Concerning Spheric Geometry.

By David A. Rothrock.

(Abstract.)

In this paper is developed a system of analytic geometry upon the surface of a sphere, in which the axes of reference are great circles and the coördinates of a point are arcs of great circles. With a proper choice of axes, the equations of the loci known as spheric straight line, spheric circle, spheric ellipse, spheric hyperbola, spheric parabola defined metrically as in plane analytics, appear in a form analogous to their equations in the plane.

The paper also investigates other loci of more complex character, together with a discussion of the notion of spheric pole and polar, radical axis, etc. A summary of the literature upon this system of geometry is also included.

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On the Representations of a Number as the Sum of Consecutive Integers.

By T. E. Mason.

(Abstract.)

Theorem:

If we define a series of consecutive integers so as to include zero and negative numbers and if we consider a number itself as a series of consecutive integers with one term, then a number

$$m = 2^a \cdot p_1^{a_1} \cdot p_2^{a_2} \cdots\cdots p_r^{a_r},$$

where the p's are the odd prime factors of m and the a's the power to which they occur, may be expressed as the sum of a series of consecutive integers in

$$2(a_1+1)\ (a_2+1)\cdots\cdots(a_r+1)$$

ways. When $m = 2^n$ it may be so expressed in two ways.

One-half of the total number of series will have an even number of terms and one-half will have an odd number of terms.

One-half of the total number of series will consist of all positive terms and one-half the number of series will contain zero or zero and negative terms.

We shall now apply this theorem to express 15 as the sum of consecutive integers.

$$15 = 3 \times 5.$$

The number of series will be

$$2(1+1)\ (1+1) = 8.$$

No. of terms.	*Mid terms.*	*Series.*
1	15	15
3	5	4+5+6
5	3	1+2+3+4+5
15	1	−6−5−4−3−2−1+0+1+2+3+4+5+6+7+8
2	7,8	7+8
6	2,3	0+1+2+3+4+5
10	1,2	−3−2−1+0+1+2+3+4+5+6
30	0,1	−14−13....−4−3−2−1+0+1+2+3+4+5+....+14+15

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[18—29034]