

Rapid Estimation of Forest Parameters Using Monareal and Polyareal Combination Sampling¹

THOMAS W. BEERS, Purdue University

It is always exciting to get something for nothing. Think how excited Walter Bitterlich must have been when he realized that he could accurately estimate basal area per acre without measuring a single tree!

I won't pretend to uncover any concepts comparable in scope to Bitterlich's but I will expand upon a something-for-nothing technique briefly presented by Grosenbaugh (3), which can be used to obtain estimates of certain forest parameters with very little or no individual tree measurements. The estimates to be described find occasional usefulness in stand description, prescription, and analysis: They are (1) diameter of the tree of average basal area (quadratic mean diameter) and (2) arithmetic average tree diameter. Either of these can be obtained by appropriate combination of fixed-size plots (monareal plot sampling) and variable-size plots (polyareal plot sampling). Estimates of quadratic and arithmetic average tree height can be obtained similarly but will not be described in detail.

A brief description of monareal and polyareal plot sampling is in order first.

Monareal plot sampling is the case where fixed-size circular, rectangular, or square plots are used as the sampling unit. Each tree tallied or measured represents the same number of trees per acre regardless of the tree size. Therefore, for example, one tree tallied on a one-tenth acre plot represents 10 trees per acre.

The most widely used example of polyareal plot sampling is "Bitterlich Sampling," known also by a multitude of other names, but most descriptively by "horizontal point sampling." In this type of sampling, the plot size and therefore the number of trees represented by each tree tallied is not constant but varies with tree diameter. For example, if the basal area factor being used is ten, a tallied 12-inch tree represents 12.73 trees per acre while a tallied 24-inch tree represents 3.18 trees per acre.

The other types of polyareal plot sampling are horizontal line sampling, vertical point sampling and vertical line sampling. Further description is not appropriate here but can be found in Grosenbaugh (3) or Husch (4). A recent discussion of monareal and polyareal sampling has been prepared by Beers and Miller (2).

Quadratic Mean Tree Diameter

An estimate of the quadratic mean diameter, that is the diameter of the tree of average basal area, can be obtained by the following procedure:

1. Purdue University Agricultural Experiment Station Journal Paper 2649.

1. Visit an arbitrary number of locations within the stand to be assessed.
2. At each location make two tallies, first a count of the trees which qualify (this is the usual "Bitterlich count," "angle count" or "point count") using a pre-selected basal area factor, and second a count of the trees within the circular plot concentric with the point where the "angle count" was made and whose radius depends on the basal area factor being used. It is convenient to designate the radius of this plot as $\frac{1}{k}$ where k is the angle gauge constant.

For example, if basal area factor = 10, the angle gauge constant = $\frac{1}{33} = .030303$, and the circular plot radius = $\frac{1}{.030303} = 33$ feet.

This condition, where plot radius is the reciprocal of the gauge constant, is referred to as the "neutral situation."

3. After sufficient locations have been visited, calculate the average point tree count and the average plot tree count per location.
4. Calculate the quadratic mean diameter from the formula

$$\text{ave. diameter (quadratic)} = 12 \sqrt{\frac{\text{ave. point tree count}}{\text{ave. plot tree count}}}$$

or refer to a table such as Table 1 where the solution is given for a limited range of tree counts.

The formula upon which Table 1 is based can be developed as follows:

$$\text{ave. basal area per tree} = \frac{\text{ave. B.A. per acre}}{\text{ave. number of trees per acre}}$$

Now if the numerator is estimated using horizontal point sampling (Bitterlich sampling) and the denominator using conventional circular plots and assuming one location is visited, we have²

$$\begin{aligned}\text{ave. B.A. per tree} &= \frac{F \text{ (point tree count)}}{\frac{43560}{\pi R^2} \text{ (plot tree count)}} \\ &= \frac{10890 k^2 \text{ (point tree count)}}{43560 \text{ (plot tree count)}} \\ &= \frac{\pi R^2 k^2 \text{ (point tree count)}}{4 \text{ (plot tree count)}}\end{aligned}$$

where

$$F = \text{basal area factor} = 10890 k^2,$$

$$k = \text{angle gauge constant} = 0.0095827 \sqrt{F},$$

$$\text{and } R = \text{circular plot radius in feet.}$$

². A basic discussion of the following fundamental relationships can be found in Beers and Miller (1).

