MATHEMATICS AND THE OTHER SCIENCES.

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American universities grant the doctorate in 20 different sciences according to information given out by the National Research Council. To a mathematician naturally the question arises as to how many of these 20 sciences make use of mathematics in some form that requires a knowledge of the calculus for a proper interpretation of certain results. That mathematical knowledge beyond the calculus is necessary for a proper understanding of such sciences as physics, chemistry, astronomy, engineering and meteorology will be granted by all who are sufficiently well acquainted with those sciences, but that certain phases of such sciences as zoology, botany, psychology, anatomy, pathology, physiology and agriculture are amenable to mathematical treatment is probably not so well known. The mathematical developments and applications in some of these sciences will be discussed in this paper, and a few examples of treatment cited, but no attempt will be made to do more than suggest the wide variety of application and to indicate something of the possibilities of future developments.

Some time ago the following fable appeared in *Science*.[1] It is entitled "The Genealogy of Theory."

"Suggestion, an eager Boy, met a winsome Maid, Credulity by name, and begat Plausibility, an enchantress. She mated with a chance acquaintance, Coincidence, and bore him Belief, a stalwart Youth who set out to conquer the World. But across his shield was blazoned the bar sinister. Reason had not consecrated either union."

Mathematics is the science of pure reason and until the facts of any science can be marshalled and subjected to the critical processes of pure reason not much progress of an exact nature can be expected. This does not necessarily imply a mechanistic attitude, but it does imply a most critical attitude. The more nearly mathematical in form a law is stated the more exact it is and the more understandable it becomes, and it can be subjected to further mathematical treatment and interpretation. Some object that the vital processes are not governed by mechanistic laws, and of course no one can decide that with the meager knowledge of the laws of the vital processes now available. Nor is it probable that a science like botany will ever be treated in a formal mathematical way as Spinoza attempted to treat his *Ethics*. But certainly a scientist with a fundamental knowledge of botany and a reasonable working knowledge of mathematics will discover many phases of botany which are governed by mathematical laws. Probably the reason so little is known of the mathematical laws of botany is that

[1] Science, Vol. 61, (mis.) 1925, p. 567.

"Proc. Ind. Acad. Sci., vol. 36, 1926 (1927)."

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very few botanists know any mathematics and almost never is a mathematician trained in botany. But these conditions are changing.

One of the leading college textbooks in genetics[2] states the following: "A knowledge of mathematics is a valuable asset because it is often necessary to subject the data of heredity and variation to mathematical treatment in order to interpret them properly. For the elementary study of genetics, a knowledge of the methods of dealing with simpler algebraic problems is sufficient; for advanced study a knowledge of the differential and integral calculus is highly advantageous." Again, the German biologist Wilhelm His says, "The ultimate aim of embryology is the mathematical derivation of the adult from the distribution of growth in the germ." And D'Arcy W. Thompson, the British naturalist, who has attempted to show the mathematical workings of Nature in his book *On Growth and Form*, says "The living and the dead, things animate and inanimate, we dwellers in this world and this world wherein we dwell, are bound alike by physical and mathematical law." These statements should suffice to show that any natural scientist with sufficient mathematical training will find his research field white to harvest. Helmholtz is a splendid example of such a scientist, for his scientific training in anatomy, physiology, physics and mathematics enabled him to make noted advances in electricity, hydrodynamics, light and sound, some of which he applied in developing instruments of great use in medical diagnostics.

Many of the laws well known to the mathematician, physicist and chemist have numerous applications in the natural sciences. Probably the best known is the compound interest law in which the form of growth is such that the rate of increase or decrease at any instant is proportional to the magnitude at that instant of that which is increasing or decreasing. This law is met with in physics in the study of electric currents and the decrease of radioactivity, and in chemistry where it is known as Guldberg and Waage's law or the law of mass action. Thus $S_t = S_0 e^{-kv}$ where $v = t + (t^2/2p)$, where S_t is the area of a wound at time t, S_0 the initial area of the wound, and k and p constants, is the equation representing the healing of wounds. It has been used to study the merits of various antiseptics and dressings, and a serious deviation from the curve is often indicative of infection before the infection becomes apparent otherwise. If bacteria are allowed to grow freely in the presence of unlimited food their number at time t is given by the equation $N = Ce^{kt}$. This law is frequently met with in vegetable growth. The law of mass action is also applicable to the dissociation of oxyhaemoglobin. The growths of animal and human populations have been studied as applications of this law but owing to environmental, economic and other restrictive conditions several modified exponential equations have been used with better success to represent these growths.

