On Possible Nuclear Synthesis Of Helium at Low Densities

TORSTEN ALVÄGER, RODNEY DARRAH and DENNIS KAWATKA
Department of Physics, Indiana State University,
Terre Haute, Indiana 47809

Abstract

The possibility of nuclear synthesis of helium from hydrogen in a plasma of very low density (density about 10^{-23} gram per cubic centimeter) is discussed. It was found that under suitable conditions the helium to hydrogen mass ratio may reach 0.01. It is assumed that the energy source driving the synthesizing reactions comes from matter-antimatter annihilations.

Introduction

Synthesis of elements is usually assumed to occur in regions of high density of matter. As far as the formation of helium is concerned, which is the topic of this paper, there are especially two high density regions that have been considered (3): 1) the interior of certain stars such as the sun, and 2) very massive objects of unknown origin that are in a process of collapse or explosion such as the hypothetical universal fireball. Both these environments can lead to the creation of a substantial amount of helium from protons or a mixture of protons and neutrons. It is possible that the correct solution involves one or both of these theories. It is also possible that other synthesizing mechanisms have been of importance.

In this paper we consider the formation of helium from hydrogen in a plasma of very low density, a density of the order of 10^{-23} g/cm^3 which corresponds to a particle density of about one per cm^3. This somewhat unusual condition for element synthesis is of interest to investigate (2, H. Alfven, personal communication) in connection with the cosmological model of Alfven and Klein (1). In the universe considered by these authors, matter and antimatter should enter in a symmetrical way. Such a world would undoubtedly contain regions where a certain amount of mixing of matter and antimatter, primarily protons and antiprotons, occur. Especially there should be such areas at the border lines between matter and antimatter systems. The energy generated in annihilation processes may be used for heating the plasma to temperatures where nuclear reaction can take place.

It is hardly possible or necessary at the present time to specify in detail what an annihilation region may look like. The emphasis here is rather to investigate, in a first approximation, the conditions that would be necessary to be able to form elements out of hydrogen.

The following general assumptions are made:

1) There exist regions of matter-antimatter of such a low material density that annihilation processes do not give rise to explosion. This implies, in general, densities of the order of 10^{-23} g/cm^3 or

less. At such densities reactions with short-lived material can be neglected. That means, among other things, that neutrons are of no importance in the formation of the elements. Of course, there may exist regions of high densities where annihilation processes give rise to explosion phenomena. It is not excluded that elements are synthesized in such regions. That possibility is not considered in this paper, however.

2) The energy released in proton-antiproton annihilation reactions can be used to heat the plasma to a very high temperature. The electrons emerging as final products in the annihilation processes have an energy spectrum that is peaked at about 30 MeV. We assume that the energy spectrum of the protons participating in the synthesizing reactions may also peak around this value. We also assume that the plasma may cool rapidly.

3) The plasma may be contained by magnetic fields. The upper limit on the necessary size of the magnetic field is given by the condition that the field energy should not exceed the particle energy density K or $H^2/8\pi < K$. With a proton energy of 30 MeV and a particle density of one per cm^3, the field strength H is about 0.04 gauss. This value is not unrealistically large if restricted to small regions of space.

4) In the initial phase, only hydrogen-antihydrogen was present. Only the production of heavier, matter particles is discussed: not antimatter particles.

Nuclear Reactions and Their Rates

From hydrogen, there are basically two possibilities for forming deuterium directly

$$H + H = D + e^{+} + \nu \qquad [1]$$

$$H + H = D + \pi^{+} \qquad [2]$$

The first of these reactions is supposed to play an important role in various stars, as the sun, where the central temperature is not too high. Reaction [2] is generally regarded being of less interest as far as nucleosynthesis is concerned. The reason is that protons with energies required to produce pions are believed to be rare at places where nucleosynthesis is supposed to occur. Under the general conditions considered in this paper, reaction [2] is the only one that is useful. Reaction [1] is excluded because the reaction rate is too slow to allow any appreciable amount of deuterium to be formed in the dilute plasma discussed here.

The threshhold for reaction [2] is about 70 MeV in the center of mass system. This means that temperatures of the order of $10^{11} - 10^{12}\,°K$ are necessary to make deuterium. However, it is only this nucleus that requires this high temperature. The formation of He^3 and He^4 can proceed through low energy reactions in steps resembling those in other synthesizing theories.

