Wave Velocity
(Abstract)

A. L. Foley, Indiana University

It will be recalled that Newton's original equation for the velocity of wave motion, $V = \sqrt{\frac{E}{D}}$, when applied to the atmosphere, gave a result roughly 20% too low. One of the early attempts to explain the discrepancy was the assumption that in any particular path through the atmosphere the molecules themselves occupy about one-sixth the distance and that sound is transmitted through them at a very much greater velocity than through the space between them. Speculation ceased when Laplace called attention to the fact that the formula gives very good results if one uses the adiabatic instead of the isothermal elasticity in applying it. The general agreement between the velocities obtained experimentally and those given by the corrected Newtonian equation was so good that scientists considered the question to be finally settled and paid little or no attention to anyone proposing a different solution.

The writer confesses that he cannot understand the demand for adiabatic elasticity in the case of a very weak sound wave, one in which the amplitude of molecular motion is but a fraction of the diameter of the molecule (some authorities say as little as 1/30 of that diameter) in a gas with molecular distances many, many times the molecular diameters. Is it possible that the Laplace-Newton equation is not the last word on the subject? At any rate, it is not the *only* word. In 1860 Earnshaw[1] proposed a solution which, unlike Newton's, demands a variation in sound wave velocity with changes in pitch and intensity. This is in direct contradiction to the usual text book statements that the velocity of sound in the atmosphere is independent of barometric pressure, pitch, and intensity. To be sure, some texts qualify the latter point by saying "independent of intensity except for sounds of very great intensity at points near the source." Many attribute a greater velocity near the source to a mass motion of the air near the source, as is known to be produced sometimes, for instance, when a cannon is fired. But, so far as the writer knows, rarely has the question been raised as to the equality of the velocities of waves of moderate and of low intensity. In discussing the question Earnshaw says:

"With respect to the velocity of sound, which has hitherto been found experimentally to exceed the velocity obtained by theory, it is shown that the value obtained by approximative methods is the minimum limit of sound velocity, so that the actual velocity will always be greater, the excess depending upon the intensity and genesis of the sound. It is shown that all the parts of a wave do not travel at the same rate,—a circumstance which leads to the formation of a bore in the front of the wave. Several previously unexplained phenomena, which have been recorded by different experimentalists, such as double reports of fire-arms heard at a great distance, the outrunning of one sound by another observed by Capt. Parry, the comparative powers of different gases of transmitting sounds, and the laws of transmission of sound from one medium to another, are accounted for in this paper, and directly deduced from the integral of the equation of wave-motion.

[1]Earnshaw, Samuel, 1860. On a new theoretical determination of the velocity of sound. Phil. Mag. 19:449; 20:37, 186. Abstract, Proc. Roy. Soc. 9:590, 1857-59.

"The error committed in calculating the velocity of sound, was not the leaving out the consideration of the development of heat, but the supposing the medium of air to be *continuous*. I am surprised to find the result so much affected by a circumstance which appears trifling,—and the more so, as the radius of the sphere of sensible molecular action is known to be, though finite, very small. The assumption of continuity is therefore by no means so allowable as we should be inclined *a priori* to supose; and its effect on the motion of an elastic medium is very much greater than was to be expected." (Sheffield, May 9, 1860.)

Regnault[2] appears to have been the first to make any extended experiments on the question of the velocity of sound at different distances from the source, which, of course, means measurements at different sound intensities. In 1864 Regnault announced that he had found the average velocity of the sound produced by firing a cannon to be a little more than 0.2% greater over a distance of 1280 meters from the source than when measured over a distance 2445 meters from the source.

Space will not permit even naming all those who have experimented on this question. It is significant, however, that all agree on the statement that sound velocities *do* depend upon sound intensities. In 1889 Threlfall and Adair measured the velocity of sound in water. The calculated velocity was about 1500 meters per second, while the observed velocity was 1732 meters per second when the sound was produced by firing a charge of 9 ounces of gun cotton, increasing to 2013 meters per second when the charge was increased to 64 ounces. The author of this paper, using a photographic method that enabled him to determine the *instantaneous* velocity of the sound produced by an electric spark,[3] found the velocity to be more than three times as great at a distance of 3.4 mm. from the spark as it was at a distance of 34 cm., 100 times as far from the spark.

The coming spring and summer the writer expects to study the question further (making use of a sound-ranging outfit loaned him by the United States Government), to determine, as accurately as the method will permit, the relation between sound velocity and intensity.

The assumption that Newton's equation demands a constant velocity for all intensities tacitly assumes that the elastic coefficient of a medium is independent of the strain so long as the strain is well below the elastic limit. The writer has under way an experiment to test the accuracy of this assumption for very small displacements. To the writer the interest attaching to the problem comes from a possible inference that may be drawn respecting the velocity of light waves of very low intensity. It is admitted at once that conclusions arrived at from a study of wave velocities in material media may not apply at all to light waves in the ether. On the other hand it is to be said that the application of Newton's equation to the ether has resulted in several verifiable deductions.

Really, why should we assume that the velocity of light waves (photons, corpuscles, or what have you) when they left the source millions of years ago, is the same as it is now when their energy is so small that it must be condensed by exceedingly large mirrors or lenses

[2]Regnault, V., 1868. Mem. de l'Acad. Pair **37**:1, 3. Compt. Rend., **66**:209. Phil. Mag. Ser. 4, **35**:161, 1868.
[3]Foley, Arthur L., Phys. Rev., N. S., **16**:449-463.

and integrated over a period of several hours or days in order to make any impression on a sensitive photographic plate. But this is precisely what is assumed by those who, finding a minute shift of the spectral lines toward the red, conclude that our universe is expanding. It is assumed, likewise, by those who would explain the shift as due to a gravitational attraction for radiation. The writer does not presume to pass upon either of these theories. However, he would call attention to the fact that it is highly improbable that the shift is due to both causes. One or the other of the two hypotheses must be rejected; possibly, on the results of further studies, both of them may be discarded. At this time the writer regards them as hypotheses only, hpyotheses that must await the future for verification or rejection.