

A More General Approach to the Concept of Threshold Population

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Abstract

There have been several efforts to operationally define or empirically measure Christaller's theoretical notion of the lower limit of the range of a good. The most widely used model for estimating this notion, referred to as the threshold population, is the one developed by Berry and Garrison nearly a decade ago. Although Berry and Garrison's model utilizes least squares averaging techniques, it is basically a deterministic model which provides single valued threshold population estimates. In this paper, Berry and Garrison's model is generalized so that for a given level of probability the threshold population for a specific central function is expressed as an *interval* rather than as a *single value*.

The generalized model is used to estimate the threshold population intervals for forty-two different types of central functions found in forty-four communities in Southwestern Michigan. In addition to estimating the threshold intervals, the model also appears to be useful in re-assessing the hitherto accepted notion of "centrality" for certain central functions such as gas stations, grocery stores, and independent auto repair shops. Moreover, the model also serves to specifically point out, but not to solve, the well known problems associated with correctly classifying or identifying multi-functional establishments such as hardware and department stores.

A notion of particular importance in the development of central place theory is that of the range of a good (2,3,4,5). Regarding the theoretical implications of this notion, Christaller states that every good has a range which

"... is a ring around a central place. It has an outer (*upper*) limit and an inner (*lower*) limit. The upper limit of a particular good is determined by the farthest (*economic*) distance from which it can be obtained from this central place; and indeed, beyond this limit, it will either not be obtained, or it will be obtained from another central place. The lower limit of the range of central goods is . . . determined by the minimum amount of consumption of this central good needed to pay for the production or offering of this central good (7)."²

The term "threshold population" has been introduced as a descriptive synonym for Christaller's lower limit of the range of central goods and it is with respect to this concept that this paper is concerned (2,3).

While there is a rather large body of literature devoted to closely related aspects of central place theory, only a relatively small number of research efforts have been directed toward an empirical estimation of the range of a good (5). One of the earliest attempts to establish, at least implicitly, threshold population levels for selected economic activities in small communities was done by Hoffer (8). Arbitrarily estab-

¹ The assistance of Gary Thompson, Michael Biechler, Henry Coppock, David DeTemple, Lewis Easterling and Cyrus Young in gathering these data is gratefully acknowledged.

² The italics are my own.

lishing ten population size groups of small urban centers, Hoffer determined the percentage of the communities within each group that contained a specific economic function. Although Hoffer's work does not concern itself directly with the minimum numbers of people required to support these functions, he postulates

"... three types of specialty stores: drug stores, grocery stores and hardware stores are apt to exist in a town having a population of less than 500. A town having a population of 1,000 is much more complete from the standpoint of a variety of services offered but some of the services which cannot exist in a town of that size have to locate in larger places." (8)

The most well known recent attempt to determine sets of threshold populations is by Berry and Garrison (2). Using an exponential growth model,³ threshold populations were estimated for a set of fifty-two central functions in Snohomish County, Washington. As in almost any empirical research, the estimates which the model provides reflect the time period and the regional setting where it is carried out. Nevertheless, the close adherence of the model to the theoretical constraints and assumptions of central place theory attest to its conceptual validity. Or, as Berry and Garrison state

"... (this) threshold measure thus provides only a crude approximation to a complex notion, but since better measures have yet to be suggested, it is put forth here as a first approximation to the concept of inner range." (3)

The Basic Model

The threshold population model to be developed here is a semi-logarithmic linear trend model, mathematically equivalent to the exponential growth model used by Berry and Garrison (2). The model is developed to test the hypothesis that there is a functional relationship between the dependent variable, or the logarithm of the population size of the members of a set of central places, and the total number of each specific central function offered in each of those communities which comprise the set of central places. Expressed symbolically, this hypothesis reads

$$Y_j = \log P = f(X_k) ; j = 1, 2, \dots, N$$

$$i = 1, 2, \dots, M$$

$$k = 0, 1, \dots, Q$$

where j is one of N central places with population Y ; i is a single specific central function X found in community j , such as a grocery store; M is the total number of different kinds of central functions found in the set of N central places; and k is the number of specific central functions found in any one community j , i.e. three, or up to as many as Q , grocery stores in central place j .

