The Mathematics of Haul.

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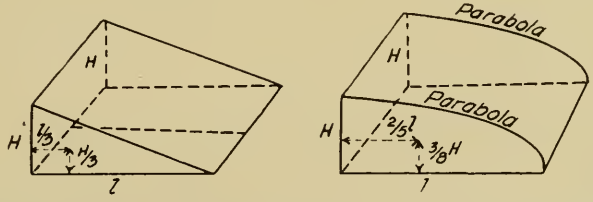
Haul or the average distance earth is moved when taken from excavation and placed in embankment has been the source of much discussion at different times for many years. For a review of the literature on the subject, "Overhaul," the writer will call attention to the "Proceedings of the American Railway Engineering and Maintenance of Way Association," for 1906, vol. 7, pages 357 to 428. Among the contributors to the subject will be found Italians, French and Germans, but it seemed to excite more interest among our American engineers.

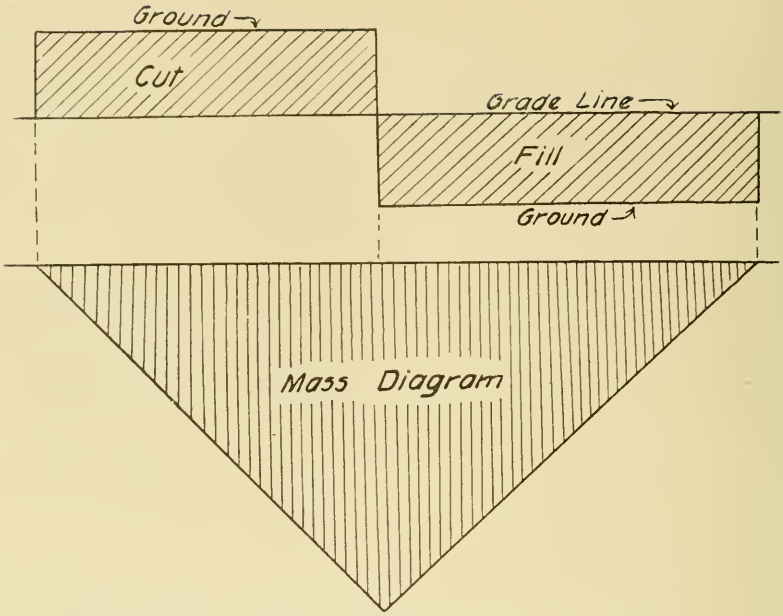
The methematics of haul deals, of course, with the methods of computing haul and overhaul, but it is the purpose here to discuss more particularly the means for locating the center of mass. These centers of mass may be located by any one of four methods, two algebraic and two graphical. All four methods for locating the center of mass fail completely for the volumes adjoining the grade point unless several extra intermediate sections are taken.

In all the calculations a close rapid approximation was used at these points

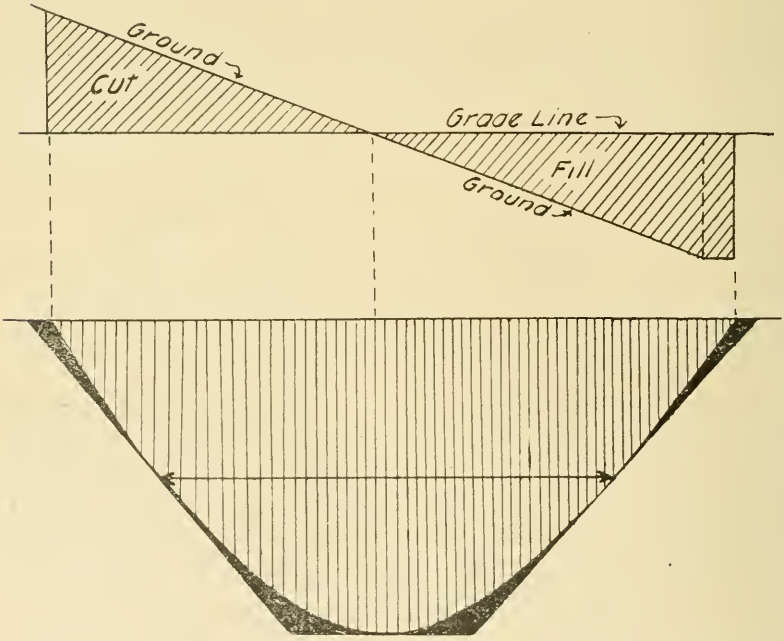


When the ground was a plane surface, the volume next the grade point was assumed at least a wedge, and the center of gravity then taken $\frac{1}{4}$ the length of the wedge from its base. When the ground was a parabola in longitudinal section, the center of gravity was taken $\frac{2}{5}$ of the length between section and grade point from the section.