Thus, He^3 may be produced in

$$H + D = He^3 + \gamma \qquad [3]$$

$$D + D = H + T = H + He^3 + e^- \nu \qquad [4]$$

$$D + D = n + He^3 \qquad [5]$$

While He^4 may be formed in

$$D + He^3 = H + He^4 \qquad [6]$$

$$He^3 + He^3 = 2H + He^4 \qquad [7]$$

and some reactions involving Be7 and Li. These are not considered to be of importance here. In general, it is also necessary to take into account reactions such as

$$H + D = 2H + n \qquad [8]$$

$$H + He^3 = D + 2H \qquad [9]$$

$$H + He^4 = D + 2H + n \qquad [10]$$

and others in which high energy particles split heavier elements into lighter ones. These reactions are of less importance for temperatures well below 10^9 °K.

It is also possible that reactions such as

$$H + D = He^3 + \pi^\circ$$

$$D + D = He^4 + \pi^\circ$$

can play an active role. They are not considered in this discussion, however.

The number of reactions per unit volume per second produced by two nuclear species denoted by subscripts i and j is given by

$$r = n_i \; n_j \; (1 + \delta_{ij})^{-1} \langle \sigma v \rangle_{ij}$$

where n_i and n_j are number densities of species i and j respectively. δ_{ij} the kronecker delta and $\langle \sigma v \rangle_{ij}$ as usual stands for

$$\langle \sigma v \rangle_{ij} = \int f(v\ T)\ \sigma(v)\ \ v\ dv$$

f(v T) is the Boltzman distribution function for one nucleus of temperature T, v their relative velocity and σ the cross section.
$\langle \sigma v \rangle_{ij}$ can also be written as

$$\langle \sigma v \rangle_{ij} = 6.2 \times 10^{-24}\ A^{1/2}\ T_9^{-3/2} \int \sigma\ E\ \exp\ (-11.6E/T_9) dE \qquad [11]$$

where E, the energy, is expressed in MeV, σ in barns and T_9 stands for temperature in units of 10^9 °K. A is the reduced mass number $= A_i A_j / A_i + A_j)$.

The quantity $\langle \sigma v \rangle_{ij}$ can be found from relation [11] when the cross section is known. In Figure 1, $\langle \sigma v \rangle_{ij}$ is given as a function of temperature for various reactions of interest here. In those cases where sufficient experimental data is available, the integral in relation [11] has been explicitly calculated. In other cases, approximate expressions for $\langle \sigma v \rangle_{ij}$ as given by Wagoner, Fowler and Hoyle (4) have been used.

