Combination Frequencies and Modulation

MILO WELLS, Lakewood Ohio*

The result of combining waves of different frequencies led to a rather famous dispute commonly known as the Helmholtz-Koenig controversy. The question which arose concerned the objective reality of beat notes. One group maintained that beat tones did not exist as separate oscillations but that they were detected as such by the ear. On the other hand, the opposite opinion held that beat tones had physical existence. With the advent of radio telephony a number of years later the controversy appeared anew in discussions of the process of modulation.

The purpose of the study reported in this paper was to examine the wave pattern resulting when two sine waves were combined under such conditions as to produce modulation in an oscillating circuit. It was hoped that by employing combining waves of sufficiently low frequencies the resulting pattern could be resolved on the screen of a commercial cathode ray oscilloscope which had a linear sweep circuit.

Grid modulation was first employed. The mutual or transfer characteristic may be represented in the neighborhood of a point (E_c, I_c) by a Taylor's series as follows:

$$I = I_c + A(E - E_c) + B(E - E_c)^2 + \dots \quad \text{(I)}$$

Now suppose that we impress upon the grid two sinusoidal voltages of frequencies $f_1 = p/2$ and $f_2 = w/2$. Substituting in (I) and using the well-known trigonometric reduction formulas

$$\sin^2 a = \tfrac{1}{2}(1 - \cos 2a)$$
$$\sin a \sin b = \tfrac{1}{2}[\cos(a - b) - \cos(a + b)]$$

We have the expression for the current

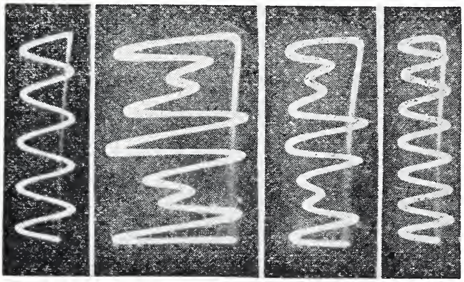
$$I = I_c + \tfrac{1}{2} BE_1^2 + BE_2^2 + AE_1 \sin w t + AE_2 \sin p t - \tfrac{1}{2} BE_1^2 \cos 2 w t - \tfrac{1}{2} BE_2^2 \cos p t + BE_1E_2 \cos (w - p)t - BE_1E_2 \cos (w + p)t + \dots \quad \text{(II)}$$

There are four non-harmonic frequencies resulting and also the second harmonic of each of the parent frequencies as well as terms of zero frequency. The terms of frequency $(w - p)/2$ and $(w + p)/2$ are the side bands. The above theory assumes that the transfer characteristic is nonlinear. If the linear portion of the curve applies, all square and higher terms drop out, including also the product term so that simple addition and not modulation is the result.

These two cases are shown in figure 1. Figure 1a and figure 1d are two sine waves whose frequencies are in the ratio of 5 to 7. Figure 1b is the wave form resulting in the output circuit when the grid is biased on the linear portion of the mutual characteristic, that is in this case,

* Connected with Kemet Laboratory Co., Cleveland, Ohio.

minus 3 volts. Figure 1c is the pattern resulting when the grid is biased on the non-linear portion of the mutual characteristic which is minus 9 volts. In each case the amplitudes of the combining frequencies were equal. By means of a wave meter it was determined that the only frequencies present in the output were the two input frequencies for the first case. It should be pointed out that the load was a pure resistance. In the second case, however, there were two new frequencies of 900 c.p.s. and 5400 c.p.s. in addition to the parent frequencies of 2250 c.p.s. and 3150 c.p.s. Although photographs are not shown the same thing was done with parent frequencies of 900 c.p.s. and 3150 c.p.s. That is a frequency ratio of 2 to 7.



a b c d

Fig. 1.

