ON THE ALTERNATING CURRENT DYNAMO. BY W. E. GOLDSBOROUGH.

Consider the case of a simple alternator having but one armature coil that rotates in a magnetic field of uniform intensity about an axis at right angles to the direction of the lines of force. If successive instants of time during one revolution of the coil are counted from the instant that the coil passes a line drawn through its axis of rotation and perpendicular to both the axis of rotation and the direction of the magnetic flux, the value of the induction piercing the coil at any instant during one cycle is expressed by the equation

$$N = N_{max} \cos wt, \qquad (1)$$

in which N_{max} equals that portion of the flux that passes through the coil at the instant the plane of the coil is at right angles to the direction of the lines of force and w represents its angular velocity. The instantaneous value of the E. M. F. generated in the coil will be, by Faraday's law

$$e = -\frac{dN}{dt} = w N_{max} \sin wt,$$

$$= E \sin wt \qquad (2)$$

since its maximum value

$$E = w N_{max} \qquad (3)$$

If the coil is closed through a circuit of resistance R_1, inductance L_1 and capacity C_1, the resistance and inductance of the coil itself being R and L respectively a current i will begin to circulate and we can write the equation of E. M. Fs. of the circuit in the form

$$e = (R + R_1)\, i + (L + L_1)\frac{di}{dt} + \int\frac{idt}{C_1}.$$

From this expression we can derive the equation of the current in terms of the constants of the circuit and the maximum value of the E. M. F. developed in the coil and obtain

$$i = \frac{E}{\sqrt{[R + R_1]^2 + [\frac{1}{C_1 w} - (L + L_1)w]^2}}$$

$$\sin\left\{wt - \tan^{-1}\left[\frac{1}{C_1 w(R + R_1)} - \frac{(L + L_1)w}{R + R_1}\right]\right\} \qquad (4)$$

which expresses the instantaneous value of i as soon as a condition of cyclic stability has been attained.

Equations (1), (2) and (4) are the general equations that cover the working of alternating current dynamos; they have been subjected to graphical analysis, the results of which are exhibited in the figure opposite page 80, and are discussed in the following paragraphs:

Suppose a circuit in which the inductance is zero, the capacity infinite and the resistance variable, to be subjected to the influence of a simple harmonic E. M. F. that is generated by an alternator having a constant armature inductance for all values of armature current, a constant field excitation and a constant speed. Under these conditions the virtual value of the E. M. F. at the brushes of the alternator just before the circuit is closed will be,—

$$\overline{E} = w\,N_{max} \div \sqrt{2}\,; \tag{5}$$

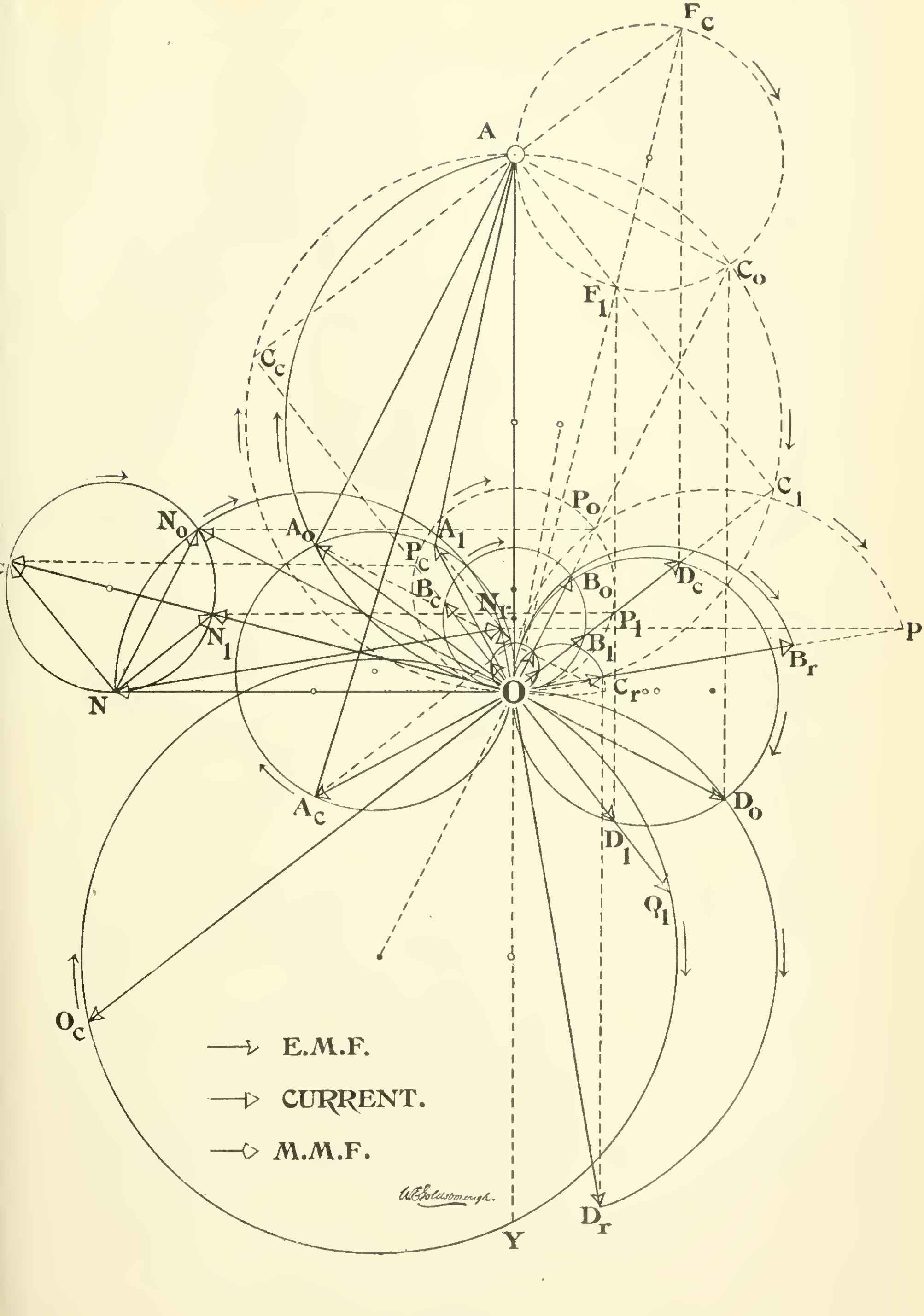
which is represented by the vector OA in the figure. The vector ON is laid off at right angles to OA to represent the value of the M. M. F. producing N_{max}. It is drawn 90° in advance of the E. M. F. it induces in accordance with the relation exhibited in equations (1) and (2). At the time of closing the circuit suppose the external variable non-inductive resistance to have a value R_1, and that the constant armature resistance has a value R and the constant armature inductance a value L. Then the equation of the current will assume the form:

