AUDIO FREQUENCY TRANSFORMER IMPEDANCE

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Several writers have developed the theory and equations for ideal transformers[1] having no distributed capacity, no leakage reactance, no eddy current, no hysterisis loss, and no copper losses. These conditions may be approximately obtained in commercial power transformers but not in audio frequency interstage and output transformers.

Some equations developed by the theory of the ideal transformer are not even close approximations. Some experimenters have for example assumed that an output transformer has a constant voltage ratio for all values of input and output impedance. This is not the case as will be shown later. Certain manufacturers print "information" in their catalogues for transformers such as, "primary impedance 15,000 ohms, secondary impedance 40,000 ohms, ratio 3:1." Just what such information is worth is rather hard to understand since such things depend entirely upon frequency, voltage, load, and the relative strength of a-c and d-c currents traversing the coils of the transformer. Other writers[2] neglect entirely any distributed capacity in output transformers. In a general way this may be a safe assumption. Fig. 1 shows that in the case of this particular type of transformer with a step down ratio of 2:25:1, the distributed capacity has some effect on the impedance values of the transformer.

In the bibliography will be found quite a number of references to the theory and operation of transformers. While apparently good theory, in most cases of practical application the predicted and actual results do not agree very well. Part of the variation in transformer theory is due to the assumptions concerning the proper equivalent circuit upon which to base the theoretical equations. No attempt will be made here to reproduce or criticise these circuits varied as they are. A circuit proposed by Webb and Diamond,[3] Fig. 2, seems to agree with the experimental results obtained for open circuit readings. Many practical considerations such as the correct value of inductance, capacity, resistance, reactance, and impedance for various parts of the circuit are difficult to obtain. The difficulty arises from the attempt to isolate such constants as distributed capacity, leakage, and a-c resistance from the inductance values of the coils with which they are associated.

In theory an ideal transformer should transpose a certain load from

[1] (a) R. R. Ramsey, Proc. Ind. Ac. Sci.

(b) K. S. Johnson, "Transmission Circuits for Telephonic Communication." Chapter VI, "Ideal Transformers." See Bibliography, "Audio Transformer Theory."

[2] R. C. Hitchcock and W. O. Osborn, "Output Transformer Design," Part I, "Electronics," Nov., 1930, p. 381. Part II, "Electronics," Dec., 1930, p. 427.

[3] J. S. Webb and H. Diamond, "Testing of Audio Frequency Transformers." Proc. I. R. E., Sept., 1927, Vol. 15, p. 767.

the secondary to an equivalent load in the primary circuit. This effective load should be the impedance of the secondary circuit multiplied by or divided by the transformer ratio n^2, $(n = \sqrt{Zs / Zp})$. If the secondary load is a pure resistance then $R = Rp + n^2Rs$ and $X = Xp - n^2XS$. These equations may be found in any standard text on electrical engineering. In an audio frequency transformer such equations apply more closely to the output transformer because the secondary has little distributed

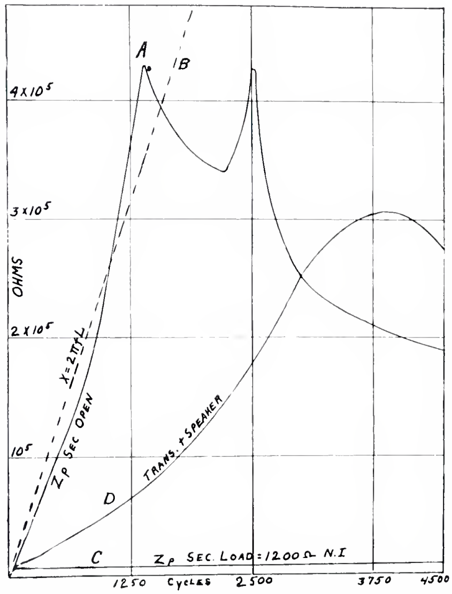

Fig. 1. Impedance curves for output transformer.