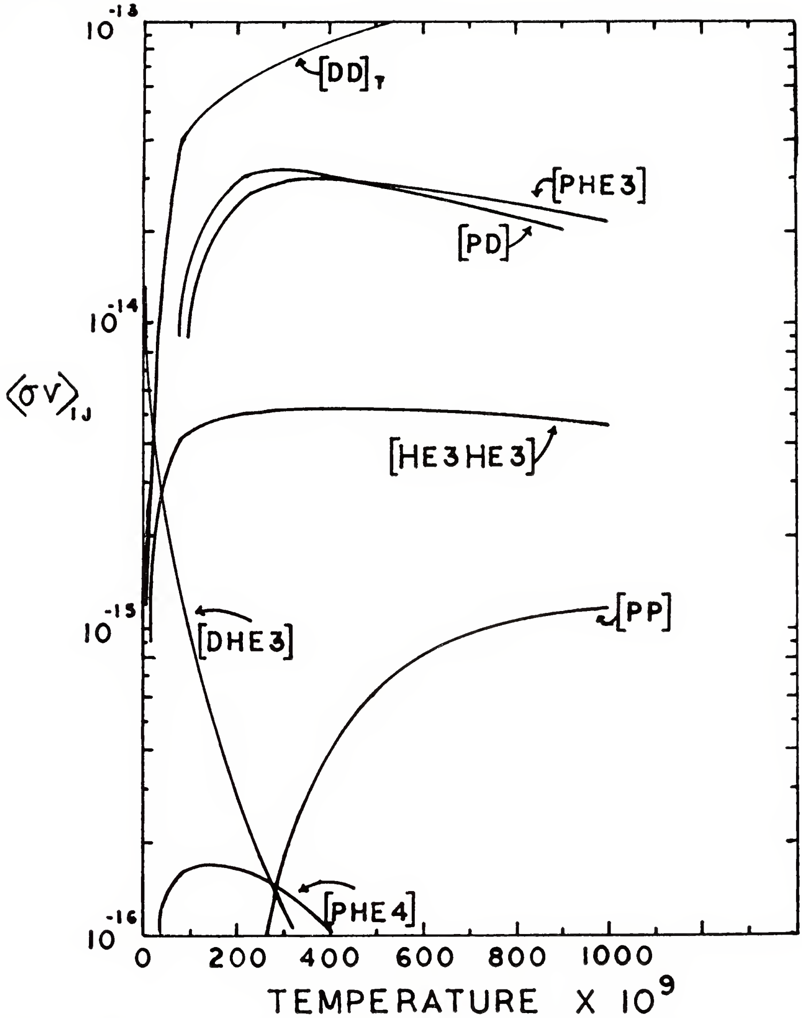

FIGURE 1. *The curves represent the values of the reaction rates as a function of temperature. The reaction rates $<\sigma\ v>ij$ (c.f. equation [11]) were calculated from temperatures of O^0K to $1000 \times 10^9\ {}^0K$.*

In describing the abundance of a given nucleus it is convenient to use the mass fraction defined by the relation

$$X(k) = A_k \frac{n_k}{N}$$

where A_k is the mass number for the species k and n_k is the number density. N is the Avogadros number.

The rate equation for the formation of helium out of hydrogen may now be written up. The equation for deuterium, for instance, is

$$\frac{1}{2}\frac{dX(D)}{dt} = X^2(H)\ [H\ H] + \frac{1}{3}\ X(H)\ X(He^3)\ [H\ He^3] + \frac{1}{2}\ X(H)\ X(He^4)\ [H\ He^4] - X(H)\ X(D)\ [H\ D] - X(H)\ X(D)\ [H\ D] - \frac{1}{4}\ X^2(D)\ [D\ D] - \frac{1}{4}\ X^2(D)\ [D\ D]_{He^3} - \frac{1}{3}\ X(D)\ X(He^3)\ [D\ He^3] \qquad [12]$$

where the different terms refer to reactions [2, 9, 10, 8, 3, 4, 5 and 6] respectively. The expression [ij] stands for $<\sigma v>_{ij}\ \rho\ N$.

