CONCERNING THE OPTICAL PRINCIPLES WHICH ARE INVOLVED IN THE HALO PHENOMENON[1]

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An observation which was published by John Ruskin in 1851 might have prevented much subsequent error in regard to the phenomenon of the ordinary halo if scientists had continued investigation along the lines suggested. This observation occurs in the first volume of *The Stones of Venice* (Ch. 21, Treatment of Ornament).

> "When the sun rises behind a ridge of pines, and those pines are seen from a distance of a mile or two, against his light, the whole form of the tree, trunk, branches and all, becomes one frostwork of intensely bright silver, which is relieved against the clear sky like a burning fringe, for some distance on either side of the sun."

Ruskin remarks that, among literary men, probably only Shakspeare and Wordsworth had noticed this fact. The allusion by Shakspeare is as follows:

> "But when, from under this terrestrial ball,
> He fires the proud tops of the eastern pines."
> *(Richard II).*

In the panorama mentioned, which contains the pine trees, the observer is seeing two opposite segments of a halo around the sun. This will become theoretically clear if we choose, as a primary type of reflector, a small sphere which has a glossy surface, for example, a marble of china-ware. Place this sphere at a distance of about two feet in front of an unshaded desk lamp L of one (frosted) bulb, the observer being six feet from the lamp. Arrange, also, to move the sphere, at choice, along the line perpendicular to the observer's line of sight toward the lamp. When the sphere is in front of the light no illumination is reflected from the former to the observer. The sphere is then in its completely shaded phase. If moved a considerable distance to one side, the ball shows reflected light only in the form of a faint virtual image of the lamplight. However, when the ball is moved so that it is in the new-moon phase as seen by the observer, the case is very different. Intense illumination in the form of a spot of light P, a so-called brilliant point[2], appears on the sphere as a feature of its crescent illumination. This point maintains its brightness as the sphere is moved back and forth along its line of motion for a short definite distance. W. H. Roever first gave the definitions of brilliant points. His principal definition is as follows:

> A point P is said to be a brilliant point of a surface T with respect to a source of light P_1 and an observer's eye P_2, if the internal bisector of the angle $P_1 PP_2$ is the normal to this surface at the point P.

Thus the observer P_2 sees, in effect, at a brilliant point P, the light itself or a section of it, whereas, when our sphere is moved to one side, what is seen is a virtual image, that is, the light greatly reduced in size and intensity by the curvature of the surface. This is an elementary observation but its significance as a scientific observation has been over-

[1]Read in outline before the Academy December 4, 1931.

[2]Roever, 1908. Trans. Amer. Math. Soc. 9:245-279. Roever, 1922. Amer. Math. Monthly 29:149-156.

looked more or less. No astronomer, for instance, has ever recognized this as the principle according to which Venus is so brilliantly lighted when she is an evening star. *Primarily* the halo is a result of brilliant point reflection from droplets of water-vapor or of ice, floating in the air between the observer and the sun. This is made clear as follows (FIG. 1):

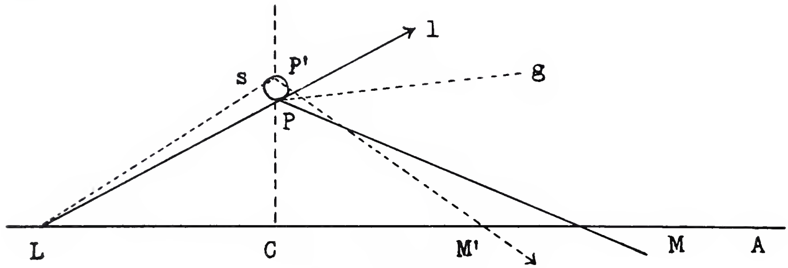

FIG. 1. See text.