Now, since the diameter associated with any basal area is found by

$$\text{diameter} = \sqrt{\text{B.A.} \left(\frac{576}{\pi} \right)}$$

then

$$\text{ave. diameter (quadratic)} = \sqrt{\text{ave. B.A.} \left(\frac{576}{\pi} \right)}$$

Substituting the formula developed above for average B.A. we have

$$\begin{aligned} \text{ave. diameter (quadratic)} &= \sqrt{\frac{\pi R^2 k^2 (\text{point tree count})}{4 (\text{Plot tree count})} / \frac{576}{\pi}} \\ &= 12 R k \sqrt{\frac{\text{point tree count}}{\text{plot tree count}}} \end{aligned}$$

$$\text{ave. diameter (quadratic)} = 12 \sqrt{\frac{\text{point tree count}}{\text{plot tree count}}}$$

where plot radius, R , is equal to $\frac{1}{k}$. Average point and plot tree counts

can be used in the formula if more than one location is visited.

Arithmetic Mean Tree Diameter

The ordinary arithmetic average tree diameter, which does not give as much weight to the larger size trees, can be estimated by a similar combination of monareal and polyareal plot sampling—again with no tree measurements needed. Here, however, it is necessary to use horizontal line sampling, desirably in conjunction with a rectangular, fixed-width plot. Horizontal line sampling differs from horizontal point sampling (Bitterlich sampling) in that trees are viewed and checked for qualification on one or more often on both sides of a line of some measured length. The trees are viewed, using an angle gauge, with the line of sight perpendicular to the fixed line segment. The rectangular plot is established using the sample line segment as the long central axis and measuring a specified distance on either side.

An estimate of the arithmetic average tree diameter can be obtained by the following procedure:

1. Visit an arbitrary number of sample line segments, say one chain in length.
2. Along each line make two tallies, first a count of the trees which qualify using a pre-selected diameter factor (analogous to basal area factor in conventional Bitterlich sampling), and second a count of the trees within the rectangular plot whose length is the length of the sample line segment and whose half-width is $\frac{10}{12k}$. For example, using an angle gauge having a basal area factor (F) of 10,

$$\text{diameter factor (inches)} = f = 12 \sqrt{10F} = 120,$$

$$\text{gauge constant} = k = \frac{1}{33} = .030303,$$

$$\text{and plot half-width} = \frac{10}{12k} = \frac{330}{12} = 27.5 \text{ feet.}$$

3. After sufficient line segments have been traversed calculate the average line tree count and the average plot tree count per location.
4. Calculate the arithmetic average diameter from the formula average diameter $= 10 \left(\frac{\text{line tree count}}{\text{plot tree count}} \right)$, or refer to a table such as Table 2.

The formula upon which Table 2 is based was developed as follows:

average diameter per tree $= \frac{\text{sum of diameters per acre}}{\text{number of trees per acre}}$, but if the numerator is estimated using horizontal line sampling and the denominator using a fixed-size rectangular plot, we have

$$\begin{aligned} \text{average diameter} &= \frac{f \text{ (line tree count)}}{\frac{43560}{2w(66L)} \text{ (rect. plot tree count)}} \\ &= \frac{\frac{3960k}{L} \text{ (line tree count)}}{\frac{330}{wL} \text{ (rect plot tree count)}} \\ &= 12 w k \left(\frac{\text{line tree count}}{\text{plot tree count}} \right) \end{aligned}$$

where

$$f = \text{diameter factor} = \frac{43560 (6k)}{66L} = \frac{3960 k}{L}$$

k = angle gauge constant

w = rectangular plot half-width (in feet)

and L = length of plot or length of line segment (in chains).

Letting the plot half-width be determined by the angle gauge being used, specifically equal to $\frac{10}{12k}$, then the formula becomes

$$\text{average diameter} = 10 \left(\frac{\text{line tree count}}{\text{plot tree count}} \right)$$

and a table can be prepared which is applicable to any combination of angle gauge and plot width leading to the neutral situation (i.e., plot

half width $= \frac{10}{12k}$).