A set of M scatter diagrams are prepared by plotting Y_j and k for each X_i as an ordered pair of numbers, and a linear regression equation

³ The exponential growth model used by Berry and Garrison is of the form $P = ABx$

is determined for each scatter diagram by the least squares method. Each regression equation is based on N observations with unique parameters $\log A$ and $\log B$ which estimate the relationships between the population sizes of the set of central places and the number of specific central functions offered in these communities. Equation one expresses this relationship as

$$Y = \log P = \log A + X_1 (\log B) \quad (1)$$

In the following analysis, forty-two different central functions found in the study area are investigated, hence forty-two individual regression equations are derived. Each equation is based on forty-four observations, the number of central places in the study area. But since the notion under investigation here is the threshold population, or the minimum number of people required to support a single central function, equation one is evaluated with each X_1 set equal to one. One condition required for estimating threshold populations in this manner is that at least one central place must contain more than one of the specific central functions in question. If this condition is not met, the parameters $\log A$ and $\log B$, and consequently the threshold population estimate, are meaningless.

However, as with any least squares fit, the predicted threshold populations are subject to errors of estimate. That is, for a given level of probability, the estimated threshold population for each central function varies about the "true" threshold population. In a more realistic sense then, the threshold population should actually be considered as an *interval* rather than a *single value*. Thus, the threshold interval for each central function or X_1 is operationally defined by the standard confidence limits for least squares estimates in "equation," or more precisely inequality, two

$$Y + t_{\frac{1}{2}\alpha} S_{y \cdot x_1} \left\{ \frac{1}{N} + \frac{(X_1 - \bar{X}_1)^2}{(N-1) S_{x_1}^2} \right\}^{\frac{1}{2}} < \mu_{y \cdot x_1} < Y + t_{1-\frac{1}{2}\alpha} S_{y \cdot x_1} \left\{ \frac{1}{N} + \frac{(X_1 - \bar{X}_1)^2}{(N-1) S_{x_1}^2} \right\}^{\frac{1}{2}} \quad (2)$$

where $\mu_{y \cdot x_1}$ is the "true" threshold population or a specific central function and Y is the estimated threshold population from equation one with each x_1 set equal to one. t is the sampling distribution, and the confidence coefficient $\alpha = .05$, $S_{y \cdot x_1}$ is the standard error of estimate and $S_{x_1}^2$ is the variance associated with the total number of individual functions found in the study area towns. It should be stressed here that the threshold confidence interval presented above corresponds *only* with Christaller's lower limit of the range of a good and does not in any way allude to the theoretical upper limit or maximum range of a good.

If a central function is found repeatedly in places with a population size below the indicated minimum threshold level or is absent from a community where the population size exceeds the maximum threshold

limit, four alternative explanations are advanced. First, the function may not be a central one. This is to suggest that in Berry and Garrison's ordered ranking of central goods, the goods at the lowest end of the hierarchy may have such a low population threshold that they may be offered independent of central places (4). Christaller recognizes that certain goods

"... are of such a local nature that we cannot call them central goods; they are goods which are offered for sale in every village (*and indeed, in the rural countryside itself*), e.g., the food and home wares demanded by the households or the services of the elementary schools." (7)⁴

Of course, with such a low population threshold, these goods also appear in the smallest of central places.

Secondly, the function may be present in a town, but not as a unique central activity. For example, consider a hardware store which offers a wide variety of central functions such as general hardware items, paint, household appliances, sporting goods, lawn and garden supplies, toys, heating and plumbing fixtures, farm supplies, pet supplies and electrical apparatus. Paradoxically then, the hardware store may be thought of as a single identifiable central function and at the same time, it must be considered as a collection of several functions operating as a single retail outlet. This same multiple offering of functions is also present in general merchandise stores, supermarkets, large drug stores and department stores. Thus, if a single central function is offered by a multiple function retail outlet, it may preclude the occurrence of an individual store attempting to singularly offer that function. This notion manifests itself as a problem of classification. Thomas suggests this problem can be partially circumvented by categorizing the offering of central goods in terms of establishments, functions and functional units (10).

