Conductivity of Circular Openings in Helmholtz Resonators

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In analogy with the flow of electricity in conductors and liquids in pipes, the conductivity of the neck of a resonator is given as (disregarding end effects) $\frac{\pi R^2}{L}$ where R is the radius of the neck and L the measured length. Since the ends of the neck also have an effect on the conductivity, the effective length is greater than the measured length. This added length is a measure of the inertia of the air in the immediate neighborhood of the ends of the neck.

It has been noticed that the expressions for the conductivity of the neck of a Helmholtz resonator as given by different authors do not agree. They do agree in general that it is given by $\frac{\pi R^2}{L_1}$ where L_1 is here the effective length and is given by L plus a suitable end correction. It is in computng this end correction that the discrepancies occur.

Barton[1] gives the end correction as $\frac{\pi R}{4} = .785R$, Morse[2] gives $\frac{16R}{3\pi} = 1.171R$, Wood[3] uses 0.6 R, and Datta[4] and Olson and Massa[5] assume no end correction whatsoever. It is correct to make this last assumption if L and L_1 are very large and R is relatively small, but with standard Helmholtz resonators this is never the case.

Rayleigh[6] takes a circular opening in an infinitely thin wall and finds that the conductivity of this orifice is 2R. Since conductivity is the reciprocal of resistance, he adds this conductivity to that of a neck of finite length L as follows:

$$\text{resistance} = \frac{1}{2R} + \frac{L}{\pi R^2}$$
$$\text{conductivity (K)} = \frac{\pi R^2}{L + \frac{\pi R}{2}} \tag{1}$$

It is evident that Barton's value of $\frac{\pi R}{4}$ assumes a correction at one end of the neck only. Morse has a correction at both ends, but he has used a value which is approximately correct only when L is very large. Wood's value of 0.6 R is that given by a correction at one end with no baffle present. Rayleigh's value of $\frac{\pi R}{2}$ assumes a correction at both ends of the neck and holds for small values of L.

In this study to determine the correct value for L_1, the apparatus consists of the following: (1) a set of ten spherical Helmholtz resonators, (2) a telephone receiver driven by a beat frequency oscillator, (3) a sound meter, and (4) a baffle board about three feet square.

[1]Barton, E. H., 1908. A textbook of sound. Macmillan.
[2]Morse, P. M., 1936. Vibration and sound. McGraw-Hill.
[3]Wood, A. B., 1930. A textbook of sound. Macmillan.
[4]Datta, A. C., 1917. A textbook of sound. Blackie and Son.
[5]Olson and Massa, 1934. Applied acoustics. Blakiston.
[6]Rayleigh, 1896. The theory of sound. Macmillan.

The resonator neck extends through a hole in the baffle so that its outside end is flush with the top surface of the baffle. The telephone receiver and microphone from the soundmeter, mounted on a swinging boom, are passed back and forth across the opening in the resonator. As the frequency of the receiver diaphragm approaches the resonance frequency of the resonator, an increased intensity will be shown by the soundmeter. When this is a maximum, the two frequencies coincide.

The following table gives the data and results of these experiments. Here the experimental K is found from the resonator formula,

$$N=\frac{C}{2\pi}\sqrt{\frac{K}{V}} \tag{2}$$

where N = fundamental resonance frequency, C = velocity of sound in air, V = volume of resonator, and the theoretical value is from Equation (1).

TABLE I.—Comparison of Theoretical and Experimental Values for K

Volume of Resonator $cm.^3$	Fundamental Frequency N vib/sec.	Length of Neck L cm.	Radius of Neck R cm.	Experimental Conductivity K c.g.s. units	Theoretical Conductivity K c.g.s. units
6220.0	129	0.25	1.93	3.58	3.57
1180.0	250	0.47	1.49	2.47	2.48
363.0	370	0.37	1.04	1.68	1.70
153.5	503	0.21	0.76	1.30	1.29
98.0	612	0.23	0.76	1.28	1.28
73.0	722	0.22	0.76	1.29	1.28
52.0	858	0.24	0.76	1.28	1.27
39.5	982	0.08	0.69	1.28	1.29
27.5	1080	0.20	0.64	1.08	1.07
21.0	1220	0.21	0.64	1.05	1.06

TABLE II.—End Corrections for Closed Pipes

	N vib/sec.	λ cm.	R_1 cm.	R_2 cm.	d cm
Pipe 1	420	62.0	59.2	18.2	2.30
Radius = 2.85 cm.	480	71.4	51.3	15.6	2.25
Theoretical end	540	63.4	45.3	13.6	2.25
Correction $d_1=\frac{R}{4}=2.24$ cm.					
Pipe 2	420	81.2	59.8	19.2	1.10
Radius = 1.3 cm.	480	71.4	52.5	16.8	1.05
$d_1=1.02$ cm.	540	63.6	46.7	14.9	1.00
	600	57.2	41.9	13.3	1.00

From this table it is evident that Rayleigh's value for the correction is right, but the question arises as to whether or not it is the same at both ends of the neck. It is necessary to make the obvious assumption that the outside end correction would be the same for a "closed" pipe as for a resonator. This outside correction for the pipe can be found since there is no inside correction; by taking the difference between this value and Rayleigh's value for the resonator, the inside end correction for the resonator can be obtained.

To determine this outside end correction, two pipes of different radii are used. One end of each is closed with a rubber stopper, which can be moved to vary the length of the pipe. The rest of the equipment and procedure are the same as described before.

Two resonance points R_1 and R_2 are found in each of the pipes for each frequency

$$R_1 = \frac{3\lambda}{4} - d \tag{3}$$

and

$$R_2 = \frac{\lambda}{4} - d \tag{4}$$

where λ is the wave length of the sound from the diaphragm and d is the experimental end correction.

Table II shows that the experimental and theoretical values for the end correction of the pipes check within experimental error. From this it can be assumed that the end correction of a resonator is evenly divided between the two ends of the neck when a baffle is used.