Other Estimates

Quadratic or arithmetic average stand height can be rapidly estimated using vertical point sampling or vertical line sampling in combination with the appropriate type of monareal plot.

Vertical sampling (which can be accomplished either at a point or along a line) is not nearly as well known as horizontal sampling. Basically, however, they are the same in that sampling is being performed with the probability of tree selection in proportion to some element of tree size. While diameter (or D^2) is the pertinent element in horizontal sampling, height (or H^2) is the pertinent element in vertical sampling. Thus for a tree to "qualify" in vertical sampling its height must be greater than some constant, pre-determined gauge angle.

Using field procedures similar to those previously described, the following formulas are appropriate using the indicated combinations of vertical sampling and plot sampling:

$$\text{ave tree height (quadratic)} = R (\tan \Phi) \sqrt{\frac{\text{vertical point tree count}}{\text{circular plot tree count}}}$$

$$\text{ave. tree height (arithmetic)} = w (\tan \Phi) \left(\frac{\text{vertical line tree count}}{\text{rectangular plot tree count}} \right)$$

where

R = circular plot radius (in feet),

w = rectangular plot half-width (in feet),

$\tan \Phi$ = vertical gauge constant.

Literature Cited

1. BEERS, T. W. and C. I. MILLER. 1964. Point Sampling: Research Results, Theory, and Applications. *Purdue Univ. Agr. Exp. Sta. Res. Bul.* 786, 56 pp.
2. ————. 1965. Polyareal plot sampling: terminology, symbolism and formulism. Manuscript submitted for publication in *Journal of Forestry*.
3. GROSENBAUGH, L. R. 1958. Point-sampling and Line-sampling: Probability Theory, Geometric Implications, Synthesis. *So. For. Exp. Sta. Occ. Paper* 160. 34 pp.
4. HUSCH, B. 1963. **Forest Mensuration and Statistics**. The Ronald Press Company.

TABLE 1. Quadratic Mean Tree Diameter by Horizontal Point and Circular Plot Tree Counts Assuming the Neutral Situation^a.

