

Measurement of the Thermal Neutron Diffusion Parameters of Water by the Pulsed Neutron Method¹

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Pulsed neutron techniques have been used to measure diffusion parameters of thermal neutrons in water at 26°C. Preliminary results are $212 \pm 11 \mu\text{sec}$ for the mean thermal lifetime, $36,985 \pm 1630 \text{ cm}^2 \text{ sec}^{-1}$ for the average diffusion coefficient, and $5021 \pm 2490 \text{ cm}^4 \text{ sec}^{-1}$ for the diffusion cooling coefficient. From these results the thermal absorption cross section of hydrogen and the thermal diffusion length in water are found to be $322 \pm 17 \text{ mb}$ and $2.82 \pm 0.09 \text{ cm}$, respectively. Moderators with geometric bucklings ranging from 0.000 cm^{-2} to 0.600 cm^{-2} were used for the lifetime measurements. Harmonic effects limit the precision of the results.

Introduction

Knowledge of the diffusion parameters of thermal neutrons in water is of fundamental importance in the design of nuclear reactors. These parameters, namely, the mean thermal lifetime, the thermal absorption cross section, the diffusion coefficient, the diffusion length, and the diffusion cooling coefficient, may all be measured by either static or dynamic methods. The pulsed neutron technique (2, 9), is capable of measuring all of these diffusion parameters in a relatively straightforward manner and with good accuracy. Basically, the experiment consists of irradiating the sample with a short burst of neutrons and measuring the time decay constant of the fundamental mode of the thermal neutron flux as a function of the size and shape of the sample, that is, the geometric buckling. The measurement of the diffusion parameters of thermal neutrons in water by the pulsed neutron technique was begun about a year ago at Valparaiso University and some of the results are presented below.

Theory of Measurement

The diffusion of thermal neutrons in a homogeneous source-free medium is described by the equation

$$D_0 \nabla^2 n(\vec{r}, t) - \frac{1}{T} n(\vec{r}, t) = \frac{\partial n(\vec{r}, t)}{\partial t} \quad (1)$$

where

$n(\vec{r}, t)$ = the number of thermal neutrons per unit volume at position \vec{r} and time t

D_0 = the thermal diffusion coefficient

T = the mean thermal lifetime.

1. Much of the instrumentation used in these measurements was purchased out of a grant from the Division of Reactor Development of the U. S. Atomic Energy Commission. This assistance is gratefully acknowledged.

This equation has the general solution

$$n(\vec{r}, t) = \sum_{ijk}^{\infty} A_{ijk} F_{ijk}(\vec{r}) e^{-a_{ijk} t} \quad (2)$$

where

$$a_{ijk} = \frac{1}{T} + D_0 B_{ijk}^2 - C B_{ijk}^4, \quad (3)$$

$F_{ijk}(\vec{r})$ and B_{ijk}^2 are, respectively, the eigenfunctions and corresponding eigenvalues of the Helmholtz wave equation

$$\nabla^2 F(\vec{r}) + B^2 F(\vec{r}) = 0$$

subject to the boundary condition that $F_{ijk}(\vec{r})$ vanish on the extrapolated boundaries of the system. Theory indicates that these extrapolated boundaries are at a distance

$$\epsilon = 2.13 D_0 / \bar{v} \quad (4)$$

beyond the physical boundaries of the medium, where \bar{v} is the average neutron velocity. The lowest eigenvalue is usually denoted by B^2 (without subscripts) which is called the geometric buckling. In the thermal neutron velocity distribution the faster neutrons tend to leak out of the medium more rapidly than the more slowly moving neutrons, leaving a velocity distribution characterized by a temperature somewhat below that of the medium. This effect is known as diffusion cooling and is measured in terms of the diffusion cooling coefficient C appearing in Eq. (3).

For an isotropic point source of neutrons located below but on the axis of a cylindrical moderator, the eigenfunctions and eigenvalues in Eqs. (2) and (3) have the form

$$F_{j,k}(r, z) = J_0(\alpha_j r/R) \sin(k\pi z/H)$$

$$B_{j,k}^2 = (\alpha_j/R)^2 + (k\pi/H)^2$$

where

$$\alpha_j = \text{the } j\text{th zero of the } J_0 \text{ Bessel function}$$

$$k = 1, 2, 3, \dots$$

$$R = R_0 + \epsilon$$

$$H = H_0 + 2\epsilon.$$

R_0 and H_0 are the physical radius and height of the cylinder, respectively, and ϵ is the extrapolation distance given by Eq. (4). The radial and axial coordinates (r, z) are measured from the base of the cylinder. Now, the neutron distribution corresponding to the fundamental mode becomes