The third alternative explanation questions the "population size" of a central place. Christaller recognizes that in addition to the town population, the rural population in the complementary area is necessary for the support of the central functions within the central place. Beckmann has succinctly formalized this notion by showing that the size of a city is proportional to the rural population which it serves (1). Bunge points out that Berry and Garrison did not consider this notion and suggests that their threshold populations for the several functions may be underestimated (6). The model presented here, like the one used by Berry and Garrison does not consider the rural population in the "population size" of a central place. But the rural population in the area under consideration in this study is quite sparse; so sparse in fact that there is good reason to believe that the threshold estimates offered here are not in serious error. However, in areas where the rural population is densely distributed, cognizance of this fact should be incorporated into the model.

The fourth explanation suggests that the central place may simply be in a state of functional disequilibrium. It is recognized that when a

⁴ The italics are my own.

central place experiences rapid population growth or decline the corresponding addition or loss of central functions does not coincide perfectly with the population change. Since this notion is a time varying process, and the collection of data reflects only one point in time, then the presence or absence of certain central functions from a particular place may only be temporary.

An Empirical Test of the Model

Forty-four communities in Allegan, Barry, Calhoun and Kalamazoo counties of Southwestern Michigan are considered in this study (see Fig. 1). The population sizes of these communities range from 3,125 in

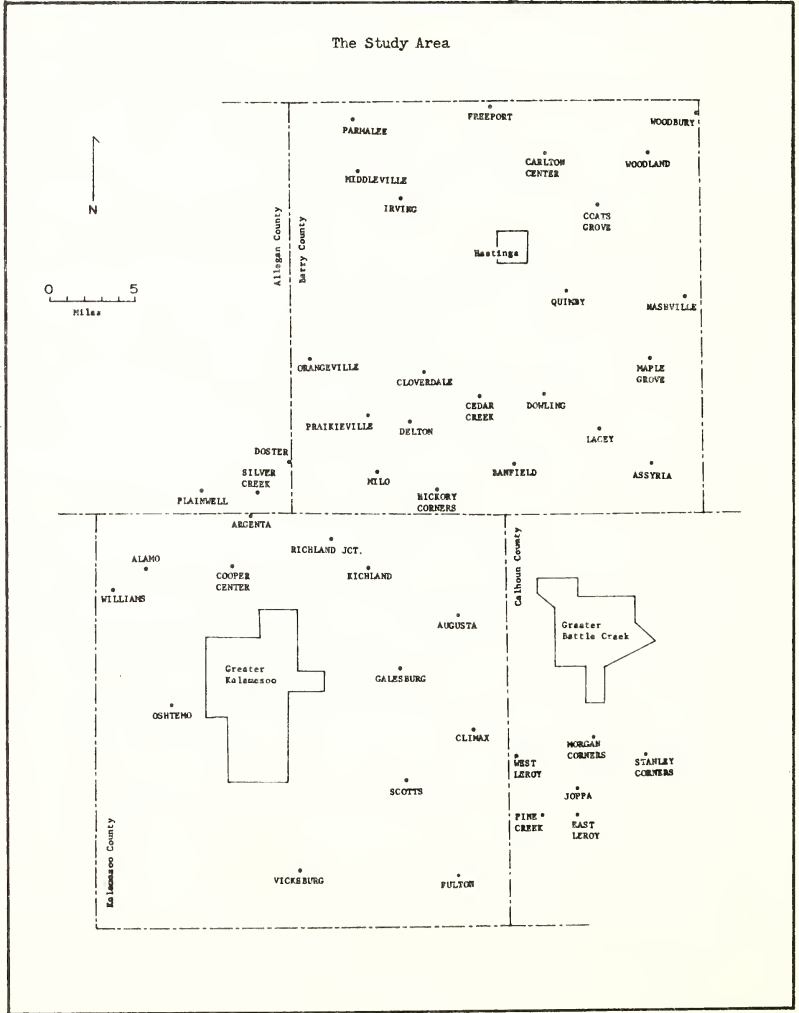


Figure 1.