capacity, the current is relatively large, the impedance of the load is small in comparison to the input impedance of an amplifier tube, and the primary impedance is ordinarily not as large since the transformer is more often designed for use in the plate circuit of a power tube. In both types of transformers however the primary or secondary resistance in some cases removes a transformer entirely from the ideal class. Kirke[4] in discussing the difference between input and output transformers in a very good article on microphone amplifiers assumes that the only difference in the two types is that the output transformer can be considered free from effects of distributed capacity. In Fig. 2 the various constants are as follows: Rp, primary and secondary resistance and losses; L_l, leakage inductance; L, primary and secondary inductance referred to the primary circuit; C, capacity of primary, capacity of secondary and mutual capacity lumped; Rs, secondary load.

For an output transformer C is small and Rs has a larger effect on primary conditions. In practically all cases the curves of voltage

[4] H. L. Kirke, E. W. and W. E., July, 1928, Vol. V, p. 361.

ratio against frequency gave reasonably flat amplification dropping slightly above 4,000 cycles. These curves for the better types of transformers may be found in the articles referred to in this paper or any of the standard texts on radio. The curves are in most cases taken by using the vacuum tube voltmeter as the measuring device under actual load conditions and as such represent rather faithfully the operation of the transformer. No curves are reproduced in this paper although data has been taken on each transformer used in this study. In the better designed audio transformers, constant voltage amplification seems to be a goal attained by most manufacturers.

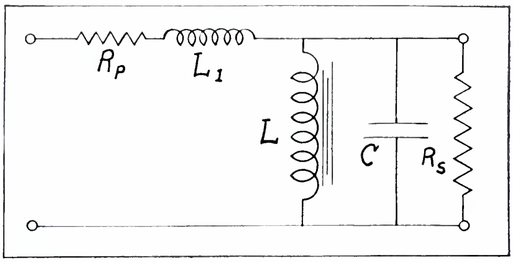

Fig. 2. Equivalent circuit of audio transformer.

Much has been written concerning matching impedances recently. This is for two purposes: first, to obtain maximum power; second, to prevent distortion by reflections. Theoretically, these two objectives are simultaneously obtained if $Z_S = Z_O$, where Z_S is the impedance of the source and Z_O is the input impedance of the transformer.[5] When a condition is set up ($Z_S = Z_O$) such that no reflections occur the value Z_O is known as the characteristic or surge impedance of the circuit. This value as given by Bradfield and John[6] for an impedance network is $Z_O = \sqrt{Z(\text{open}) \times Z(\text{closed})}$ where Z(open) is the impedance of the transfer network with its output terminals open and Z(closed), the impedance with the output terminals short circuited. If an ideal transformer is considered as an impedance network[7] we should obtain the same relation $ZO = \sqrt{Z(\text{open}) \times Z(\text{closed})}$.

The writer decided to see how closely reactance, inductance, and resistance values of actual transformers would agree with those predicted by ideal theory. It was hoped that some information could be obtained concerning the matching of impedances and if "characteristic impedance" had any meaning as applied to the actual transformer in which ideal conditions are not obtainable. (The writer seriously doubts whether ideal conditions would be desirable.)

The remainder of this paper will be divided into two parts, (A) data concerning output transformers and (B) data concerning input transformers.

[5] G. W. Pierce, "Electric Oscillations and Waves," p. 304.
[6] Bradfield and John, "Telephone and Power Transmission," Chapter II.
[7] T. E. Shea, "Trans. Networks and Wave Filters," Chapter 3, and pp. 325-334.

A. OUTPUT TRANSFORMERS

The output transformers used in this study cannot be claimed to be entirely representative of their class because in the writer's opinion such a transformer does not exist. The transformers used were: a Western Electric $n = 2.25:1$; a Magnavox $n = 16:1$; and a Silver-Marshall $n = 1:1$, where n is the average voltage transformation ratio.