$$n_{11}(r, z, t) = A_{11} J_0(2.405r/R) \sin(\pi z/H) e^{-at} \quad (5)$$

with

$$\alpha = \frac{1}{T} + D_0 B^2 - C B^4 \quad (6)$$

and

$$B^2 = (2.405/R)^2 + (\pi/H)^2. \quad (7)$$

Measurement of α , the decay constant of the fundamental mode, for a variety of cylindrical sample sizes yields data which may be fitted by the method of least squares to Eq. (6), using an iteration procedure with Eqs. (4) and (7) so that the extrapolation distance ϵ can also be found from the data. This then determines the diffusion parameters T , D_0 and C . Since $T = (v\Sigma_a)^{-1}$, where Σ_a is the macroscopic absorption cross section, the absorption cross section for "1/v" absorbers may be found from the mean thermal lifetime. The value of B^2 for which the decay constant $\alpha = 0$ is equal to minus the square of the reciprocal of the thermal diffusion length L . From Eq. (6), therefore,

$$L^2 = D_0 T [1 + C(D_0^2 T)^{-1}]$$

when $4C \ll D_0^2 T$.

Experimental Arrangement

The experimental arrangement for the neutron lifetime measurements is shown in Fig. 1. The pulsed neutron generator is Model 150 -1H manufactured by the Texas Nuclear Corporation. Neutrons of 14 Mev energy are produced essentially isotropically by the interaction of 150 kev deuterons on a tritium target with a possible source strength of about 4×10^{10} n/sec. These fast neutrons are slowed down to thermal energies as a result of successive collisions with the hydrogen and oxygen nuclei in the water moderator. Neutron pulse widths and repetition rates are continuously variable from 1 μ sec to 10 msec and from 10 cps to 100 kcs, respectively.

The cylindrical moderator is located a few inches above the tritium target with its axis of symmetry passing through the target. In order to isolate the system from the large water tank which serves as a partial neutron shield, the cylinder is completely surrounded by a cadmium sheet 20 mils thick. Cadmium of this thickness is a virtually perfect absorber of thermal neutrons.

Neutrons were detected by means of a neutron-sensitive scintillation probe manufactured by the Nuclear-Chicago Corporation (Model DS8-10). The 96 percent enriched B^{10} , Zn(Ag) activated crystal had a diameter of $\frac{3}{8}$ inch, a thickness of 0.04 inch, and was mounted at the end of a lucite light pipe 42 inches long. Since lucite has very nearly the same thermal neutron properties as water, this detector introduces a negligible perturbation into the system and acts essentially as a point detector. By locating this thin crystal detector on the mid plane of the cylinder and at a radial distance equal to $(2.405 R/5.520)$ from the axis, contributions to the count rate from the first radial and first axial harmonic modes are eliminated.

Pulses from the detector are amplified and fed into a 20-channel time analyzer manufactured by the Fairport Instrument Company. Channel

BLOCK DIAGRAM FOR NEUTRON LIFETIME MEASUREMENTS IN WATER MODERATORS

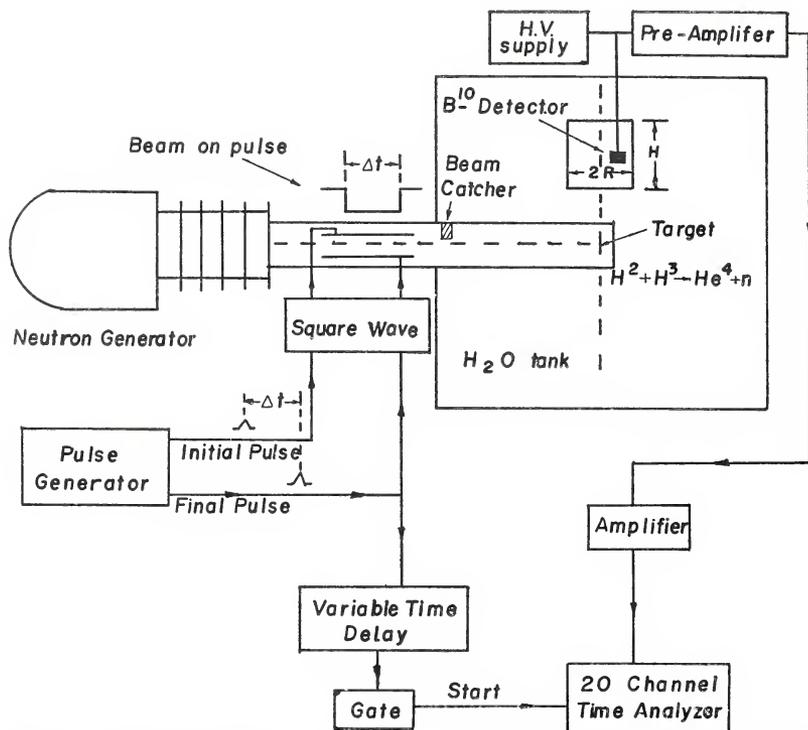


Figure 1. Experimental arrangement and instrumentation for neutron lifetime measurements.