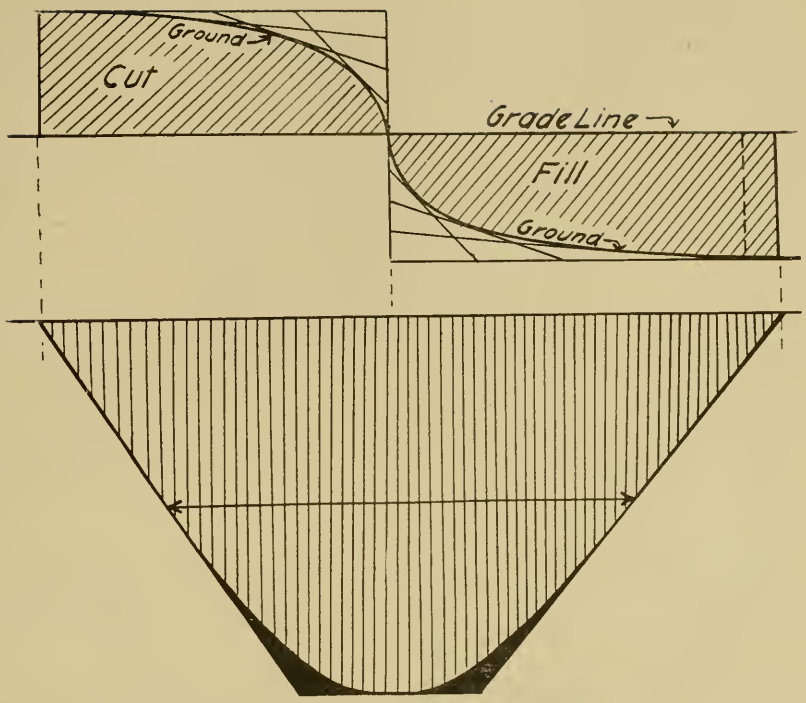
The four methods of computation were carried on under the conditions of the three general types of profile, i. e.:



Case I, all four methods of computing haul agree perfectly.



Case II. Under this condition the four methods of computing haul give their widest variance.



The above figure is Case III, and shows the ground a parabola in longitudinal section. This case most nearly coincides with the actual conditions, for haul, of any of the three cases, and any variance discovered here in the results of the four methods are about what would occur in actual practice, while those discovered under Case II are limiting values.

It being the object of the writer to discover the greatest variance that could occur, thus obtaining limiting values, most of the computations were under Case II, with a few test investigations under Case III, to obtain values that would be encountered more often in practice.

Method No. 1 depends upon the general form that the center of gravity of individual prismoids is located a distance from the mid-section toward the larger section a distance

$$X = \frac{1}{6}\frac{A - A'}{A - A'} \text{ and Haul} = \frac{\Sigma \text{Moments}}{\Sigma \text{Volumes}}.$$

For all practical purposes it is exact.

Method No. 2 depends upon the general form that the center of gravity of each individual prismoid is located in from one end a distance equal to

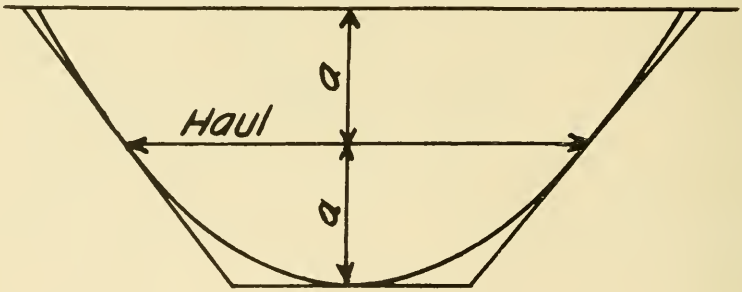
$$\overline{X}_{A'} = l \frac{A}{A + A'};$$

That is to say, it is located inversely proportional to the end areas, and

$$\text{Haul} = \frac{\Sigma\text{Moments}}{\Sigma\text{Volumes.}}$$

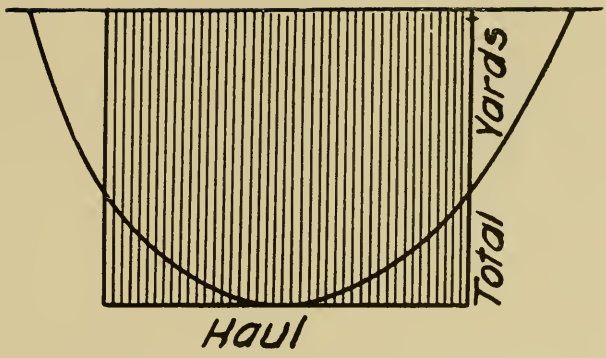
It gives results always in favor of the contractor and on very short hauls is rarely in error to exceed say 3.0 per cent. and on long hauls rarely if every exceeds 0.5 per cent.

Method No. 3 depends upon the general proposition that the position of half mass point is approximately the position of the center of mass and graphically looks like figure below.



Haul equals the mean length of the two sides of the given tropezoid and the pay haul equals the aera of the tropezoid. This method gives results always in favor of the contractor and on very short hauls is rarely in error to exceed say 4.0 per cent., and on long hauls rarely in error to exceed say 6.0 per cent.

Method No. 4. the last one treated in this report, depends for its results upon the area of the mass diagram.



The pay haul is equal to the area of the rectangle which has for its base the haul, and its altitude the total yardage or maximum ordinate, the product of the two being also the area of the mass diagram.

If the points are connected by a curved line it will give practically the true result, but if the points of the diagram are connected by straight lines as is recommended by most engineers, and as was done here, it gives values always against the contractor; on short haul being in error as high as 6 per cent., and on long haul about 1.0 per cent.

Final summary in tabular form:

Method.			Max. Error in %.	
Number.	General Form.	Center of Gravity of Individual Prismoid.	Short Haul.	Long Haul.
No. 1.	$\text{Haul} = \frac{\Sigma M}{\Sigma V}$	$\bar{X} = \frac{1}{6}\frac{A-A'}{A+A'}$	Correct (Practically).	Correct (Practically).
No. 2.	$\text{Haul} = \frac{\Sigma M}{\Sigma V}$	$X_A = 1\frac{A}{A+A'}$	+ 3	+ 0.5
No. 3.	Haul = Length of chord through middle of Maximum ordinate.	.....	+ 4	+ 6
No. 4.	$\text{Haul} = \frac{\text{Total Area Diagram}}{\text{Total Yardage}}$	.....	− 6	− 1

[8—18192]