Consider a lamp L and an observer M, both fixed in position, and a sphere s on the perpendicular to LM at C. The sphere will represent a droplet in a halo which has C as its center. We have magnified the droplet for clearness. The plane of the halo will be perpendicular to LM. Let l be the ray of light which is tangent to s. Anyone, N, situated a little above l, would see the sphere in its dark-of-the-moon phase. In fact it is necessary for N to come below a line g of somewhat smaller gradient than that of l in order to see the brilliant-point P on s, and P stays visible to N as long as the latter remains within the fixed angle gPh. Below the line Ph, P lapses in brightness, becoming a virtual image reflection. The fixed observer M, being within $<\theta=hPg$, sees the brilliant-point P, one among the myriad which form the halo.[3]

Let us next find the halo's width. As we move s toward C, the vector g turns inward until it eventually intersects LM, as at A. As we continue to move s toward C, A approaches M. The position a', of s when A reaches M, is on the lower boundary of the halo. Next suppose that s is moved along its perpendicular away from C. Then h approaches M, and, if b' is the location of s when h reaches M, b' marks the upper boundary of the halo. The whole area occupied by the halo is obtained by rotating $a'b'$ around LM as an axis. It is well known and is readily verified by use of any glossy sphere for s that, when l is parallel to LM —— the case of a halo of the sun —— the angular radius of the halo is not greater than the 22°. We write $<gPl=t$.

Proposition 1. *To determine the width, W, of the halo and the diameter, $2V$, of the dark area within, each as a function of LC, CM, θ, t, only.*

Let the small circles in Figure 2 represent droplets of water vapor situated at the extreme upper and lower boundaries, respectively, of the halo. We have $LC=n$, $CM=m$, $CQ=V$. $QP=W$, (Def.), $<gPl=t=<MQK$, $<MPg=\Theta=<hQM$, (Cons.). Let $<CLQ=\beta$, $<QLP=\alpha$, $<CMQ=\delta$, $<QMP=\gamma$.

[3]The point h is the intersection with LM, of the line extending from P toward M.

Then, since an exterior angle of a triangle equals the sum of the two opposite interior angles,

$$(\alpha+\beta)=(\theta+t)-(\gamma+\delta),\ \beta=t-\delta,\ \alpha=\theta-\gamma.$$

Also θ, t are fixed angles and $\delta < t$, while by experiment, $\theta+t>\gamma+\delta$, $\delta<22°$. Let $a=\tan(\theta+t)$, $b=\tan t$, then,

$$V=n\tan\beta=n\tan(t-\delta)=(nb-n\tan\delta)/(1+b\tan\delta),$$
$$V=m\tan\delta.$$

Eliminating $\tan\delta$.

$$bV^2/m+(m+n)V/m-nb=0,$$

and the diameter of the dark area is

(1) $$2V=-(m+n)/b+\left[(m+n)^2+4b^2mn\right]^{1/2}/b$$

In like manner,

$$V+W=n\tan(\alpha+\beta)=n\tan(\theta+t-\gamma-\delta)$$
$$=\left[na-n\tan(\gamma+\delta)\right]/\left[1+a\tan(\gamma+\delta)\right],$$

$$V+W=m\tan(\gamma+\delta),$$

Eliminating $\tan(\gamma+\delta)$,

$$a(V+W)^2/m+(m+n)(V+W)/m-na=0,$$

$$V+W=-(m+n)/2a+\left[(m+n)^2+4a^2mn\right]^{1/2}/2a.$$

Hence the formula for the width of the halo is,

(2) $$W=\left[-(m+n)/2a+\left\{(m+n)^2+4a^2mn\right\}^{1/2}/2a\right]-\left[-(m+n)/2b+\left\{(m+n)^2+4b^2mn\right\}^{1/2}/2b\right].$$

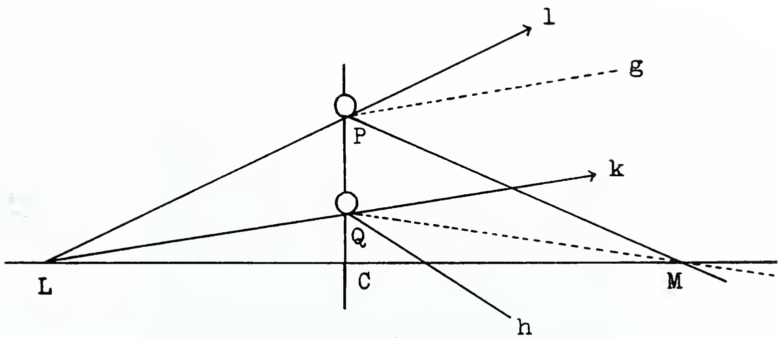

FIG. 2. See text.