widths from 10 to 10^5 μsec are available. Since the harmonic modes decay more rapidly than the fundamental mode, a variable time delay following the termination of the neutron pulse is inserted before triggering the time analyzer. Typical delay times ranged from 100 to 200 μsec .

Experimental Results

Deionized water having a resistivity of about 1×10^7 ohm-cm and a temperature of 26°C was used for all the lifetime measurements. Fig. 2 shows experimental decay curves for thermal neutrons in water for cylinders of various sizes and shapes. The actual cylinder dimensions and corresponding decay constants and geometric bucklings are listed in Table I. A plot of the decay constant versus the geometric buckling B^2 is shown in Fig. 3. The departure of the data from a straight line at large values of B^2 is a clear indication of the diffusion cooling

COUNTS PER CHANNEL Vs. TIME
AFTER NEUTRON BURST

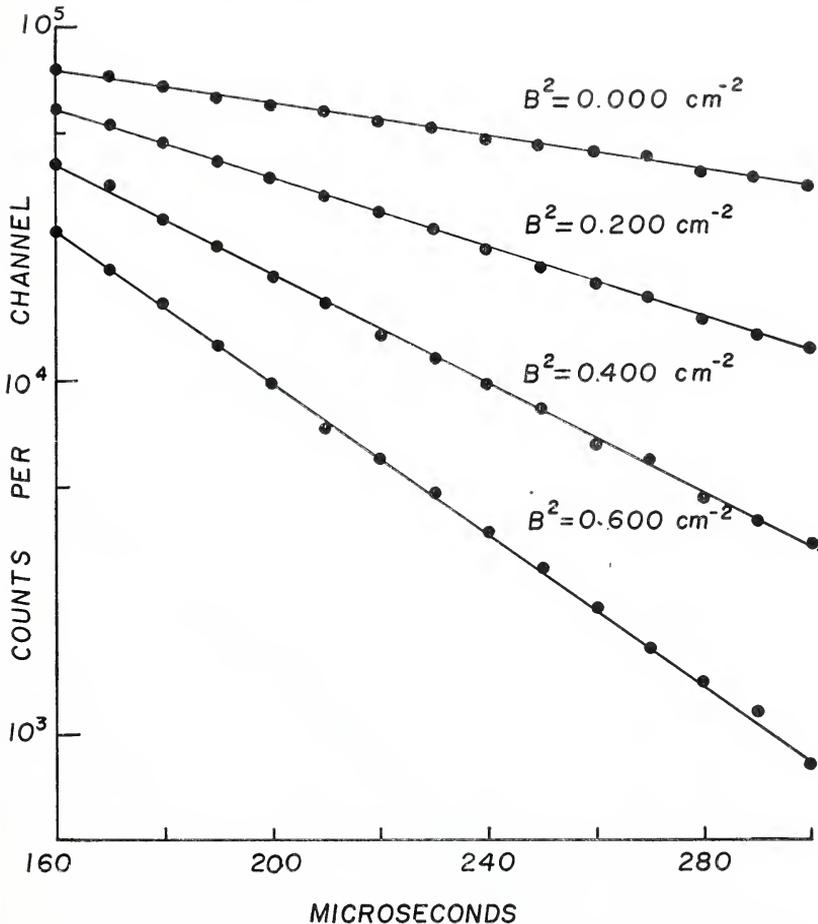


Figure 2. Semilogarithmic plot of count rate versus time after neutron burst for several different geometries.

effect. The diffusion parameters for thermal neutrons in water as obtained from this measurement are shown in Table II where other experimental results (1, 3-8) are quoted for comparison. The absorption cross section $\sigma_a(\text{H})$ is that for hydrogen since the absorption cross section for oxygen is completely negligible.