In his studies in the morphology of the blood vessels John Hunter found and stated that the angle of origin of the branch vessels varied in such a way that the circulation of blood would be just sufficient for

[2] Babcock and Clausen, Genetics in Relation to Agriculture, p. 11.

the part. Hess's law assumes that the loss of pressure is mainly due to the friction of the blood stream against the vessel walls and that the pressure varies directly as the length of the vessel and inversely as its radius, that is, $P=\left(\frac{M}{R}+\frac{N}{r}\right)K$, where M and N are the distances of the flow along the main vessel and branch vessel, respectively, and R and r are their respective radii, and K is the factor of proportionality. If now the minimum value of P be determined by means of the calculus, the following relation is found: $\frac{r}{R} = \cos x$, where x is the angle which the branch vessel makes with the main vessel. This relation shows that very small vessels like capillaries come off of larger vessels at practically right angles, branch vessels that are almost as large as the main vessel come off almost parallel to the main vessel, and all vessels of practically the same size come off the same main vessel at the same angle.

Again, in the study of the nerves it has been found that if the outer coat or myelin sheath be considered as similar in its function to the insulating coat of a submarine cable, and that if the same law be assumed to hold for the speed of transmission of an impulse as for the speed of signalling along the cable, then the ratio between the radius of the axon or nerve core and that of the myelin sheath is such as to make the speed of the impulse approximately a maximum.

There are numerous other special applications of mathematics to biology. Thus thermodynamical laws may be used in determining the amount of work done by the kidneys. The Van't Hoff-Arrhenius law for the influence of temperature on the velocity of reaction of substances in solution has applications in the conduction of impulses along a nerve, the rate of heart beat, the rhythm of the small intestine, respiration in plants, et cetera. The Schütz-Borissoff law with regard to the action of enzymes is applicable to gastric digestion in the early stages. The muscles fall into several distinct types and the work done by them can be approximately computed by mathematical methods. Still another interesting illustration is the relation between the tension and radius of curvature of a membrane under constant pressure. The thickness of wall is proportional to the tension which in turn is a function of the radius, and the variable thickness of the walls of the heart and uterus, for example, is easily explained by this relation.

So far no mention has been made of Mendelian theory which is based on probabilities and requires a knowledge of the methods of mathematical theory of statistics for a proper understanding of it. The mathematical theory of statistics involves the use of calculus, some differential equations, the laws of probability, the theory of errors, et cetera, and may be applied wherever the number of variables is great and most of the variables are beyond control. It has been used heretofore chiefly in life insurance, educational and psychological tests, genetics and agricultural experimentation as a means of interpreting data and forecasting future possibilities.

Probably the best known American author of mathematical biological studies is Prof. Raymond Pearl, of Johns Hopkins University, whose books *Studies in Human Biology* and *Medical Biometry and Statistics* are real statistical studies. In a recent work, *Elements of Physical Biology*, Prof. A. J. Lotka, of Johns Hopkins University, attempts to apply systematically to certain topics of biology, such as the growth of species and the evolution of living species, the same methods as are used in theoretical mechanics. Another biologist who has made use of mathematical methods is P. Lecomte du Nouy, of the Rockefeller Institute for Medical Research, the results of whose researches are frequently published in the *Journal of Experimental Medicine.* Du Nouy discovered the law for the healing of wounds. The book *Growth and Form* by D'Arcy W. Thompson contains a most interesting collection of illustrations of mathematical laws and curves as found in plant and animal life, and the numerous observations therein noted should serve as a starting point for some most interesting biomathematical research. Another recent book, *Biomathematics*, by W. M. Feldman, contains the solutions of a number of special biological problems, several of which have been used as illustrations in this paper. A number of other investigators are doing work along these lines but no further enumeration will be made here.

One must be cautious in drawing conclusions based on mathematical treatment, for the results are no truer than the fundamental assumptions made leading to their derivations. Gross errors may be made by the novice in the use of mathematical and statistical methods, for the proper interpretation of the results requires more than a mere knowledge of mathematical and statistical formulas. Consequently the best research work will probably be accomplished through the co-operative efforts of the mathematician, physicist, chemist and biologist, for certainly the fields of mathematics, physics, chemistry, biology, and agriculture are too broad for one man to master more than a part of any one of them. The general philosopher has passed along with the spinning wheel, the bootjack and the stage coach. On the other hand, the mathematician need only look about him for applications of mathematics, and as the Talmud has it, "He who knows mathematics and does not make use of his knowledge, to him applies the verse in Isaiah (V, 12), 'They regard not the work of the Lord, neither consider the operation of his hands'."