Using the transformer 120-C, a transformer designed to couple a 5000-8000 ohm tube to a magnetic speaker, four series of readings were taken. In Fig. 1 is represented graphically the results of these measurements. Curve A is the impedance curve of the transformer 120-C with the secondary unloaded. The maximum value at 1400 cycles represents an impedance of 430,000 ohms. The depression in the curve at 1800 cycles is caused by series resonance between L_1 and the leakage reactance and C the lumped capacity of the transformer. Above 2500 cycles the capacity C which shunts L passes the alternating current.

Curve B is a theoretical value of reactance of the open circuited transformer calculated from the bridge value of 40 henries at 100 cycles. The maximum of reactance at 5000 cycles was 1,250,000 ohms.

Curve C is the measured impedance of the primary with a pure resistance load of 1200 ohms across the secondary. Using a value $n = 2.25$ for the theoretical amplification where $n = \sqrt{Zp/Zs}$, then $Rp = R_1 + n^2R_2$. $Rp = 720 + 6080 = 7800$. Rp as measured at 1000 cycles is 7800. Actual measured value of n at this frequency was found to be 2.75. The values of Zp varied from 7300 ohms at 60 cycles to 8700 ohms at 5000 cycles.

Curve D is the impedance of the output transformer primary with a magnetic (Utah) unit on the secondary. The capacity of the unit seems to pass the current above 4000 cycles. Other output transformers gave practically the same results. Matching impedances with this combination would be an impossible task.

In Tables I and II are shown the data secured by using the circuit shown in Fig. 3. Primary and secondary voltages were read by a V. T. voltmeter. The primary alternating current was obtained by measuring

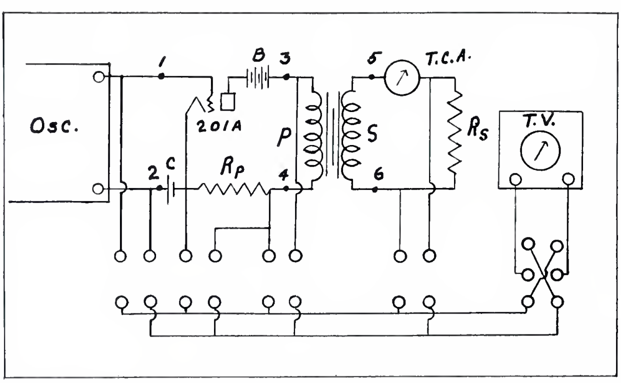

Fig. 3. Measuring circuit for transformer characteristics.

the voltage across a resistance in series with the transformer since $I_{a\text{-}c} = E_{a\text{-}c}/R$. The V. T. voltmeter registers only the a-c voltages. For the data of Table I no tube was used, all current being alternating. For Table II, a 112A tube was connected as shown. The d-c plate current varied from 4.5 to 7.5 mills. It is seen that the current ratio values are more constant and hence more valuable as a basis for transformer computations under all loads. Different primary voltages have but little effect on these ratios but change the impedance values to a large extent.

TABLE I

Voltage and current ratios for 120-C at 60 cycles.

Primary voltage 4.15 volts No d-c No tube

Z_P ohms	R_S ohms	V_S volts	I_S milliamps	I_P	A_I	A_E
2350	100	...	3.6	1.76	2.06	...
2800	200	.58	3.05	1.48	2.06	7.15
3300	300	.76	3.50	1.25	2.05	5.75
4200	500	1.02	2.04	.99	2.04	4.06
5000	700	1.13	2.00	.83	2.00	3.66
5700	900	1.27	1.93	.73	1.93	3.26
6800	1200	1.37	1.87	.61	1.87	3.02
7800	1500	1.43	1.79	.53	1.79	2.90
8600	1800	1.55	1.79	.48	1.79	2.68

TABLE II

Voltage and current ratios for 120-C at 60 cycles.

