PROGRAM OF THE SECTION ON MATHEMATICS

Chairman: C. K. Robbins, Purdue University

1. On the foundations of geometry. Karl Menger, University of Notre Dame.

J. E. Dotterer, Manchester College, was elected chairman of the section for 1938.

A Foundation of Projective Geometry

Karl Menger, University of Notre Dame

We assume a system of elements denoted by A, B, . . . and two operations which associate with any two elements A and B, an element $A+B$ and an element $A.B$. The operations may satisfy the following conditions:

I. Associativeness: $A+(B+C)=(A+B)+C$, $A.(B.C)=(A.B).C$.

II. Existence of two indifferent elements V ("vacuous" element) and U ("universal" element) such that for each element A we have:

$$A+V=A, \qquad A.U=A,$$
$$A.V=V, \qquad A+U=U.$$

III. A weakened distributive law:

$$A+(A+B).C=A+(A+C).B, \qquad A.(A.B+C)=A.(A.C+B)$$

From these assumptions we easily deduce that both operations are commutative and, for each A and B, satisfy the condition $A+(A.B)=A=A.(A+B)$. Consequently, if we have $A+B=A$ for two elements A and B, then we also have $A.B.=B$, and conversely. If for two elements A and$B$ both formulas $A+B=A$ and $A.B=B$ hold, then we call B a part of A. The part relation defined in this way has the ordinary properties. From C part of B, and B part of A, it follows that C part of A, etc.

We furthermore easily derive the formula $A+A=A=A.A$ for each A, by virtue of which there are neither multiples nor powers in the formulas of the algebra of geometry derived from the postulates I, II, III. We get thus: A part of A for each A; besides: V part of A, and A part of U on account of postulate II. Those elements which are different from V and do not contain any other parts than themselves and V shall be called points. Those elements which are different from U and are not part of any other element than themselves and U shall be called hyperplanes. Our definition makes precise the famous first words of Euclid's Elements: "Point is that which has no parts."