Proposition 2. *To determine the diameter, 2V′, and the width, W′, after the light, L, has been removed to an infinite distance* ($n\to\infty$). In this case the rays which come up to the rear of the canopy are parallel ($\alpha\to B\to O$. This is the problem of the halo around the sun or moon. We have only to find the limit of W and 2V above, as $n\to\infty$.

The radicals in (2) may be expanded into absolutely convergent binomial series. The first terms of these series cancel the respective first terms within the brackets and all terms of each series, after the second, approach zero as $n \rightarrow \infty$. The second terms of the series combine into,

$$C = (a-b)mn/(m+n).$$

Hence,

(3) $$W' = \lim_{n \rightarrow \infty} C = (a-b)m.$$

Likewise,

(4) $$2V' = \lim_{n \rightarrow \infty} O = \lim_{n \rightarrow \infty} 2bmn/(m+n) = 2bm.$$

The following table shows the respective values of 2V, W, calculated on the basis of formulas (1), (2), (3), (4), from the indicated assigned values of m, n, Θ, t. The unit of length is one inch.

m	n	Θ	t	2V	W	Photo.
80	10	27°	10°	3.1	4.8	Fig. 3
73	24	27°	10°	6.3	9.3	Fig. 4
71	42	27°	10°	9.2	13.2	Fig. 5
71	∞	27°	10°	25.0	41.0	....

When $n = \infty$ the angle subtended at the midpoint M of the camera film by the radius of the dark region within the halo, is about 10°. To this we must add 12° to obtain the angle which subtends the radius of the circle of maximum illumination.

The halo of 46 degrees. Under Roever's definition, concave reflecting surfaces can also give brilliant points. Let s in Figure 1 be a transparent sphere (droplet). A ray from the light will enter the sphere and be relayed by refraction so that it is reflected against a point P' in the concave side of the spherical surface. It emerges from the sphere and reaches the observer M'. The formulary (1),—, (4) still holds. When the angles Θ, t (cf. (2)), are $\Theta = 24°$, $t = 35°$ this point P' is found to be a brilliant point to an observer within Θ. Thus we obtain another known type of halo. The radius of its circle of maximum light subtends, at M', the angle 46°, $(n = \infty)$.

Description of an experiment. The accompanying Figures 3, 4, and 5 are dark-room photographs (time exposures) of artificial halos, which were obtained as follows: The pictures show reflected light from a plane canopy of glass beads, the source being a lamp L (cf. Fig. 1) placed on the side of the canopy opposite that of the camera. The distance from the canopy to the camera plate, (focal distance), is m. The canopy, which is 2.3 feet in width, was woven of strings or series of approximately spherical glass beads of diameter less than ⅛ inch. The beads (turquoise blue) were strung at intervals of ⅕ inch on black, threestrand cotton yarn, to form the series. The series were woven in various designs to form the canopy, but mostly with one series going around in spiral fashion at intervals of ½ inch, other series being strung as diameters all the way across the canopy.

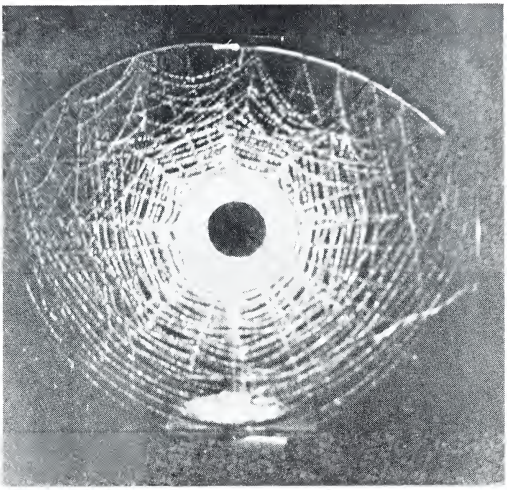

FIG. 3. See text.