PLOT TREE COUNT	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	12.0	17.0	20.8	24.0	26.8	29.4	31.7	33.9	36.0	37.9	39.8	41.6	43.3	44.9	46.5	48.0	49.5	50.9	52.3	53.7
2	8.5	12.0	14.7	17.0	19.0	20.8	22.4	24.0	25.5	26.8	28.1	29.4	30.6	31.7	32.9	33.9	35.0	36.0	37.0	37.9
3	6.9	9.8	12.0	13.9	15.5	17.0	18.3	19.6	20.8	21.9	23.0	24.0	25.9	26.8	27.7	28.6	29.4	30.2	31.0	31.0
4	6.0	8.5	10.4	12.0	13.4	14.7	15.9	17.0	18.0	19.0	19.9	20.8	21.6	22.4	23.2	24.0	24.7	25.5	26.2	26.8
5	5.4	7.6	9.3	10.7	12.0	13.1	14.2	15.2	16.1	17.0	17.8	18.6	19.3	20.1	20.8	21.5	22.1	22.8	23.4	24.0
6	4.9	6.9	8.5	9.8	11.0	12.0	13.0	13.9	14.7	15.5	16.2	17.0	17.7	18.3	19.0	19.6	20.2	20.8	21.4	21.9
7	4.5	6.4	7.9	9.1	10.1	11.1	12.0	12.8	13.6	14.3	15.0	15.7	16.4	17.0	17.6	18.1	18.7	19.2	19.8	20.3
8	4.2	6.0	7.3	8.5	9.5	10.4	11.2	12.0	12.7	13.4	14.1	14.7	15.3	15.9	16.4	17.0	17.5	18.0	18.5	19.0
9	4.0	5.7	6.9	8.0	8.9	9.8	10.6	11.3	12.0	12.6	13.3	13.9	14.4	15.0	15.5	16.0	16.5	17.0	17.4	17.9
10	3.8	5.4	6.6	7.6	8.5	9.3	10.0	10.7	11.4	12.0	12.6	13.1	13.7	14.2	14.7	15.2	15.6	16.1	16.5	17.0
11	3.6	5.1	6.3	7.2	8.1	8.9	9.6	10.2	10.9	11.4	12.0	12.5	13.0	13.5	14.0	14.5	14.9	15.4	15.8	16.2
12	3.5	4.9	6.0	6.9	7.7	8.5	9.2	9.8	10.4	11.0	11.5	12.0	12.5	13.0	13.4	13.9	14.3	14.7	15.1	15.5
13	3.3	4.7	5.8	6.7	7.4	8.2	8.8	9.4	10.0	10.5	11.0	11.5	12.0	12.5	12.9	13.3	13.7	14.1	14.5	14.9
14	3.2	4.5	5.6	6.4	7.2	7.9	8.5	9.1	9.6	10.1	10.6	11.1	11.6	12.0	12.4	12.8	13.2	13.6	14.0	14.3
15	3.1	4.4	5.4	6.2	6.9	7.6	8.2	8.8	9.3	9.8	10.3	10.7	11.2	11.6	12.0	12.4	12.8	13.1	13.5	13.9
16	3.0	4.2	5.2	6.0	6.7	7.3	7.9	8.5	9.0	9.5	9.9	10.4	10.8	11.2	11.6	12.0	12.4	12.7	13.1	13.4
17	2.9	4.1	5.0	5.8	6.5	7.1	7.7	8.2	8.7	9.2	9.7	10.1	10.5	10.9	11.3	11.6	12.0	12.3	12.7	13.0
18	2.8	4.0	4.9	5.7	6.3	6.9	7.5	8.0	8.5	8.9	9.4	9.8	10.2	10.6	11.0	11.3	11.7	12.0	12.3	12.6
19	2.8	3.9	4.8	5.5	6.2	6.7	7.3	7.8	8.3	8.7	9.1	9.5	9.9	10.3	10.7	11.0	11.4	11.7	12.0	12.3
20	2.7	3.8	4.6	5.0	6.0	6.6	7.1	7.6	8.0	8.5	8.9	9.3	9.7	10.0	10.4	10.7	11.1	11.4	11.7	12.0
21	2.6	3.7	4.5	5.2	5.9	6.4	6.9	7.4	7.9	8.3	8.7	9.1	9.4	9.8	10.1	10.5	10.8	11.1	11.4	11.7
22	2.6	3.6	4.4	5.1	5.7	6.3	6.8	7.2	7.7	8.1	8.5	8.9	9.2	9.6	9.9	10.2	10.5	10.9	11.2	11.4
23	2.5	3.5	4.3	5.0	5.6	6.1	6.6	7.1	7.5	7.9	8.3	8.7	9.0	9.4	9.7	10.0	10.3	10.6	10.9	11.2
24	2.4	3.5	4.2	4.9	5.5	6.0	6.5	6.9	7.3	7.7	8.1	8.5	8.8	9.2	9.5	9.8	10.1	10.4	10.7	11.0
25	2.4	3.4	4.2	4.8	5.4	5.9	6.3	6.8	7.2	7.6	8.0	8.3	8.7	9.0	9.3	9.6	9.9	10.2	10.5	10.7
26	2.4	3.3	4.1	4.7	5.3	5.8	6.2	6.7	7.1	7.4	7.8	8.2	8.5	8.8	9.1	9.4	9.7	10.0	10.3	10.5
27	2.3	3.3	4.0	4.6	5.2	5.7	6.1	6.5	6.9	7.3	7.7	8.0	8.3	8.6	8.9	9.2	9.5	9.8	10.1	10.3
28	2.3	3.2	3.9	4.5	5.0	5.5	5.9	6.3	6.7	7.0	7.4	7.7	8.0	8.3	8.6	8.9	9.2	9.5	9.9	10.1
29	2.2	3.2	3.9	4.5	5.0	5.5	5.9	6.3	6.7	7.0	7.4	7.7	8.0	8.3	8.6	8.9	9.2	9.5	9.8	10.0
30	2.2	3.1	3.8	4.4	4.9	5.4	5.8	6.2	6.6	7.3	7.6	7.9	8.2	8.5	8.8	9.0	9.3	9.5	9.8	10.0

^a Quadratic mean diameter = $12 \sqrt{\frac{\text{point tree count}}{\text{plot tree count}}}$; the neutral situation exists when plot radius in feet equals the reciprocal of the gauge constant.