Primary voltage = 4.80 112A tube on primary

Plate voltage = 90 Average plate current = .006 mills

Z_P ohms	R_S ohms	V_S volts	I_S milliamps	I_P	A_I	A_E
2800	200	.72	3.60	1.72	2.09	6.65
3300	300	.90	3.00	1.45	2.06	5.34
4000	500	1.21	2.42	1.17	2.02	3.96
4800	700	1.35	1.93	1.00	1.95	3.55
5600	900	1.50	1.66	.85	1.94	3.20
6600	1200	1.67	1.39	.72	1.88	2.88
7500	1500	1.78	1.19	.64	1.86	2.70
8400	1800	1.82	1.03	.59	1.74	2.63

Table III shows the results obtained with the Magnavox output transformer by taking measurements of the primary impedance with the secondary open and closed. The image, surge, or characteristic impedance Z_O was then obtained by

$$Z_O = \sqrt{Z(\text{open}) \times Z(\text{closed})}.$$

TABLE III

Characteristic Impedance

Magnavox output transformer — $E_p = 1$ volt

f cycles	Z (open) ohms	Z (closed) ohms	$Z_O = (Z_o \times Z_c)^{\frac{1}{2}}$ ohms
60	.....	....	.....
100	3150	1280	2050
200	6200	1350	2890
300	8930	1340	3460
500	15400	1450	4730
700	20600	1600	5750
1000	29700	1800	7300
1400	42000	2180	9550
1800	56000	2550	12000
2000	65000	2780	13400
2500	91600	3300	17400

If Z_O is plotted against frequency the curve has a decided rise. It appears that if characteristic impedance is to mean anything definite the frequency should be specified. At 1000 cycles, Z (open) is 29,700, Z (closed) is 1800, and Z_O is 7300. The transformer current ratio at 1000 cycles is approximately 21. If we divide 7300 by $(21)^2$ or 441 we get a value of 16.5 ohms for X at 1000 cycles. X at 1000 cycles by measurement is 15 ohms. This shows fair agreement, but sufficient work has not been done by the writer to make any definite conclusions concerning the generality of such experiments.

From this section of the paper the writer feels safe in drawing the following conclusions: (a) Output impedance can be matched more nearly by using speaker units of low impedance values in conjunction with high ratio, step-down, output transformers; (b) characteristic impedance can be of use in impedance matching but it is variable, depending upon frequency; (c) current ratios are more constant and furnish a better basis for calculations; (d) the output audio frequency transformer more closely approaches the ideal or perfect transformer than the interstage or input transformer. The basis for this last conclusion will be found in the next section.

B. Input Transformers

The transformers used in this part of the study, all audio frequency intertube transformers, were as follows: Thordarsen, models R-400 and R-300; Sampson 3:1 ratio; Jefferson replacement (dated 11-31); Erla, Concert Grand; and an All-American type R-41, 3:1 ratio.

Data was taken for each of these transformers with the secondary open and no d-c in the primary, a General Radio (25-70,000 cycle) oscillator furnishing the a-c current in all measurements.

Load curves were taken employing circuits Fig. 4 and Fig. 5.

In Fig. 4 the impedameter was connected across Z_p. Curves were taken with R_S equal to infinity, .2 megohm and .5 megohm. Various values of C from 0-4.5 volts were used. The tube was a type 201A with

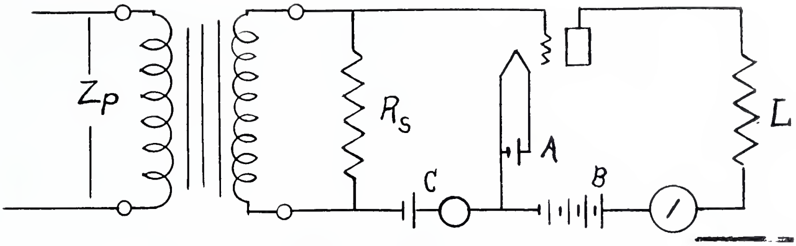

Fig. 4. Transformer circuit for primary impedance.

