MATHEMATICS

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ABSTRACTS

Retraction, Homotopy, Integral. LAMBERTO CESARI, Purdue University.—Recent studies have shown that parametric surfaces in the ordinary space under the sole hypotheses to be continuous and to have finite area have remarkable analytical and geometrical properties which proved to be adequate in analysis and calculus of variations. The author has defined an integral $I(S) = (S)\int F(p,t)$ over a surface S of finite area as a Weierstrass integral ($F(p,t)$ is any continuous function of the point p and the direction t, and $F(p,kt) = kF(p,t)$ for $k > 0$). For the definition of $I(S)$ by a limit process the surface S is divided into pieces and the direction of an average normal to each piece can be defined by the help of topological methods. A. G. Sigalov, J. Cecconi, J. M. Danskin and the author have used the integral $I(S)$ for the proof of existence theorems of calculus of variations. The author has proved and applied lemmas concerning retraction and homotopy. Every surface S whose boundary B is close to a given simple curve C has a nondegenerate retraction S_0 whose boundary B_0 is also close to C. If B is contained in C and is not nulhomotop in C then B_0 is also contained in C and is not nulhomotop in C.

Extreme Points of Convex Sets. MEYER JERISON, Purdue University.—A bounded, closed, convex set in a finite dimensional vector space is the closed convex hull of its extreme points. If $[K_n]$ is a decreasing sequence of such sets, then their intersection, K, is also such a set. It is proved here that every extreme point of K is the limit of a sequence of extreme points of the sets $[K_n]$. If $[K_n]$ is an increasing sequence, then the union is convex. Letting K be the closure of that union and assuming that it is bounded, one obtains the same conclusion as in the preceding case.

With some slight changes, notably, replacing "bounded, closed" by "compact", the results become valid in an infinite dimensional space.

Mathematics for Teachers. CHARLES F. BRUMFIEL, Ball State Teachers College.—Current dissatisfaction with the job that the high schools are doing in preparing students for college work in mathematics is certainly justified. Any improvement in the high school program must come primarily from the high school teachers themselves. The immediate task, then, should be the creation of better training programs for teachers. In Indiana, high school and elementary teachers are required to earn the Master's degree within a few years of graduation in order to retain license to teach. This requirement that teachers follow a graduate program presents an opportunity to set up courses on the graduate level for both the elementary and high school teachers that will give them a better insight into the meaning of mathematics. Conventional graduate courses leading to research are of course impossible. Courses in foundations and History are in order. Moreover, it would seem desirable to consider whole areas of mathematics, as algebra, geometry, and analysis, and reorganize materials

so that they may be presented to the high school teacher in such a way that he can relate them to his teaching and draw from them a living stimulus for his work.

Integrating Factors of $P(x,y,z)dx + Q(x,y,z)dy + R(x,y,z)dz = 0$. WILL E. EDINGTON, DePauw University. — The differential equation $P(x,y,z)\,dx + Q(x,y,z)\,dy + R(x,y,z)\,dz = 0$ is of considerable importance when integrable in interpreting solutions of partial differential equations, and frequently its solution is essential to the solution of a second order partial differential equation where Monge's method is used and it becomes one of Monge's equations used in determining an intermediate integral. In this paper an expression for an integrating factor of the above equation is put in such a form as to indicate or suggest a relation, often in functional form, that will lead to the solution of the equation. Certain exceptional cases are discussed and interpreted.

Periodic Solutions of Non-Linear Differential Equations. R. A. GAMBILL, Naval Ordnance Plant, Indianapolis.—Consider the system of differential equations

$$y_j = \rho_j y_j + \epsilon q_j\,(y_1, \ldots, y_n, t; \epsilon), \qquad j = 1, \ldots, n, \tag{1}$$

where $\rho_1, \ldots, p_n$ are complex numbers, ϵ is a small parameter, and each q_j is a holomorphic function of $y_1, \ldots, y_n, \epsilon$, for $|y_j| < A$, $|\epsilon| < \epsilon_0$, with coefficients periodic in t of period $2\pi/\omega$. The problem of interest here is to determine periodic solutions of (1) whose dominant terms have period $2\pi/\omega$ (harmonics), $2\pi n/\omega$, n an integer > 1 (subharmonics), or $2\pi/n\omega$, n an integer > 1 (ultraharmonics). General sufficient conditions are given for the existence of harmonics, subharmonics, ultraharmonics of (1) for $|\epsilon|$ small. At the same time, their approximate expressions are determined through a convergent process. The method used is a method of successive approximations which is a variant of the Poincaré method of casting out the secular terms. It is similar to the method used by L. Cesari for the determination of the characteristic exponents of linear systems with periodic coefficients, and to the method used by J. K. Hale for the determination of cycles of non-linear autonomous systems. The following results, among others, are obtained:

I. The numbers $\rho_j = ik_j\omega/m_j + 0(\epsilon)$ can be determined so that there is a periodic solution of (1) of the form

$$y_j = a_j e^{ik_j\omega t/m_j} + \epsilon w_j(t;\epsilon), \qquad j = 1, \ldots, n, \tag{2}$$

k_j, m_j positive integers, a_j arbitrary complex constants, $|a_j| < A$, and $w_j(t;\epsilon)$ holomorphic in ϵ with coefficients of period $T = 2\pi m_1, \ldots, m_n/\omega$.

II. If $\rho_j = ik_j\omega/m_j$, and if

$$S_j(a_1, \ldots, a_n) = \frac{1}{a_j T}\int_0^T q_j\,(a_1 e^{ik_1\omega t/m_1}, \ldots, a_n e^{ik_n\omega t/m_n}, t; 0)\, e^{-\rho_j t}\,dt,$$

then there is a periodic solution of (1) of the form (2) if $S_j(a_1^\circ \ldots, a_n^\circ) = 0$, and the Jacobian $|\partial S_j(a_1^\circ, \ldots, a_n^\circ)/\partial a_k| \neq 0$. This condition on S_j has been obtained by Coddington and Levinson. Cases where the above Jacobian is zero are discussed when system (1) arises from a real system of second order differential equations.