Next a Heising modulating circuit was set up. The current in the oscillating circuit is of the form:

$$I = AE \cos w t \qquad \text{(III)}$$

If the plate potential is made to vary sinusoidally then:

$$E = E_0 + E_1 \cos p t \qquad \text{(IV)}$$

and

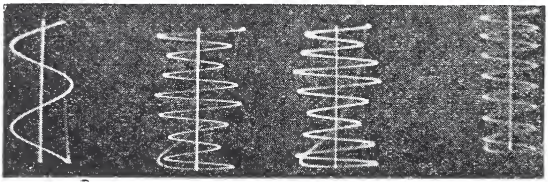
$$I = A(E_0 + E_1 \cos pt) \cos w t \qquad \text{(V)}$$

which is the equation of the curve shown in figure 2b and figure 2c. Expanding (V) and again making use of trigonometric reduction formulas, we get

$$I = AE_0 \cos w t + \tfrac{1}{2} AE_1 [\cos (w-p)t + \cos (w + p)t]$$

This expression, unlike the expression for grid modulation, does not contain the modulating frequency. The tuned grid circuit across which the oscilloscope deflecting plates where connected would reject any frequency very much different from the carrier or natural oscillation frequency of the tuned circuit. For Heising modulation the carrier frequency was 30,000 c.p.s. and the modulating frequency 7,500 c.p.s., a ratio of 4 to 1.

Although the general shape of the curve is the same for figure 1 the actual frequencies which are contained in the resultant are quite different. It may be said that the amplitude of the curve shown in figure 1b varies sinusoidally in the ratio of 7 to 2 but there is no frequency in the output corresponding to 2. For figure 1b the only frequencies in the output are in the ratio of 7 to 5. Returning now to figure 2 it will be noted that the

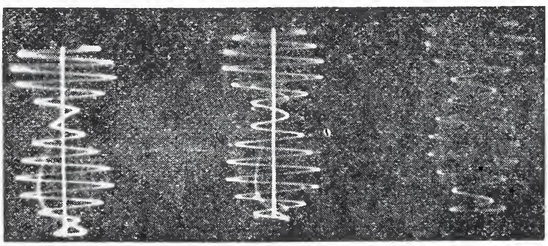


a b c d

Fig. 2.

frequency ratio is greater than 2 to 1 while the frequency ratio is less than 2 to 1 for the first case.

Figure 2 is the resultant of adding the carrier to one but not both of the side bands. This pattern was obtained by connecting the oscilloscope deflecting plates across the condenser of a wave meter loosely coupled to the tank coil of the grid circuit. The wave meter was sufficiently selective to accept any one of the three frequencies in the output, filtering out completely the other two. However, between the carrier and either side band a point could be found at which the amplitude of the carrier was equal to the amplitude of one side band or the other. Thus the patterns of Figure 3 represent simple addition and are comparable in every respect to figure 1b.



a b c

Fig. 3.

The results of the investigation may be summarized as follows:

1. The theory in regard to the formation of beat notes for sound waves has been verified for electrical waves. That is sound waves form beat notes only when combined in a non-linear medium.

2. In a similar manner electrical waves form beat notes or "side bands" only when combined in a non-linear medium. Modulation is the result of combining electrical waves in a non-linear medium.

3. A visual pattern of simple addition and one of non-linear addition appear so much alike that only an expert can distinguish between the two. As Professor Ramsey pointed out, a spectrometer such as a very selective wave meter is better adapted for the purpose.

References

1. Hazel, H., 1935. Beat Notes, combination tones and side bands, Phil. Mag. **19**:103. Digest in Am. Journ. Phys. (Am. Phys. T.) **3**:95.

2. Ramsey, R. R., 1935. The Helmholtz-Koenig controversy, Science. **81**:561; 1940. "On the combination of sine waves", Am. Journ. Phys. **8**:237.

3. Bragg, W., 1939. Combination tones in sound and light, Nature, **143**:542. Digest in Am. Journ. Phys. (Am. Phys. T.) **7**:427.

4. Aigner, F., and Kaber, C. L., 1936. Hochfrequenztechnik und Electro-akustik, **48**:59.

5. Peterson, E., and Keith, C. R., 1928. Grid current modulation. Bell System Tech. Journ., **7**:138.

6. Heising, R. A., 1921. Modulation in radio telephony. Proc. I. R. E., **9**:305.