or

(3) $F(a, BP_1^{S_1}) = F(a^{p_1^{fs_1}}, B) - F(a^{p_1^{f(s_1-1)}}, B)$.

If we let $B = P_2^{S_2}$ we get from (3)

$$F(a, P_2^{S_2}P_1^{S_1}) = F(a^{p_1^{fs_1}}, P_2^{S_2}) - F(a^{p_1^{f(s_1-1)}}, P_2^{S_2})$$

and hence by (2)

(4) $F(a, P_2^{S_2}P_1^{S_1}) \equiv 0, \bmod P_2^S$

By a similar reasoning we also get,

(5) $F\left(a, P_2^{S_2} P_1^S\right) \equiv 0 \bmod P_1^{S_1}$ and hence by (4) and (5).

(6) $F\left(a, P_2^S P_1^{S_1}\right) \equiv 0 \bmod P_2^{S_2} P_1^{S_1}$.

We now assume that for an arbitrary a the function $F(a, A)$ is divisible by A, then if P be any prime ideal not contained in A we have by (3)

$$F(a, AP^s) = F(a^{p^{fs}}, A) - F(a^{p^{f(s-1)}}, A) \text{ and hence,}$$

(7) $F(a, AP^s) \equiv 0 \bmod A$.

Now let $A = CQ^t$ where Q is a prime ideal and C prime to Q. Then,

$F(a, AP^s) = F(a^{q^{ft}}, CP^s) - F(a^{q^{f(t-1)}}, CP^s)$ where q is the rational prime divisible by Q and t the degree of Q, and since by our assumption the two terms on the right side are divisible by CP^s it follows that,

(8) $F(a, AP^s) \equiv 0 \bmod CP^s$, and hence,

(9) $F(a, AP^s) \equiv 0 \bmod AP^s$.

Hence if $F(a, A)$ is divisible by A when A contains n distinct prime factors it is also divisible by A when A contains n+1 distinct prime factors. Making use of (4) we then find that $F(a, A)$ is divisible by A for any A.

On the Class Number of the Cyclotomic Numberfield

$$K\left(e^{\frac{2\pi i}{p^n}}\right)$$

Jacob Westlund.

[By title.]

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