Notes on the Artificial Fertilization of the Eggs of the Common Clam, (*Venus Mercenaria*).

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In view of the economic importance of the common clam we endeavored to artificially fertilize its eggs during the past summer* at the United States Fisheries Laboratory, at Beaufort, North Carolina. Many clams were full of eggs or contained active spermatozoa when we first examined them in July. This condition prevailed till the 12th of September, but early in November of this year the spermatozoa were not active.

Several times during July and August we fertilized the eggs by the addition of active spermatozoa from several males and observed the maturation of the egg, the segmentation, and the early trochophore stage. In one instance (August 1, 1906) the development continued to the young veliger stage.

The female sexual element is a pear-shaped cell in which a large germinal vesicle is found. Many of the cells become spherical fifteen or twenty minutes after the addition of active spermatozoa. The eggs then show no further evidence of being fertilized till two hours after the addition of sperm, when the first polar body is cast off and this followed by the second at an interval of twenty or thirty minutes. Thirty minutes later the egg passes into the two-celled stage by a holoblastic and unequal division. The next division occurs in a plane at right angles to the first and this is followed by division in a plane at right angles to the other two.

The cells divide synchyonously up to the thirty-two-celled stage but we were unable to determine whether this continues beyond this stage.

The percentage of eggs that could be fertilized was small during July; it increased during August and September, but during November the spermatozoa were not active and the eggs could not be fertilized.

While we have not reached a definite conclusion regarding the breeding habits of the common clam we feel that these data are themselves significant.

*Summer of 1905.

Determination of all Surfaces for Which, When Lines of Curvature are Parameter Lines (u=const., v=const.), the Six Fundamental Quantities, E, F, G, L, M, N, are Functions of One Variable Only.

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The following simplifications come out of the data:

(1) $F = 0 = M$ (Since lines of curvature are parameter lines).

(2) The v — derivatives of E, G, L, N vanish. (Since the latter are functions of u only.)

(3) We may substitute for u a function defined by the equation,

$$Edu^2 = du'^2,$$

which makes E, G, L, and N functions of u′ only. Also the system of parameter curves is (as a whole) not changed, for when $u = \text{const}$, $u' = \text{const}$ also. Now if we drop the prime from u′, the substitution has exactly the effect of making $E = 1$.

Let (X_1, Y_1, Z_1) and (X_2, Y_2, Z_2) be direction cosines of tangents to the v-curve and u-curve at any point of the surface. These tangents, together with the normal to the surface (direction cosines of which are X, Y, and Z) at the point form a rectangular system of axes.

$$(1) \quad X_1 = \frac{\delta x}{\delta u}; \; Y_1 = \frac{\delta y}{\delta u}; \; Z_1 = \frac{\delta z}{\delta u} \text{ (since } E = 1\text{)}.$$

$$(2) \quad X_2 = \frac{1}{\sqrt{G}}\frac{\delta x}{\delta v}; \; Y_2 = \frac{1}{\sqrt{G}}\frac{\delta y}{\delta v}; \; Z_2 = \frac{1}{\sqrt{G}}\frac{\delta z}{\delta v}$$

Then the differential equations for the general surface (see Bianchi, 1902 Edition, p. 123) become after introducing the above simplifications,

$$(3) \quad \frac{\delta X_1}{\delta u} = LX$$

$$(4) \quad \frac{\delta X_1}{\delta v} = \frac{d\sqrt{G}}{du} X_2$$

$$(5) \quad \frac{\delta X_2}{\delta u} = o$$

$$(6) \quad \frac{\delta X_2}{\delta v} = \frac{N}{\sqrt{G}} X - \frac{d\sqrt{G}}{du} X_1$$

$$(7) \quad \frac{\delta X}{\delta u} = - L X_1$$

$$(8) \quad \frac{\delta X}{\delta v} = - \frac{N}{\sqrt{G}} X_2$$

and similar equations for Y and Z.

NOTE. These, together with the simplified Gauss and Codazzi equations, should give by integration the required surfaces. In the attempt to perform the integration, the following geometric solution was reached. I hope to complete the solution by integration later.

The v-curves make principal sections.

The equation of principal normal to the surface is

$$\frac{\xi - x}{\frac{d^2x}{ds^2}} = \frac{\eta - y}{\frac{d^2y}{ds^2}} = \frac{\zeta - z}{\frac{d^2z}{ds^2}}$$

$$\frac{dx}{ds} = \frac{dx}{ds_v} \text{ along v-curve}$$

$$= \frac{dx}{du}, \text{ since } ds_v = \sqrt{E}\, du = du$$

$$= \frac{\delta x}{\delta u} + \frac{\delta x}{\delta v}\frac{dv}{du}$$

$$= \frac{\delta x}{\delta u}, \text{ since } v = \text{const.}$$

$$= X_1$$

$$\frac{d^2x}{ds^2} = \frac{\delta X_1}{\delta u} = LX.$$

Similarly for y and z, giving

$$\frac{\xi - x}{x} = \frac{\eta - y}{y} = \frac{\zeta - z}{z}$$

which is the normal to the surface at (x, y, z).

