(Abstract.)

On the General Solution and So-Called Special Solutions of Linear Non-Homogeneous Partial Differential Equations.

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The integrals of a partial differential equation of the first order were first classified by Lagrange, who separated them into three groups, namely, the general, the complete, and the singular integrals. For a long time this classification was thought to be complete. In fact, Forsyth in his *Differential Equations,* published first in 1885, gives a supposed proof of a theorem stating that every solution of such a differential equation is included in one or other of the three classes named. This error is also carried through the second and third English editions and the two German editions, the last one being published in 1912.

In 1891 Goursat pointed out in his *Equations aux derivees partielles du premier ordre,* that solutions exist which do not belong to any of these three classes and showed indeed that the existing theory was not complete even for the simplest forms.

In November, 1906, Forsyth, in his presidential address to the London Mathematical Society, emphasized the fact that the theory is incomplete, and in his closing remark says: "It appears to me that there is a very definite need for a re-examination and a revision of the accepted classification of integrals of equations even of the first order; in the usual establishment of the familiar results, too much attention is paid to unspecified form, and too little attention is paid to organic character, alike of the equations and of the integrals. Also, it appears to me possible that, at least for some classes of equations, these special integrals may emerge as degenerate form of some semi-general kinds of integrals; but it is even more important that methods should be devised for the discovery of these elusive special integrals."

Forsyth also in an address delivered by request, at the 4th International Congress of Mathematicians, takes advantage of the opportunity offered, to again emphasize the incompleteness of the existing theory of partial differential equations of the first order.

In attacking this problem the logical place to begin is with the simplest case, namely, with the linear equation. This is the equation dealt with in the paper. It can be written in the form

$$\sum_{i=1}^{n} \chi_i \left[z, X_1, X_2, \quad . \quad . \quad . \quad . , X_n \right] = Z \left[z, X_1, X_2, \quad . \quad . \quad . \quad . , X_n \right].$$

The restrictions made on this equation are that all common factors have been removed from Z, X_1, X_2, ... , X_n; that there is also a set of values of the variables z, X_1, X_2, ... , X_n in the vicinity of which the functions X_i and Z have no branch points and otherwise behave regularly.

Forsyth, in his treatise on *Partial Differential Equations* published in 1906 goes to much labor to give solutions that are examples of the so-called special integrals. In the present paper a means is developed by which all the elusive special integrals can be readily determined and a new and complete classification is given of all the integrals of the equation.