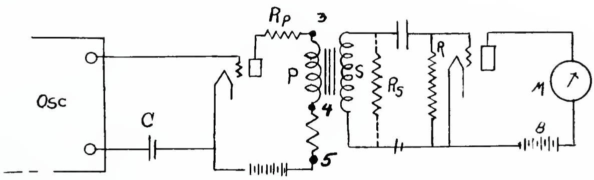

Fig. 5. Transformer impedance measuring circuit.

83 volts on the plate. Under these conditions the amplification factor determined by the dynamic method[8] was 8.2. The a-c grid voltage was determined by measuring the voltage across L with a V. T. voltmeter.

Fig. 6 shows graphically the results obtained. The most important

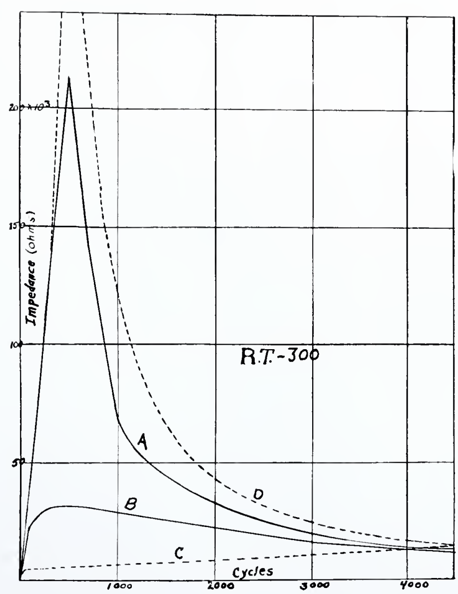

Fig. 6. Impedance curves for interstage transformer.

[8] R. R. Ramsey, "Experimental Radio," p. 77.
H. A. Brown, "Radio Frequency Electrical Measurements," pp. 228-230.

points to be observed are: Curve D, the no secondary load curve, shows definite parallel resonance at 500 cycles with an impedance at this point of 450,000 ohms; Curve A, where the amplifier tube with the current constants is used as load, shows a similar resonance point at 500 cycles with an impedance of 210,000 ohms.

For Curve B conditions were exactly the same as for Curve A, except that a .5 megohm resistance was placed across the secondary of the transformer. This flattens the impedance curve to a great degree.

For Curve C the conditions were the same as for Curve B, except that the grid bias voltage was zero. This allowed a rectified grid current of from 12-25 microamperes to flow through the secondary, thereby reducing its "effective" impedance. This decrease in secondary impedance increased the voltage amplification from 1.28 for Curves A and B to 2.67 for Curve C.

The criticism might be made that the curves obtained by using the circuit of Fig. 4 would not be representative of actual conditions since no direct current flows through the transformer primary. To investigate this point the circuit of Fig. 5 was employed. Contacts 3, 4, and 5 give a means of determining the primary current, voltage, and impedance. $R_p = 0$ in this set of readings. The circuit to the right of R_S being a V. T. voltmeter which acted as a load for the transformer, at the same time measuring the secondary voltage of the transformer. The voltage across P was kept constant at .98 volt.

The writer has noticed some written criticism that a V. T. voltmeter does not load an interstage transformer sufficiently. According to E. B. Moullin the input impedance of such a voltmeter, if R (Fig. 5) is 2 megohms, is approximately 1 megohm. The curves obtained with this

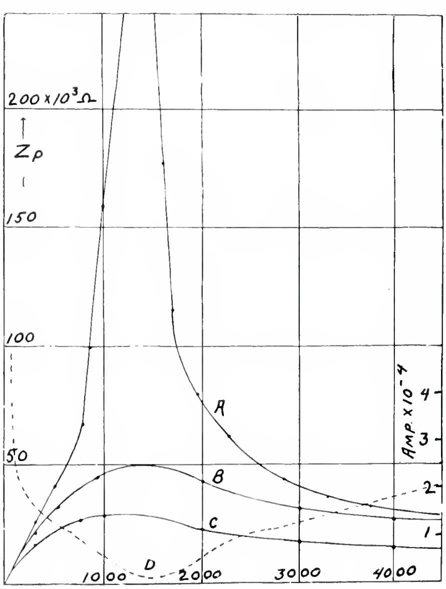

Fig. 7. Series resonance curves.

