Grain Reserve Systems: Failure of Multiobjective Decisions Under Catastrophic Intervention, A Case for Topologic Stability in Singular Mapping

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Introduction

Fundamental similarities and differences exist between management of water reservoir and grain reservoir systems. The similarities are due to their roles as temporal buffer stocks for commodity fluctuations. The differences relate to the fact that streamflow is primarily a weather related process but grain production and consumption are affected by national policies, economic incentives, technological changes and other factors in addition to the vagaries of nature. It is possible to adopt stochastic or deterministic optimization, reliability analysis and multi-objective programming to the management of grain reserves in a manner similar to their use in the management of water reservoir systems.

However, modifications are also necessary in the methods of analysis. The major area of emphasis in grain buffer stock policy models has been on the market economics, while water reservoir planning and operations models have concentrated on natural phenomena with poor long term predictability. The existence of a market for grains is the major factor affecting the transfer of simulation and optimization models to grain buffer stock management. In water resource analysis, streamflow is independent of the demand for water or its products, whereas the amount of grain produced is a function of demand for grains. Therefore, an analyst must model the economic aspects in particular. The grain market itself can be treated as implicit in historical production data series. Production then becomes a surrogate for technological changes as well as market and weather induced changes in a limited sense.

The importance of a grain reserve is in its role as an insurance against catastrophes. In particular for the U. S., where major national income comes from grain exports, the optimum management of a grain reserve system is of prime importance. The major varieties of grains in this country which are sources of staple food are wheat and corn. Different grains are close substitutes on both demand and supply sides. It is therefore desirable that a model of the grain market should include all the grains. Any effort to stabilize the price of one grain should affect the prices of other grains too. Therefore, the best method would be to include explicitly and simultaneously the different categories of grains in the model. Also, such a model should consider monthly or seasonal fluctuations and the exact location of reserves. However, methods and capacities of computational procedures stand in the way of constructing such a global model at this time. Simplifications and approximations are adopted to cope with this problem.

The First Model

The foremost objective of building a grain reservoir system is to stabilize the price variation within the limits of profitable return to the producers. At the same time, the capacity of the reservoir should be offset by (i) monetary benefits to the farmers, (ii) price stabilization above the subsidization level, (iii) reduction in payment to the farmers by the government through set aside program, or (iv) gains in social welfare by assuring certain levels of consumption at all times, particularly for the poorer sections of the population. So the two basic objectives may be stated as: (1) minimize grain price fluctuations over some time horizon, and (2) minimize the grain reserve capacity.

A multiple objective linear program that ensured a minimum return to farmers over a ten year study period (1968-77) was constructed and solved to determine the relationships between objectives 1 and 2. The results are shown in Table 1 where it is evident that an inverse relationship exists between the reserve capacities for wheat and corn and the grain price stability. For example, a wheat reserve capacity of 2.1 billion bushels ensures that wheat prices deviate only $1.86/bushel over ten years; whereas a reserve capacity of 1.95 billion bushels produces a price flucturation of $3.25/bushel. The smaller reserve capacities are not feasible because they do not provide the minimum return to farmers as specified in the model.

Together with the above study, an effort was made to correlate the historical values of exports with other related variables. However, such a model exhibited very poor performance. It was noted that with the imposition of an export embargo on grains the prices of grains like corn or wheat have slumped by about 25% in a single day in the domestic market. This only points out the inadequacy of methods used in economic analysis, based on equilibrium conditions of demand and supply as required for models such as the linear program mentioned above. The instantaneous response to external intervention in the market system certainly points to the divergencies and discontinuities which exist in a socio-economic system and which are generally ignored in the process of conventional analysis.

The Second Model

The above notion served as a motivation for trying to build a conceptual model, based on the ideas of the "Separation Theorem" as noted by Samuelson (6) and the unstable behavior of stock exchanges as noted by Zeeman (7).

We start from the separation theorem in economics, which states that "Points of stable equilibrium (in the small) are separated by points of definitely unstable equilibrium and vise versa," where equilibrium "in the small" exists, if for sufficiently small perturbations the equilibrium is stable. It can also be recalled that a critical point is structurally stable only if it is not degenerate. [A function f has a nondegenerate critical point at u, if the Hessian matrix

TABLE I. *Possible Values of Grain Reserve Capacity vs. Maximum Price Fluctuation For a Ten Year Period*

Wheat		Corn	
Capacity (million bushels)	Price Fluctuation ($/bushel)	Capacity (million bushels)	Price Fluctuation ($/bushel)
1900	no feasl. sol.	1600	no feasl. sol.
1950	3.248	1639	2.11
1975	2.872	1650	2.07
2000	2.529	1700	1.93
2025	2.196	1800	1.78
2050	1.952	1850	1.72
2075	1.860	1894	1.674
2100	1.860	1900	1.674
2212	1.860	2106	1.674

$$Hf|_\mu = \left[\frac{\partial^2 f}{\partial x_i \partial x_j} \bigg|_\mu \right]$$

is nonsingular or determinent $(Hf/_\mu) \doteq 0$. The condition that the Hessian be nonsingular is exactly the condition that the Jacobian of the mapping

$$Df = \left(\frac{\partial f}{\partial x_1} \cdots \frac{\partial f}{\partial x_n} \right) : R^n \rightarrow R^n$$

be nonsingular, which is the condition that the graph of Df meet that of the zero function transversely.]

