Methods in Solid Analytics.

By Arthur S. Hathaway.

Define the "vector" $[h, k, m]$ as the carrier of the point $(x, y, z) = P$, to the point $(x + h, y + k, z + m) = Q$, and show that the distance and direction cosines of the displacement PQ are given by functions of the vector called its *tensor* and *unit*, $T[h, k, m] = \sqrt{(h^2 + k^2 + m^2)} = n$, $U[h, k, m] = [h/n, k/n, m/n]$.

Interpret the sum $[h, k, m] + [h', k', m'] = [h + h', k + k', m + m']$ as a resultant displacement, $PQ + QR = PR$, and the product $n[h, k, m] = [nh, nk, nm]$, as a repetition of the displacement.

Define the linear functions of $q = [x, y, z]$ as the "scalars" or "vectors" whose values or components are linear homogeneous functions of the components of q, such as $ax + by + cz$, etc. Hence, for a linear function Fq, $F(q + r) = Fq + Fr$, $nFq = F(nq)$.

Hence, for a bi-linear function Fqr, $F(aq + a'q', br + b'r') = abFqr + ab'Fqr' + a'bFq'r + a'b'Fq'r'$.

A special scalar and vector bilinear function of $q = [x, y, z]$, $q' = [x', y', z']$ are defined.

$$Sqq' = xx' + yy' + zz' = Sq'q.$$

$$Vqq' = [yz' - zy', zx' - xz', xy' - yx'] = - Vq'q.$$

If θ be the angle between the displacements q, q', these functions are interpreted as,

$Sqq' = Tq \,.\, Tq' .\ \cos\theta$. $TVqq' = Tq \,.\, Tq' .\ \sin\theta$; and Vqq' is a displacement perpendicular to both q and q', in the same sense as the axis OZ is perpendicular to OX and OY, *i. e.*, on one side or the other of the plane XOY.

The use of this material is illustrated in the following examples:

$A = (2, 3, -1)$, $B = (3, 5, 1)$, $C = (8, 5, 2)$, $D = (5, 7, 11)$.

1. Find the lengths and direction cosines of AB, AC, AD.

   Ans. $TAB = 3$, $UAB = [\frac{1}{3}, \frac{2}{3}, \frac{2}{3}]$, etc.

2. Find $\cos BAC$. Ans. $SUABUAC = \frac{16}{21}$.

3. Find area of ABC and volume of $ABCD$.

   Ans. $\frac{1}{2}\, TVABAC = \frac{1}{2}\, 185$, $\frac{1}{6}\, SADVABAC = -13$.

4. Find the cosine of the diedral angle $C - AB - D$.

   Ans. $SUVABACUVABAD = \frac{-1}{37\sqrt{10}}$

5. Find the sine of the angle between AD and the plane ABC.

   Ans. $SUADUVABAC = -\frac{6}{\sqrt{185}}$.

6. Find the projection of AB on CD and the distance between them.

$$\text{Ans.}\quad SABUCD = \frac{19}{\sqrt{94}},\ SADUVABCD = \frac{78}{\sqrt{485}}.$$

7. Find the equation of the line AB.

$$\text{Ans.}\quad AP = tAB, \text{ or } \frac{x-2}{1} = \frac{y-3}{2} = \frac{z+1}{2}\ (= t).$$

8. Find the equation of the plane ABC.

$$\text{Ans.}\quad SAPVABAC = 2x + 9y - 10z - 41 = 0.$$

(a) The distance from this plane to (x', y', z') is $SAP'UVABAC$, or

$$\frac{(2x' + 9y' - 10z' - 41)}{\sqrt{185}}.$$

9. The vector whose tensor and components are the moments of AB about C and about axes through C parallel to OX, OY, OZ, is $VCAAB = [2, 9, -10]$.

10. The work done by CD in making the displacement AB is $SABCD = 19$.

Rose Polytechnic Institute,
Terre Haute, Ind.