circuit were almost identical with the curves employing circuit 4. If a 1 megohm grid leak is used in the V. T. voltmeter, the conditions are the same as for a value of R_S = .5 megohm in the circuit of Fig. 4. Using the circuit of Fig. 5 a resonant peak was obtained with a Jefferson transformer of 100,000 ohms at 1400 cycles.

Fig. 7 is a series of curves representing values for the Jefferson transformer using the circuit of Fig. 5 for A, B, and C.

In Curve A the $R_S = \infty$, C = —4.5. In Curve B, R_S = .5 megohm, C = —4.5. In Curve C, R_S = .5 megohm, C = 0.

Curve D of Fig. 7 was obtained by the circuit of Fig. 8. M in Fig. 8 was an output meter to keep the voltage from the oscillator constant at all frequencies, R was 10,000 ohm resistor, T the transformer primary and V. T. V. M. a thermionic voltmeter.

E_R divided by 10,000 gives the current through the transformer primary. In Curve D, 1×10^{-4} amperes is plotted against frequency. At 1400 cycles a minimum current very nearly zero is obtained showing that the impedance of the transformer approaches very high values at this frequency. This again indicates parallel resonance and hence the validity of the equivalent circuit, Fig. 2.

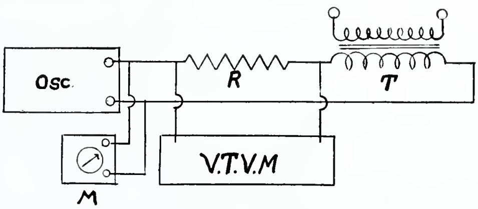

Fig. 8. Current determining circuit for series resonance.

If impedance matching is to have any meaning, input transformers should be loaded with a leak resistance. This loading decreases the effect of distributed capacity by furnishing a lower resistance to be reflected to the primary circuit by the transformer. In short the load resistance causes the transformer to act more nearly as a transformer. For example, the Sampson transformer at 1000 cycles with C = —4.5 and a load of .5 megohm has a primary impedance of 56,000 ohms. The ratio being approximately 3:1, a value of 55,000 ohms reflected resistance is obtained by dividing 500,000 by 3^2. Below and above 1000 cycles the value varies from this theoretical amount because of the high capacity of the secondary.

In conclusion of this section it may be suggested that an audio frequency input transformer may be represented by an equivalent circuit as shown in Fig. 2 and that to give more nearly ideal transformer action the secondary should be properly loaded.

CONCLUSIONS

Audio frequency output transformers are more nearly ideal transformers because they have windings of comparatively small resistance and possess a smaller distributed capacity.

The image impedance of a transformer as defined by

$$Z_O = \sqrt{Z(\text{open}) \times Z(\text{closed})}$$

is a variable, increasing exponentially with frequency.

The current ratio for audio frequency transformers is more constant than the voltage ratio for widely different loads.

The primary impedance of an output transformer is determined almost entirely by the load.

The primary impedance of an input transformer is determined by the load if large, but by the capacity of the transformer with small load or no load.

Ideal transformer equations are of very little practical importance for input transformers.

Impedance matching can be more nearly accomplished by loading input transformers with leak resistances having values from .3 megohm to 1.5 megohms, depending upon the transformer.

The a-c resistance of a transformer should not be neglected in transformer theory.

Equivalent circuits may be drawn to explain observed input transformer operation, but these circuits cannot be relied upon for accurate numerical predictions. This follows from the fact that the mutual inductance, primary capacity, secondary capacity, mutual capacity, a-c resistance, and core losses are all variable factors so interrelated that isolation of their several effects is exceedingly difficult.

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