TABLE 2. Arithmetic Mean Tree Diameter by Horizontal Line and Rectangular Plot Tree Counts Assuming the Neutral Situation^a.

PLOT TREE COUNT	LINE TREE COUNT																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	10.0	20.0	30.0	40.0	50.0	50.0	40.0	45.0	50.0	50.0	43.3	46.7	50.0	42.5	45.0	47.5	50.0			
2	5.0	10.0	15.0	20.0	25.0	30.0	35.0	40.0	45.0	50.0	33.3	36.7	40.0	32.5	35.0	37.5	40.0	42.5	45.0	47.5
3	3.3	6.7	10.0	13.3	16.7	20.0	23.3	26.7	30.0	33.3	20.0	22.5	25.0	27.5	30.0	32.5	35.0	37.5	40.0	40.0
4	2.5	5.0	7.5	10.0	12.5	15.0	17.5	20.0	22.5	25.0	14.3	17.1	18.6	20.0	21.4	22.9	24.3	25.7	27.1	28.6
5	2.0	4.0	6.0	8.0	10.0	12.0	14.0	16.0	18.0	20.0	22.0	24.0	26.0	28.0	30.0	32.0	34.0	36.0	38.0	40.0
6	1.7	3.3	5.0	6.7	8.3	10.0	11.7	13.3	15.0	16.7	18.3	20.0	21.7	23.3	25.0	26.7	28.3	30.0	31.7	33.3
7	1.4	2.9	4.3	5.7	7.1	8.6	10.0	11.4	12.9	14.3	15.7	17.1	18.6	20.0	21.4	22.9	24.3	25.7	27.1	28.6
8	1.3	2.5	3.8	5.0	6.3	7.5	8.8	10.0	11.3	12.5	13.8	15.0	16.3	17.5	18.8	20.0	21.3	22.5	23.8	25.0
9	1.1	2.2	3.3	4.4	5.6	6.7	7.8	8.9	10.0	11.1	12.2	13.3	14.4	15.6	16.7	17.8	18.9	20.0	21.1	22.2
10	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0	12.0	13.0	14.0	15.0	16.0	17.0	18.0	19.0	20.0
11	0.9	1.8	2.7	3.6	4.5	5.5	6.4	7.3	8.2	9.1	10.0	10.9	11.8	12.7	13.6	14.5	15.5	16.4	17.3	18.2
12	0.8	1.7	2.5	3.3	4.2	5.0	5.8	6.7	7.5	8.3	9.2	10.0	10.8	11.7	12.5	13.3	14.2	15.0	15.8	16.7
13	0.8	1.5	2.3	3.1	3.8	4.6	5.4	6.2	6.9	7.7	8.5	9.2	10.0	10.8	11.5	12.3	13.1	13.8	14.6	15.4
14	0.7	1.4	2.1	2.9	3.6	4.3	5.0	5.7	6.4	7.1	7.9	8.6	9.3	10.0	10.7	11.4	12.1	12.9	13.6	14.3
15	0.7	1.3	2.0	2.7	3.3	4.0	4.7	5.3	6.0	6.7	7.3	8.0	8.7	9.3	10.0	10.7	11.3	12.0	12.7	13.3
16	0.6	1.3	1.9	2.5	3.1	3.8	4.4	5.0	5.6	6.3	6.9	7.5	8.1	8.8	9.4	10.0	10.6	11.3	11.9	12.5
17	0.6	1.2	1.8	2.4	2.9	3.5	4.1	4.7	5.3	5.9	6.5	7.1	7.6	8.2	8.8	9.4	10.0	10.6	11.2	11.8
18	0.6	1.1	1.7	2.2	2.8	3.3	3.9	4.4	5.0	5.6	6.1	6.7	7.2	7.8	8.3	8.9	9.4	10.0	10.6	11.1
19	0.5	1.1	1.6	2.1	2.6	3.2	3.7	4.2	4.7	5.3	5.8	6.3	6.8	7.4	7.9	8.4	8.9	9.5	10.0	10.5
20	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0

^a Arithmetic mean diameter = $10 \left(\frac{\text{line tree count}}{\text{plot tree count}} \right)$; the neutral situation exists when rectangular plot half-width equals $10/12 k$ where k is the gauge constant.

