terms of the power series expansion are needed to give this accuracy. Other estimates are even more accurate. Approximations of this type may be useful in high speed computing where storage of information is a problem.

Some convergent sub-series of the harmonic series. HOWARD K.

HUGHES, Purdue University.—If from the series $\sum_{n=1}^{\infty} \frac{1}{n}$ one omits every term whose denominator contains a given digit, say 9, in any manner, the remaining terms form a series which converges. In case 9 is used, the sum is less than 23.

On the concept of continuity of functions $f(x,y)$. ARTHUR ROSENTHAL, Purdue University.—The continuity of a single-valued function $f(x,y)$ at a point $p_0 = (x_0, y_0)$ can be characterized by means of the continuity of f along certain classes of curves through p_0 . In this respect the following result is obtained: If f is continuous at p_0 along every curve through p_0 which is (at least) once continuously differentiable, then f is also continuous at p_0 as a function of (x,y) . Yet f can be continuous along every

curve through p_0 which is (at least) twice differentiable without being continuous at p_0 as function of (x,y) .

Remarks on graduate study in mathematics. RALPH HULL, Purdue University.—A brief discussion of some of the problems associated with graduate study in mathematics, from a departmental point of view, and with particular emphasis upon the problem of selecting and counselling Ph.D. candidates.

Some problems and results of additive number theory. PAUL ERDOS, Notre Dame University.—Let $0 < a_1 < a_2 < \dots$ be an infinite sequence of integers. Denote by $f(n)$ the number of solutions of $n = a_1 + a_1$. Turán and the author conjectured that if $f(n) > 0$ for all sufficiently large n then $\limsup f(n) = \infty$. This conjecture seems very difficult to prove. Another conjecture of theirs is the following: Let $a_1 < a_2 < \dots$ be an infinite sequence of integers. Then for every constant c

$$\limsup \left| \sum_{k=1}^n f(k) - cn \right| = \infty.$$

About twenty years ago Sidon asked the author if he could construct a sequence of integers so that, for all n , $f(n) > 0$ but $f(n)/n^\epsilon \rightarrow 0$, for all $\epsilon > 0$. Recently the author succeeded in proving the existence of such a sequence. In fact his sequence satisfies $f(n) < c \log n$.

The general solution of some linear partial differential equations with constant coefficients. MICHAEL GOLOMB, Purdue University.—The general solutions, in the sense of all solutions from a specified function class, are known only for very few partial differential equations. A solution of this problem for the equation $P(\partial/\partial x, \partial/\partial y)u = 0$, where $P(\xi, \eta)$ is an arbitrary homogeneous polynomial with constant coefficients, recently established by Adam Schmidt, is extended to equations of the same form, where $P(\xi, \eta)$ is no longer homogeneous, but is a product of arbitrary linear and quadratic factors, and to equations of the form $P(\partial/\partial x, \partial/\partial y)u = f(x, y)$.

On the asymptotic stability of solutions of nonlinear differential equations of parabolic type. R. NARASIMHAU, Indiana University.—A sufficient condition is obtained for the asymptotic stability of the zero solution of the nonlinear parabolic differential equation,

$$U_t = \sum_{i,j} A_{ij} U_{x_i x_j} + \sum_i b_i U_{x_i} + F(x, t, u); \quad i, j = 1, 2, \dots, n.$$

The result is extended to systems of such equations and under more restrictive assumptions, to the case where $F = F(x, t, u, u_x)$. The above results include as special cases the existing theorems on stability of parabolic differential equations.

Sampling distribution of ranges from an arbitrary discrete population. IRVING W. BURR, Purdue University.—Use and importance of the range as a measure of variability in small samples is steadily growing. A method is given for finding the exact sampling distribution for any discrete population with finite range. It depends upon certain combinations of the n 'th

powers of sums of consecutive probabilities. The calculations are quite feasible for samples of five or less, if the population probabilities are taken to the nearest .01 or .005.

The hydraulic pendulum. C. TRUESDELL, Indiana University.—The work of Newton and other early writers on the motion of a finite amount of water in a curved tube has been incorporated in elementary texts. More important, but apparently little known, is Euler's determination of the cavitation criteria for these motions, for if the pressure falls to zero the dynamical solution becomes illusory. Euler's method is entirely general. Here it is applied to the motion of a section of water of arbitrary length in a circular tube. When the pressure remains positive, it is obvious that the motion is that of an appropriate equivalent simple pendulum. Necessary and sufficient conditions for the pressure to remain positive are obtained.

Discussion of a state-wide mathematics competition. LESTER H. LANGE, Valparaiso University.—An annual mathematics competition for mathematics students in the colleges of Indiana was sponsored by the Mathematics and Physics Club of Valparaiso University in the spring of 1953. A report on that competition is given and it is proposed to discuss the desirability of such a competition and to enlist the aid of all the colleges in both the formulation of the examination and the encouraged participation of their students.