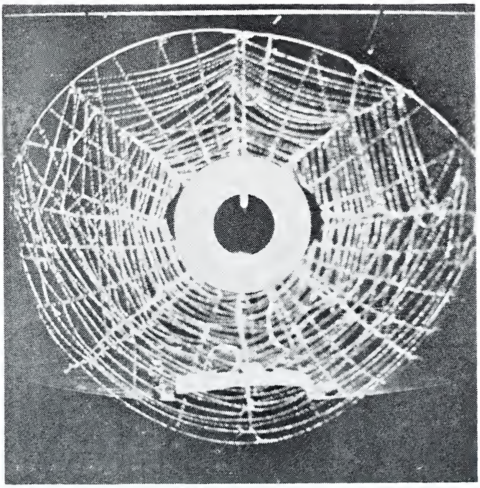

FIG. 4. See text.

One purpose of the construction was to show photographically the individual brilliant points which combine into a halo. On account of the screen at the center, only light which is reflected toward the camera from the beads is intense enough to affect the camera film. Some regret must be expressed for the use of the yarn, since its fuzz caused enough diffraction near the center[4] of the canopy almost to obliterate, in the photographs, the dark interior of the halo. The use of copper wire of a caliber equal to that of the aperture in the beads would eliminate diffraction. It is obvious from the theory, however, that there is a dark circle at the center of the canopy since the beads there are in the dark-of-the-moon phase. Aside from this defect each picture clearly shows a halo. The author considers the photographs as being successful in view of the fact that, in this case, the reflecting wall is only one bead (droplet) in thickness.

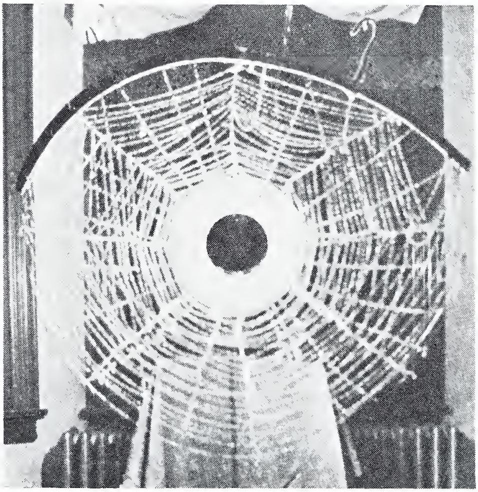

FIG. 5. See text.

The camera distance was about 75 inches, and corresponding to three distances, n, of the light from the canopy, actual measurements of the configurations of the halos on the canopy verified the corresponding calculations, which are summarized in the accompanying table.

The diametral series of beads are regularly eclipsed, that is, pass into the dim virtual image reflection, at the outer edge of the halo in Figures 3 and 4. Note that the halo has a bright part and that, in all cases, there is a gradual decline of its intensity in its outer portions. The bright part corresponds to the angle, (within Θ), of maximum illumination of the brilliant point on a bead. The dark circle is comparatively small when n is small, but, when the incident rays are parallel, the diameter of this interior circle subtends, at the camera plate M an angle of about 20°. Then the dark circle is recognizable as that to be seen in an ordinary halo. It is useful to consider the figure on the canopy as if pro-

[4]These regions of maximum diffraction have been removed from the pictures. Problem in technique: Find the circle of maximum illumination in Figure 5.

jected from M upon a distant wall. What the canopy presents to the eye is brilliant rather than obscure.

Discussion.—In any landscape where there are trees with glossy leaves, or wet leaves lighted by the morning or afternoon sun, brilliant-point illumination of portions of this foliage will be conspicuous. Such a tree, when nearly between the observer and the sun, shows a bright and charming illumination, and the general appearance of such a landscape would be very different if the optical principles were different from those which we have described. This fact is of importance to landscape painters. When a tree is pictured, against the light of an afternoon sky, as something entirely dark against a bright back-ground, it impresses us subconsciously at once as something unreal. We are accustomed to a fringe of brilliant-point on such a tree. This remark is a severe criticism of a good many paintings. Corot, among others, was a master with brilliant-point light.

Contrary to what is asserted in the present article, in quite all of the previously existing theory concerning halos it is assumed, and, as far as the present writer has been able to find, without adequate demonstration, that refraction of light through ice crystals is the *primary* cause.