Another mathematical property taken into account is that transversality is a stable property and transverse crossings are themselves stable. However, when dealing with a family of functions, where the paramaters are allowed to vary, a structurally stable family can include individual functions with degenerate critical points. The surrounding members of the family exert some form of calming influence on the degenerate functions, which is called "unfolding" by Rene Thom. As an example, the family of functions

$$f = \frac{x^4}{4} + \frac{a}{2} x^2$$

does not have structurally stable critical points, as the mapping $(x,a) \rightarrow Df(x,a)$ where $Df = \frac{df}{dx}$ intersects the zero plane nontransversally because the intersection is not a manifold. However, by adding a perturbation term bx to f,

$$f + p = \frac{x^4}{4} + a\frac{x^2}{2} + bx$$

the intersection of the mapping $D(f + p)$ with the zero plane becomes a manifold, i.e. a curve in "n" dimensional space with no intersections. And the critical points of this family of functions as obtained by mapping in the (x, a, b) space for different values of the parameters a and b become structurally stable.

The family

$$\frac{x^4}{4} + a\frac{x^2}{2} + bx$$

is called the cusp catastrophe and the mapping $(x, a, b) \rightarrow Df(x, a, b)$ is called the catastrophic manifold. Singular points of this mapping are those points with vertical tangents when x is assumed as the vertical axis. Therefore, it can be concluded that by adding a perturbation term, the critical points of a family of functions acquire some form of topologic stability.

Rene Thom showed in his "Catastrophe Theory" that "Structurally stable smooth change of coordinates can occur in a R parameter family only in finite ways." These families of mappings are called catastrophic manifolds. Because repeatability and reproduceability are the main ideas of any modeling effort, we shall use the above concept together with the "Separation Theorem", trying to present a conceptual model. The problem becomes the exploration of equilibrium surfaces which are connected to the stability changes of the system, as the control parameters slowly evolve. The correlation between the Catastrophe Theory and the Bifurcation Theory, embedded in the perturbation concept, can help remove the qualitative nature of the Catastrophe Theory.

We now consider this situation of a sudden restriction in export commitments by government, and its effects on the price stability of the grain market. It is assumed that initially we have a stable system. Therefore, in the chronological dynamic model we considered before, the state of the system at a particular discrete time interval was supposed to be related to the state of the system at a previous time interval and other related variables. The shock to which the system is subjected may cause a sudden shift in the equilibrium position to another equilibrium position, through a short lived process of dynamic instability.

Our main concern is to investigate these two states of equilibrium separated from each other, or the entire process of comparative statics. Here we confront the problem of establishing the form and mechanism of this intermediate path, where a bifurcation occurs through external perturbations. The concept of structural stability in Catastrophe Theory, may form a link between the static stationary states and truly dynamic path through time under external shocks. The whole process is assumed to be structurally stable, repeatable or reproduceable. The main objectives are to construct some sort of a predictive model, to design effective corrective measures, to make the rate of change of the price index for grain positive, and to remedy the catastrophic slump in the price index.

The following variables are defined.

P_i = price index for grains (may be for aggregated average of all varieties or a particular one such as wheat)

$$T = \frac{dP_i}{dt} = \dot{P}_i = \text{rate of change of price index}$$

$T = 0 \rightarrow$ initial stable condition

$T > 0 \rightarrow$ export market dominates

$T < 0 \rightarrow$ domestic market dominates

$$K = \frac{\left(\begin{array}{c}\text{Excess stock expected to be left over}\\ \text{after domestic consumption and}\\ \text{reserve commitments}\end{array}\right) - \left(\begin{array}{c}\text{Export commitments}\\ \text{to other}\\ \text{countries}\end{array}\right)}{(\text{Total domestic consumption})}$$

X = excess demand in domestic market

$X = 0 \rightarrow$ no excess demand in domestic market

The following hypotheses based on the behavior of the economic system are put forward.

Hypothesis 1: T responds to K and X much faster than K and X respond to T. This implies that price stability is affected much faster by a change in the proportion of expected net excess of grains to that of domestic consumption and the excess demand in the domestic market.

Hypothesis 2: If K is small, T is a continuous, monotonic increasing function of X, passing through the origin. Therefore, when the ratio K is small, T will try to increase with a positive change in excess domestic demand. This is true for both $T < 0$ and $T > 0$.

