Hard cast-iron shoe.

Heavy pressure.

Speed increasing from 10 to 65 miles per hour.

Coefficient of friction decreasing from 25 per cent. to 12 per cent.

3. The coefficient of friction of cast-iron brake shoes is practically constant with variations in temperature of shoe and wheel within the limits of the experiments.

Diamond Fluorescence.

[Abstract.]

By Arthur L. Foley.

A year ago I presented to the Academy an account of an experiment with a diamond and a photographic dry plate (Proceedings of Academy, 1899, p. 94). Later experiments have confirmed the theory presented. It has been found that a low temperature is favorable to the success of the experiment.

A Theorem in the Theory of Numbers.

By Jacob Westlund.

Let n be any prime number and let

$$S_k = 1^k + 2^k + 3^k + \ldots\ldots\ldots\ldots + (n-1)^k.$$

Then

$S_k \equiv 0$, mod n, when $k \not\equiv 0$, mod $(n-1)$ and $S_k \equiv -1$, mod n, when $k \equiv 0$, mod $(n-1)$.

Proof. Consider the congruence.

$$x^{n-1} - 1 \equiv (x-1)\ (x-2) \ldots\ldots\ldots\ldots (x - \overline{n-1}), \text{ mod } n.$$

This congruence is evidently satisfied by the $n-1$ incongruent numbers.

$$1, 2, 3, \ldots\ldots\ldots\ldots\ldots\ldots (n-1).$$

But the congruence is of the degree $n - 2$, since it may be written

$$+ a_1 x^{n-2} - a_2 x^{n-3} + a_3 x^{n-4} \ldots\ldots\ldots\ldots - a_{n-1} - 1 \equiv 0, \text{ mod } n, \text{where}$$

$$a_1 = 1 + 2 + 3 + \ldots\ldots\ldots\ldots + (n - 1)$$

$$a_2 = 1 . 2 + 1 . 3 + \ldots\ldots\ldots\ldots + 2 . 3 + \ldots\ldots\ldots\ldots$$

$$a_3 = 1 . 2 . 3 + 1 . 2 . 4 + \ldots\ldots\ldots\ldots$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$a_{n-1} = 1 . 2 . 3 \ldots\ldots\ldots\ldots (n - 1).$$

Hence, since the number of roots of a congruence with prime modulus can not be greater than the modulus, the given congruence must be identical. Hence,

$a_1 \equiv 0$, mod n.

$a_2 \equiv 0$, mod n.

$a_{n-2} \equiv 0$, mod n.

$a_{n-1} \equiv 1$, mod n.

But from the theory of symmetric functions we have the following relations:

$$S_1 - a_1 = 0.$$

$$S_2 - S_1 a_1 + 2a_2 = 0.$$

$$\ldots\ldots\ldots\ldots\ldots\ldots$$

$$S_{n-2} - S_{n-3} a_1 + \ldots\ldots\ldots\ldots - (n - 2) . a_{n-2} = 0.$$

$$S_{n-1} - S_{n-2} . a_1 + \ldots\ldots\ldots\ldots + (n - 1) . a_{n-1} = 0.$$

$$S_n - S_{n-1} . a_1 + \ldots\ldots\ldots\ldots + S_1 . a_{n-1} = 0.$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Hence,

$S_1 \equiv 0$, mod. n.	$S_{2n-3} \equiv 0$ mod. n.
$S_2 \equiv 0$, mod. n.	$S_{2n-2} \equiv -1$ mod. n.
....	$S_{2n-1} \equiv 0$ mod. n.
$S_{n-2} \equiv 0$, mod. n.	....
$S_{n-1} \equiv$ 1 mod. n	
$S_n \equiv 0$, mod. n.	
....	

or

$S_k \equiv 0$, mod n, when $k \not\equiv 0$ mod $(n - 1)$ and $S_k \equiv - 1$, mod n, when $k \equiv 0$ mod $(n - 1)$.