$$i = \frac{E}{\sqrt{(R + R_1)^2 + L^2 w^2}} \sin\left[wt - \tan^{-1}\frac{Lw}{R + R_1}\right] \tag{6}$$

and its virtual value—

$$\overline{I} = \frac{\overline{E}}{\sqrt{(R + R_1)^2 + L^2 w^2}} \tag{7}$$

which we can represent by the vector OB_o lagging $\tan^{-1} \dfrac{Lw}{R + R_1}$ degrees behind OA. This armature current will react upon the magnetizing forces due to the constant field excitation, and by virtue of the inductance of the armature will produce an M. M. F. in phase with itself which is represented by the vector NN_o, drawn parallel to the current vector from the positive extremity of ON. This armature M. M. F. sets up a cyclic magnetization developing a counter E. M. F. OD_o lagging 90° degrees behind the current, and there is a loss of effective E. M. F. due to the armature resistance that is shown by the short E. M. F. vector in phase with OB_o, therefore the total loss of E. M. F. in the armature will be the resultant of these two vectors or OA_o. The effective E. M. F. that overcomes the resistance of the non-inductive external circuit will be the vector A_oA, since it completes the E. M. F. triangle on OA and is in phase with the current


E.M.F.
CURRENT.
M.M.F.
W.E. Goldsborough.

OB_o. The total effective E. M. F (OC_o) that overcomes the total ohmic resistance ($R + R_1$) of the circuit, is due to the cyclic magnetization set up by the M. M. F. vector ON_o. ON_o is the resultant of ON and NN_o and as shown by the geometry of the figure it is 90° in advance of the current, and therefore of A_oA, as it should be. The projection of NN_o on ON is the component of the armature M. M. F. that acts against the field magnetization, *i. e.*, it is *a measure of the armature reaction.* The projection of NN_o on OA is likewise a measure of the cross-magnetizing action of the armature.

Having constructed the initial diagram we can now follow out what takes place when the resistance of the external circuit is varied. Suppose R_1 is reduced to a value R_r. The current vector head B_o will move out along the semi-circle OB_oB_r until equilibrium is again established in the circuit by the current reaching its maximum possible value under the new conditions.* The vectors OA and ON retaining their positions, all the other vectors involved will reach their final values corresponding to the new current by following the arcs of the circles passing through their positive extremities to the positions designated by the common subscript letter (r). The correctness of the variations indicated can be readily verified by an inspection of the geometry of the figure in connection with equation (7).

In the present case R_1 has been reduced to zero; in other words the subscripts (r) indicate what takes place when a machine whose armature inductance is large, as well as constant, is short circuited. A_o moves up to A, and the E. M. F. at the brushes is zero. The current assumes an angle of lag of almost 90° behind the total internal armature E. M. F. OA, the *armature reaction* almost counterbalances the M.M.F. of the fields, and the resultant M. M. F. ON_r is just sufficient to develop the E. M. F. OC_r that overcomes the resistance of the armature.