Hypothesis 3: If K is large, this will cause some instability in the whole system. Thus for a large K, where net excess is large, perturbations in the form of an exogeneous shock may cause a change in the equilibrium state of the system, i.e. a jump from one stable state to another. The evidence of postulation of such a sudden crash is inherent in the "Separation Theorem." The basic ideas in the three hypotheses may be summarized by Figures 1 and 2.

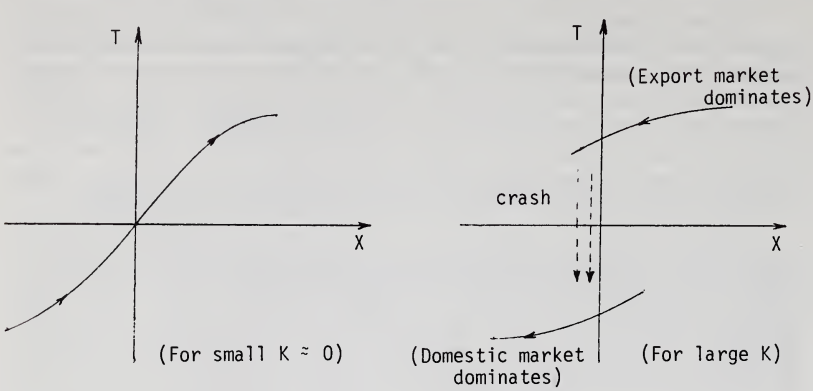

FIGURE 1. *Synthesis of the Hypotheses*

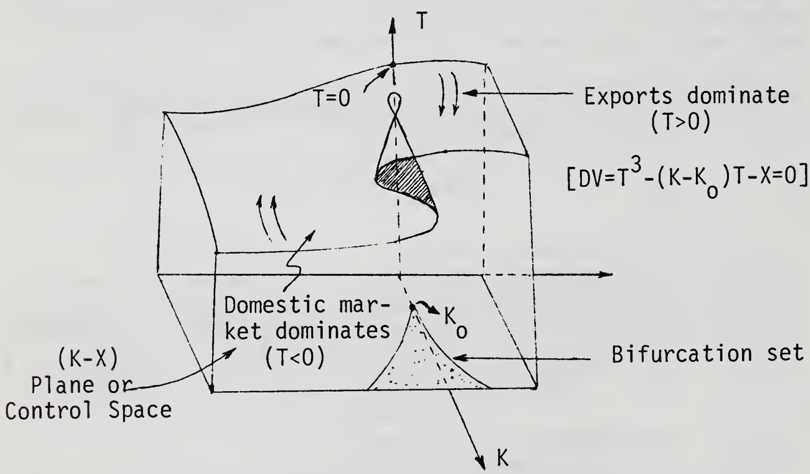

FIGURE 2. *The Behavior and Control Surfaces*

The behavior surface is represented by the function

$DV = T^3 - (K - K_0)T - X$ [K_0 = some normal value of K]

where $V = \frac{T^4}{4} - \frac{(K - K_0)}{2}T^2 - XT$ represents the state of the system in some form.

The mapping of $(T, K, X) \leftarrow DV(T, K, X)$ represents the nature of flow in the behavior surface due to changes in the control parameters K and X. The singular points of this mapping are enclosed within the bifurcation set shown in the control space. The basic conclusion we can dervie from the above hypotheses is that in order to stabilize prices, measures should be adopted to decrease K and increase X when a sudden collapse is imminent. However, transition back, as shown by arrows, is a slower process. Also, since catastrophic conditions fortunately do not occur very often, callibration may be difficult to obtain. We may look upon the catastrophic manifolds in the simplest sense as an effective way of describing a system with variable degrees of hysteresis.

Summary and Conclusions

The entire economic system of grain production, distribution, consumption, storage and export is by no means a simple isolated entity. The crucial interactions of various social, political and economic forces are fundamental to the system. Therefore, an intuitive understanding of the entire process based on logical assumptions should yield a better model than other much more sophisticated analytic techniques. It is strongly believed that the above process of exploration of singularities of mapping and structural stability in an economic system should give a better idea of the interactions of crucial variables at crucial states, and should help to identify effective corrective measures, both quantitatively and qualitatively. The conceptual model based on the ideas of Catastrophe Theory was presented as a supplement to the previously described multi-objective optimization model, which fails to operate meaningfully under catastrophic conditions marked by sudden changes in the stable configuration of the system. The assumptions involved are no doubt partly intuitive and based on some forms of topologic indefiniteness. Also, it is plausible to find limitations of many of the assumptions made. However, it is strongly believed that a further and closer look should help remove these deficiencies and render the above approach in modeling an important addition to all other commonly adopted methods of analysis.

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