A THEORETICAL LOWER LIMIT TO THE MASS OF A STABLE ASTEROID

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Since the time of Sir Isaac Newton's discovery of universal gravitation* it has been assumed in astronomy, and often stated, that a projectile, fired from the surface of the earth with a force large enough to counteract, ultimately, the effect of gravitational attraction, would assume a stable orbit, continuing to rotate around the earth.

Yet, no one has observed a stable planetary body of small dimensions in motion upon an ellipse of small eccentricity. One of the moons of Mars (Deimos) is approximately five miles in diameter.

In this paper there is derived a formula for the product of the masses, of an asteroid and the Sun, which shows that a lower limit to the mass of the former exists. A table of numerical verifications is being prepared.

In accordance with a method which the author has† previously developed, properties of a central orbit are studied by means of a transcendental curve whose equation is simple, a segment of which is in practical coincidence with a segment of the orbit. Since the central attraction for any astral orbit can be determined from a segment C of it, the method is analogous to the theory according to which the elements of a planetary orbit are computed from three observations.

The transcendental curve is obtained from an equality between certain integral invariants, viz:—

$$\varphi \int dr/p(r) = \theta + \alpha, \tag{1}$$

where,

$$p(r) = ar^n + br^{n-1} + \cdots + l, \tag{2}$$

and α, φ are arbitrary constants. Let $n = 2$. Then the integral of (1) is,

$$r = v\tan(e\theta + \beta) - u, \tag{3}$$

in which

$$e = m/2\varphi,\quad m = \sqrt{4ac - b^2},\quad u = b/2a,\quad v = m/2a,\quad \beta = e\alpha.$$

If e, β, u, v, are appropriately chosen, the curve (3) will coincide, over a part of its length, with an ellipse. Note that the choice of φ controls e when we wish the selection of u, v to be arbitrary.

By hypothesis the central attraction F is constant at the center; hence (3), considered as the equation of C, determines F. For, a well-known equation of a central‡ orbit is,

$$d^2w/d\theta^2 + w = F/\gamma^2 w^2, \quad (w = 1/r), \tag{4}$$

and by substitution in (4) from the equation (3) of C,

$$F = 2\gamma^2 e^2 \left[\frac{u/v^2}{r^2} + \frac{3u^2/v^2 + 1/2e^2 + 1}{r^3} + \frac{3u^3/v^2 + 3u}{r^4} + \frac{(u^2+v^2)^2/v^2}{r^5} \right].$$

The curve (3) approximates to a planetary orbit, an ellipse with small eccentricity,

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*Isaac Newton (1642-1727).

†Proc. Ind. Acad. Sci., Vol. 39, p. 243.

‡Ziwet, *Theoretical Mechanics*, p. 128.

when v is a small constant and u is large. The radius vector r remains nearly equal to u. If we write $u/r = 1+\delta$, $\delta^2 \doteq 0$, then F may be reduced to,

$$F = 16\gamma^2 e^2 \left[\frac{(1+{}^3/{}_2\delta)u/v^2}{r^2} + \frac{1/2e^2+1}{8r^3} + \frac{3u}{8r^4} + \frac{2u^2+v^2}{8r^5} \right].$$

The last two terms will be negligible in the present problem since r is large. The second may represent a relativity correction or other perturbation if e is properly chosen. We can select the arbitrary γ so $16\gamma^2 e^2 = M$, where M is the gravitational constant in Newton's law for the case of two attracting planets. This law states that F varies directly as the product of the masses.

In the problem of this paper, then, $(1+{}^3/{}_2\delta)u/v^2$, is the product of the mass of the Sun and the mass of the asteroid, and since u is large and v small, this product is necessarily large accordingly. However, it would not be large if the asteroid were of the dimensions of a meteor, assuming usual units of measurement. *Hence there is a lower limit to the mass of any asteroid in stable motion.*

Concerning any body smaller than this limiting size, in solitary motion upon a planetary ellipse of small eccentricity, clearly its motion is necessarily chaotic.