Returning to the initial conditions, suppose we increase the value of L_1 from zero to some value L_1, *i. e.*, suppose we introduce inductance into the external circuit. The virtual value of the current will then be expressed by the equation

$$\bar{I} = \frac{E}{\sqrt{(R + R_1)^2 + (L + L_1)^2 w^2}} \qquad (8)$$

and it will lag behind the internal E. M. F. $\bar{E}$ or OA, by an angle

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*See Bedell and Crehore's Alternating Currents, page 223.

$$\phi = \tan^{-1}\left(\frac{L + L_1}{R + R_1} w\right). \tag{9}$$

Referring to the figure, the new positions assumed by the variable vectors, owing to the introduction of L_1, are designated by the subscript letter (1). The current will decrease and its vector head move along the circle $OB_cB_oB_1O$ until a state of equilibrium exists between the forces involved. The E. M. F. that overcomes the resistance and inductance of the armature will decrease also and move to the position OA_1, its vector head following the circle $OA_cA_oA_1O$, and the E. M. F. at the collector rings will first decrease and then increase to a final value A_1A. The introduction of inductance into the external circuit brings the E. M. F. at the collector rings and the total internal E. M. F. (OA) more nearly into phase; it, however, causes a lag angle F_1OB_1 to be introduced between the collector E. M. F. and the current. The inductance E. M. F. of the armature decreases along the circle $OD_cD_oD_1O$ to a value OD_1 and the inductance E. M. F. of the external circuit increases from zero along the circle YQ_cOQ_1Y to a value OQ_1. The resultant M. M. F. will be ON_1, and it is seen that while the armature reaction has remained very nearly constant the cross-magnetizing effect has been reduced about 50 per cent.

From our initial conditions as indicated by the subscript letter (o) we can also study the effects produced by the introduction of capacity into the external circuit. If the value of C_1 is reduced from infinity to some value C_c, the virtual value of the current will change to

$$\bar{I} = \frac{\bar{E}}{\sqrt{(R + R_1)^2 + \left(\frac{1}{C_c w} - Lw\right)^2}} \tag{10}$$

and the angle between OA and the current will have a value

$$\phi = \tan^{-1}\left[\frac{1}{(R + R_1)C_c w} - \frac{Lw}{(R + R_1)}\right] \tag{11}$$

In consequence of this change the current vector will assume the position OB_c and the other variable vectors will move to their corresponding positions shown by the subscript letter (c). The current in its new position is not only in advance of the E. M. F. (A_cO) at the brushes, but is also in advance of the E. M. F. OA, since it has moved from B_o to a maximum value when passing OA, and then decreased in value.[1]

1. See Bedell and Crehore's Alternating Currents, p. 297.

The collector E. M. F., on the other hand, steadily increases as the capacity decreases till it reaches a value A_cA much greater than the open circuit E. M. F. of the machine. A resonant effect comes into play here after the capacity of the line neutralizes the inductance of the armature that is very well illustrated by the figure: the line A_cA will be a maximum when it passes from A through the center of the circle $OA_cA_0A_1O$, and will represent the greatest difference of potential that can possibly exist between the brushes so long as R and R_1 remain unchanged in value. This rise in potential is due to the current being in *advance* of the vector OA, for the position of the armature M. M. F. vector is also advanced, and NN_c increases the total flux in the air-gap instead of diminishing it. The cross-magnetizing action of the armature, however, remains approximately the same.

The introduction of capacity into the line causes the inductance E. M. F. of the armature to move to the position D_c, and the reactance E. M. F. of the external circuit to decrease through zero and then increasing, assume a position Q_cO, considerably in advance of the collector E. M. F., and 90° in advance of the current OB_c.

The arrows indicate the relative direction of motion of the vectors as the resistance is varied from infinity to zero, or as the reactance is carried from zero capacity to an infinite inductance.

By following out a similar line of constructions the effects produced by variations of the armature inductance can be studied, and by successfully varying the resistance, inductance, capacity and frequency constants, and constructing corresponding diagrams, a large variety of problems involving the simultaneous variation of several terms can be successfully treated.

A Method of Measuring Permeability. By A. Wilmer Duff.

[Abstract.]

The most common method of measuring the permeability of iron, or the ratio in which the presence of iron strengthens the magnetic field, is to make a ring of the specimen, cover it with two layers of wire, one connected with a source of current to magnetize the ring, the other with a ballistic galvanometer to measure the quantity of electricity induced in this secondary coil by making or breaking the primary current. The galvanometer is calibrated by means of a straight calibrating coil consisting of a non-magnetic core similarly wound with a primary