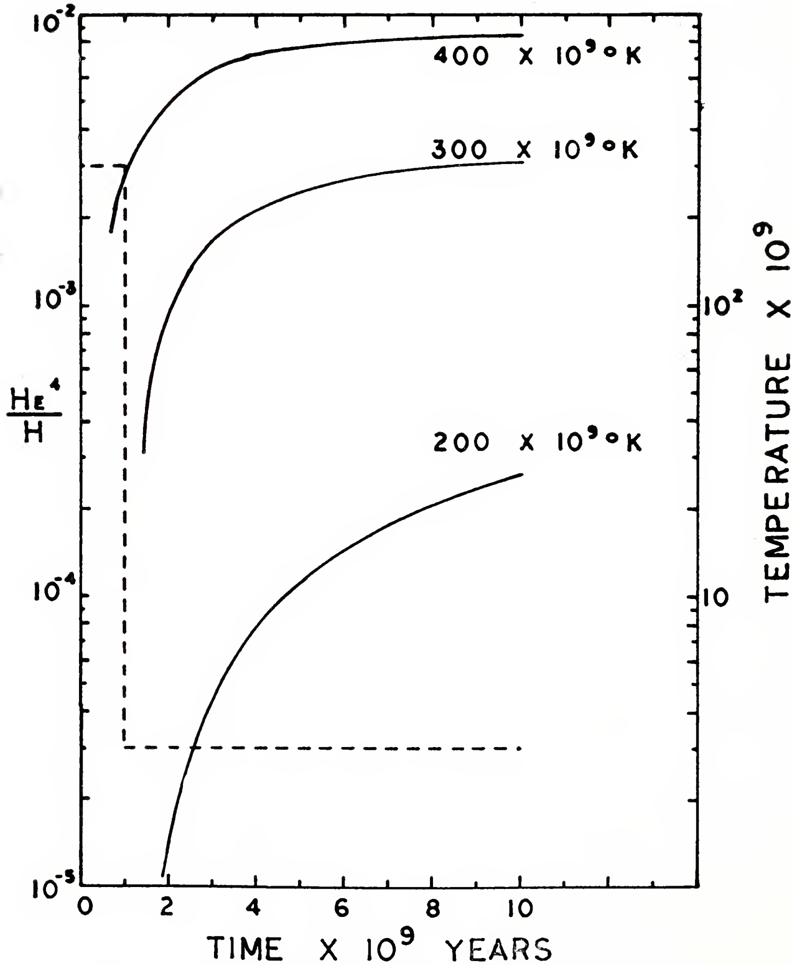

FIGURE 2. *Helium to hydrogen mass ratio as a function of time for three different initial temperatures: $400T_9$, $300T_9$ and $200T_9$ respectively ($T_9 = 10^9$ °K). The dotted curve shows the choice of the temperature variation: a short, high temperature phase followed by a long, low temperature phase (equal to $3T_9$ in all three cases). $He^4/H = X(He^4)/X(H)$.*

Some Results

The rate equations, *i.e.* [12], and the appropriate equations for H, He^3 and He^4 have been solved numerically for various temperatures. It goes without saying that favorable conditions for helium formation should involve a high temperature phase of the plasma when deuterium is formed, followed by a low temperature phase when helium 4 is allowed to be synthesized without the destructive reactions [9] and [10] present. In general it is found that the output of He^4 is increased if this temperature variation is repeated in a periodic fashion. The time scale involved in the calculations has in general been 10^{10} years.

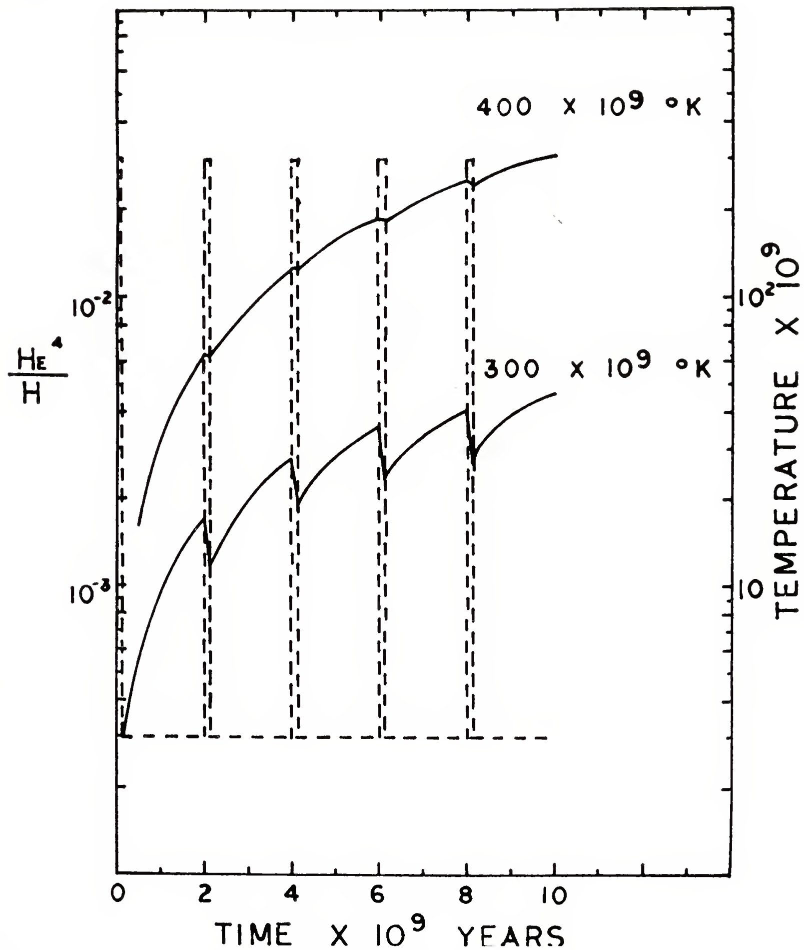

FIGURE 3. *Helium to hydrogen mass ratio (solid curves) as a function of time for a temperature variation as shown by the dotted curve. The two solid curves correspond to initial temperatures of $400T_9$ and $300T_9$ respectively ($T_9 = 10^9$ °K). $He^4/H = X(He^4)/X(H)$.*

Figure 2 shows the result from a special choice of the temperature variation; a short, high temperature phase is followed by a long, low temperature phase. The three curves correspond to three different initial temperatures.