Plainwell to 16 in Milo. The number of variable central functions ranges from 105, again in Plainwell, to one found in six separate communities.⁵

The data used in this study were collected by field survey and the primary central activities carried on in each community were recorded. For the most part, the central activities investigated correspond rather closely to Thomas' definition of an establishment, or the primary activity carried on in an individual building (10). Hardware stores, therefore, were identified as a single function and the multiple function offerings within them were not counted separately. The same criterion was applied to other multi-functional units such as supermarkets, general merchandise stores and variety stores. However, exceptions were permitted where a gasoline function was operated in conjunction with a small town grocery store or hardware store and a dry cleaning pick up station was present in another local business activity. Also repair functions were counted only if they operated independently from a major sales function such as furniture reupholstery and repair, shoe repair and auto repair. Wherever possible, an attempt was made to keep classification of retail and business services compatible with the SIC classification listed in the 1963 Census of Business. With this notion in mind and following the rules adopted by the 1958 Census of Business, the small town general store was classified as a grocery store (11). For a complete listing of the functions found in the study towns, see Table 1.

Of the forty-two functions which appear more than once in at least one of the study places, ten were present in all towns whose population size exceeded the minimum threshold limit. Twenty-three functions were found in communities whose population size was below the minimum threshold and some twenty-six functions were absent from a few towns whose population size exceeded the maximum threshold limit.

Nearly 50% of the communities with a population size under the minimum threshold level for food stores and churches contained at least one. The same holds true for over 40% of the communities with regards to a gasoline function and approximately 20% of the towns contained an auto repair function in disrespect for its lower threshold limit. Finally, just over 16% of the communities had an elementary school in deference to its lower population threshold. Of the four alternative explanations offered above for these deviations, the first one seems appropriate. Christaller specifically indicates that a food function and the elementary school may not be central functions. Certainly, the automobile has become an ubiquitous necessity in the United States and the demand for automobile services and maintenance can be considered equally ubiquitous. In fact, the empirical evidence offered here appears to support the notion that the demands for auto services are even greater in their local nature than those for elementary schools.

An analysis of the other end of the spectrum suggests that another

⁵These figures refer only to those central functions which vary in number from place to place and therefore do not necessarily indicate the total number of central functions in these central places.

with new car sales. Also, modern service station complexes usually offer rather extensive repair services in addition to their regular function of pumping gasoline and other auto services. Complete lines of sporting goods are found in hardware stores, certain dry goods and general merchandise stores, as well as larger drug stores. Thus, while the sporting goods function is present in the larger communities, it does not exist as a single shopping entity. The same argument also applies to household appliances and heating and plumbing functions.

Two rather obvious non-central functions are pointed out in this section of the analysis. Bait shops, as individual shopping entities are absent in all of the largest towns in the study. In fact, the only bait stores in the study area are located in those towns situated immediately adjacent to popular local fishing areas. The other non-central function absent from over half of the larger communities is the fuel dealer (considered here to be a fuel oil dealer or a bottled gas dealer). Not only is this function subject to rather strict county zoning ordinances, but by urban standards, this type of fuel function is land use extensive and is forced into rent competition where it cannot successfully compete. Secondly, this good is generally ordered by phone or mail to be delivered to the consumer and according to Christaller this method does not require a central place (7). Therefore, one would expect to find fuel dealers at most any location, subject to a communication and transportation constraint.

The functions which are present in all towns whose population sizes are above the minimum threshold limit are clearly central functions. Furthermore, each function is readily identified and not likely to be misclassified or duplicated in another central function. In this study, these functions are variety stores, dairy products stores, drug and apothecary stores, florists shops, banks, newspaper and printing establishments, laundromats, dry cleaning establishments, funeral homes, and auto parts stores.

In summary, some reflections on the model appear to be in order. The generalized model presented here suggests a more comprehensive operational definition for Christaller's notion of local variation in the lower limit of the range of a good. In addition, the model provides a means of critically evaluating the centrality of a specific function under local conditions as well as accentuating the inadequacies in recording and classifying the occurrence of central functions in central places. However, the ultimate criterion for evaluating a model, or for that matter an operational definition, is its usefulness of understanding the phenomenon under investigation within theoretical constraints. To this end, the more general model presented here appears to be useful in operationally fulfilling one of the elusive assumptions upon which central place theory is predicated.

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