The v-curves are also plane curves.

Torsion $\dfrac{1}{T} = \sqrt{\left(\dfrac{d\lambda}{ds}\right)^2 + \left(\dfrac{d\mu}{ds}\right)^2 + \left(\dfrac{d\nu}{ds}\right)^2}$ where λ, μ, ν are direction cosines of bi-normal to curve (here X_2, Y_2, Z_2)

$$\frac{dX_2}{ds_v} = \frac{dX_2}{du} = \frac{\delta X_2}{\delta u} = o \text{ and similarly } \frac{dY_2}{ds} = o = \frac{dZ_2}{ds} \therefore \frac{1}{T} = o.$$

The u-curves have constant radius of curvature.

The equation of radius of curvature is,

$$\frac{1}{\rho^2} = \left(\frac{d^2x}{ds^2}\right)^2 + \left(\frac{d^2y}{ds^2}\right)^2 + \left(\frac{d^2z}{ds^2}\right)^2$$

$$\frac{dx}{ds} = \frac{dx}{ds_u} = X_2$$

$$\frac{d^2x}{ds^2} = \frac{d}{ds_u} X_2$$

$$= \frac{\delta X_2}{\delta v} \frac{dv}{ds_u}$$

$$= \frac{1}{\sqrt{G}} \frac{\delta X_2}{\delta v}, \text{ since } ds_u = \sqrt{G}\, dv$$

$$\frac{d^2x}{ds^2} = \frac{1}{\sqrt{G}} \left(\frac{N}{\sqrt{G}} X - \frac{d\sqrt{G}}{du} X_1 \right)$$

$$\text{So, } \frac{d^2y}{ds^2} = \frac{1}{\sqrt{G}} \left(\frac{N}{\sqrt{G}} Y - \frac{d\sqrt{G}}{du} Y_1 \right)$$

$$\frac{d^2z}{ds^2} = \frac{1}{\sqrt{G}} \left(\frac{N}{\sqrt{G}} Z - \frac{d\sqrt{G}}{du} Z_1 \right)$$

$$\frac{1}{\rho^2} = \frac{1}{G} \left[\frac{N^2}{G} \Sigma X^2 - 2 \frac{N}{\sqrt{G}} \Sigma X_1 X + \left(\frac{d\sqrt{G}}{du}\right)^2 \Sigma X_1^2 \right]$$

$$\Sigma X^2 = 1 = \Sigma X_1^2 + \Sigma X_1 X = o$$

$$\frac{1}{\rho^2} = \frac{1}{G} \left[\frac{N^2}{G} + \left(\frac{d\sqrt{G}}{du}\right)^2 \right]$$

$$= \text{function of } u^\delta \text{ only.}$$

$\therefore$ const for u-curves.

The u-curves are also plane curves, and therefore circles.

We may write equation of torsion in the form,

$$\frac{1}{T} = -\rho^2 \begin{vmatrix} x' & y' & z' \\ x'' & y'' & z'' \\ x''' & y''' & z''' \end{vmatrix}$$

(where primes denote derivatives with respect to s).

From last paragraph we have for u-curves,

$$x'' = \frac{d^2x}{ds^2} = \frac{1}{\sqrt{G}} \frac{\delta X_2}{\delta v}$$

$$x''' = \frac{d}{ds_u}\left(\frac{1}{\sqrt{G}} \frac{\delta X_2}{\delta v}\right)$$

$$= \frac{\delta}{\delta v}\left(\frac{1}{\sqrt{G}} \frac{\delta X_2}{\delta v}\right)\frac{dv}{ds_u}$$

$$= \frac{1}{G} \frac{\delta^2 X_2}{\delta v^2}$$

$$= -\frac{1}{G}\left[\frac{N^2}{G} + \left(\frac{d\sqrt{G}}{du}\right)^2\right] X_2 \text{ from (6), (4) and (8)}$$

$$x''' = \phi(u)\, X_2$$

$$\text{So } y''' = \phi(u)\, Y_2$$

$$z''' = \phi(u)\, Z_2$$

$$\frac{1}{T} = -\rho^2 \phi(u) \frac{1}{\sqrt{G}} \begin{vmatrix} X_2 & Y_2 & Z_2 \\ \frac{\delta X_2}{\delta v} & \frac{\delta Y_2}{\delta v} & \frac{\delta Z_2}{\delta v} \\ X_2 & Y_2 & Z_2 \end{vmatrix} = 0$$

Since the u-curves are plane and have constant radi of curvature they are circles.

Finally, the plane of each v-curve is normal to every u-circle, and therefore passes through its center. The intersection of any two v-planes determines the line of centers of the u-circles. Thus all the required surfaces are surfaces of revolution. Taking the line of centers of u-circles as z-axis and the plane of any u-circle as xy-plane, the equation of our surfaces are

$$\begin{cases} x = u \, . \cos v \\ y = u \, . \sin v \\ z = f(v) \end{cases}$$