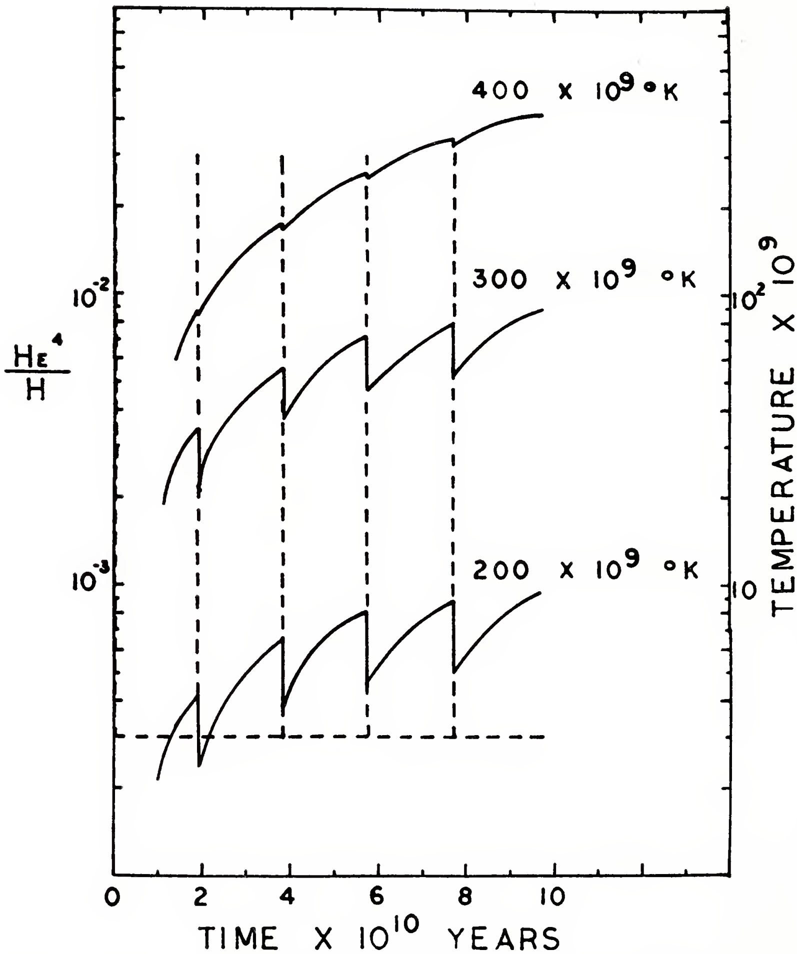

FIGURE 4. *Helium to hydrogen mass ratio (solid curves) as a function of time for a temperature variation as shown by the dotted curve. The pattern of the temperature variation is the same as shown in Figure 3, the only difference being that the low temperature phase is stretched here a factor of 10. The three solid curves correspond to initial temperatures of $400T_9$, $300T_9$ and $200T_9$ respectively ($T_9 = 10^9$ °K). $He^4/H = X(He^4)/X(H)$.*

Figure 3 gives the outcome of a somewhat more interesting case than the one shown in Figure 2. A short, high temperature phase is followed by a long, low temperature phase. This pattern is then repeated 4 times, which is approximately the optimum number for the time scale considered. It is seen that a certain improvement in the helium to hydrogen mass ratio is obtained compared to the non-periodic burning considered in Figure 2. This is more pronounced if the time scale for the low temperature period is prolonged as shown in Figure 4.

The conclusion from this preliminary study is that an appreciable amount of He^4 may be produced in low density plasma. Further investigations are necessary to perform to find the optimum conditions.

Acknowledgment

One of the authors (T.A.) wishes to thank Professor H. Alfven for encouraging discussions.

Literature Cited

1. ALFVEN, H. 1965. Antimatter and the Development of the Metagalaxy. Rev. of Mod. Phys. **37**:652-665.
2. ALVÄGER, T. 1967. Are heavy ion reactions responsible for the nucleosynthesis of uranium and thorium? Arkiv for Fysik **36**:529-534.
3. TAYLER, R. J. 1966. The origin of the elements. Rep. on Prog. in Phys. **29**:489-536.
4. WAGONER, R. V., W. A. FOWLER and F. HOYLE, 1967. On the synthesis of elements at very high temperatures. The Astroph. J. **148**:3-49.