Summary

The diffusion parameters for thermal neutrons in water have been measured by the pulsed neutron method. Results are in good agreement with recently published data, although statistical errors are relatively

TABLE I. Experimental Data

Moderator Size		$(\epsilon = 0.34 \text{ cm})$	
R_0 (cm)	H_0 (cm)	α (sec ⁻¹)	B^2 (cm ⁻²)
84	152	$(4.767 \pm .03)10^3$	0.000
5.72	14.56	$(11.60 \pm .07)10^3$	0.200
5.72	9.64	$(13.72 \pm .13)10^3$	0.250
5.72	7.63	$(15.40 \pm .14)10^3$	0.300
5.72	6.47	$(17.43 \pm .17)10^3$	0.350
5.72	5.63	$(18.79 \pm .15)10^3$	0.400
3.81	8.58	$(19.90 \pm .13)10^3$	0.450
3.81	7.05	$(22.12 \pm .17)10^3$	0.500
3.81	6.09	$(23.42 \pm .27)10^3$	0.550
3.81	5.41	$(25.18 \pm .20)10^3$	0.600

α Vs. B FOR WATER MODERATOR

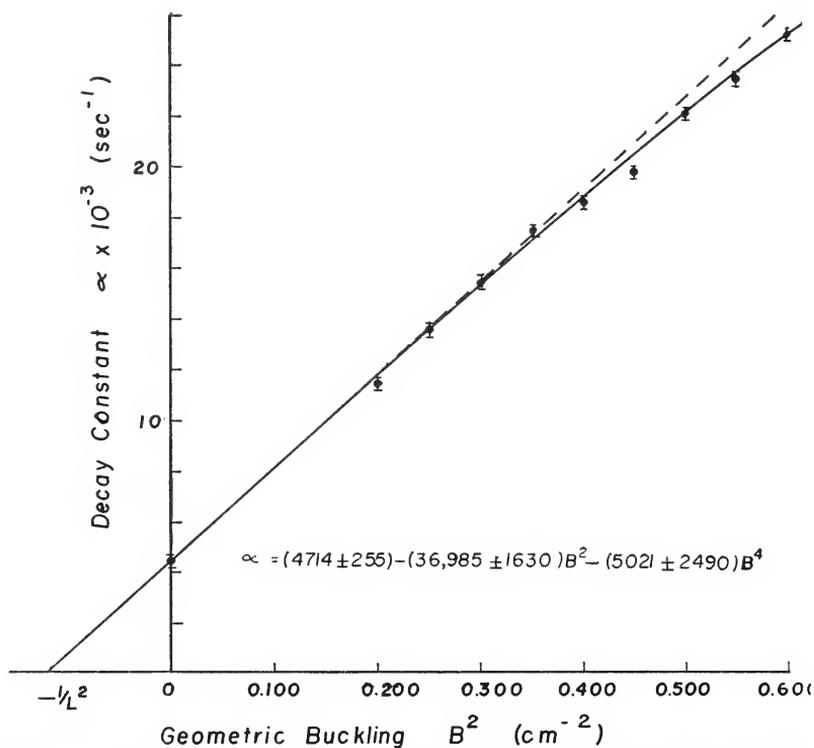


Figure 3. Least squares fit of decay constant versus geometric buckling.

large. Much of this error arises from interfering harmonic modes and work is currently in progress to reduce these harmonic contributions

TABLE II. Experimental Results

Thermal Neutron Diffusion Parameters For Water						
Reference	T (μ sec)	D_0 (cm ² -sec ⁻¹)	C (cm ⁴ -sec ⁻¹)	σ_a^H (mb)	L (cm)	
This Work	212 \pm 11	36,985 \pm 1,630	5,021 \pm 2,490	322 \pm 17	2.82 \pm .09	
8	204 \pm 2	36,340 \pm 750	7,300 \pm 1,500	333 \pm 3	2.72 \pm .03	
1	207 \pm 6	35,000 \pm 1,000	4,000 \pm 1,000	329 \pm 10	2.7 \pm .1	
3	202 \pm 6	34,850 \pm 1,100	3,000 \pm 1,000	337 \pm 10	2.66 \pm .11	
4	208 \pm 4	35,450 \pm 600	3,700 \pm 700	328 \pm 6	2.72 \pm .06	
5	209 \pm 6	35,400 \pm 700	4,200 \pm 800	326 \pm 6	2.72 \pm .08	
7	198 \pm 23	36,146 \pm 5,600	5,666 \pm 11,670			
6	210 \pm 1	36,892 \pm 400	5,116 \pm 776	325 \pm 2	2.80 \pm .02	

and to increase the neutron detection efficiency as well as the neutron source strength.

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