Reservoir Operating Rules

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Introduction

This study considers a deterministic optimization model to determine general reservoir operating rules. An algorithm that cycles through a deterministic dynamic program, a regression analysis and a hydrologic simulation has been developed. At each iteration the reservoir release becomes more constrained and the general operating rule becomes more refined. This work is the logical extension of Young's (3) study to refine the general operating rules by repeating the process with one extra constraint on the dynamic program; namely, release must be within some specified percentage of the release defined by the previously defined general operating rule.

This algorithm was tested using annual streamflow data for the Gunpowder River, Maryland, the Osage River, Missouri, and the Blacksmith River, Utah. In all cases, significant improvements in operations of the reservoirs have resulted from use of the algorithm. In the remainder of this paper the algorithm is described, some results are presented, and finally a discussion of the results and a set of conclusions are given.

Algorithm to Refine Operating Rules

A discrete dynamic program has been formulated to provide the optimal operating policies for a single reservoir over a finite time horizon of T years. The reservoir inflows for all T years and the reservoir capacity are assumed to be known. The objective function is to minimize the total losses caused by very high or very low streamflows over the T year time horizon. This can be expressed by

$$\text{Minimize} \sum_{t=1}^{T} \text{Loss}\,(R_t) \tag{1}$$

The loss function (Loss (R_t)) used in this investigation responds to reservoir outflow (R_t) only. As the deviation of outflow or release above or below some acceptable range increases, the losses associated with the release also increase.

This problem is subject to some physical constraints. The continuity or mass balance at the reservoir should be preserved at all times. The discrete version of mass balance can be written as:

$$S_{t+1} - S_t + R_t = I_t \qquad t = 1, 2, \ldots, T \tag{2}$$

where: I_t = inflow volume during year T
S_t = storage volume at the beginning of year t
R_t = outflow volume from the reservoir during year t

Depending on the reservoir system there may also be restrictions on the maximum and minimum allowable outflows and storages.

$$R_t^{max} \geq R_t \geq R_t^{min} \quad t = 1, 2, \ldots, T \tag{3}$$

$$S_t^{max} \geq S_t \geq S_t^{min} \quad t = 1, 2, \ldots, T \tag{4}$$

The solution of the dynamic program comprises optimal outflows R_t^* and optimal storages S_t^* for the T year time horizon. These optimal storages and releases are dependent on the particular sequence of inflows used in the dynamic program and, therefore, do not define operating rules for the reservoir.

However, the optimal outflows R_t^* can be regressed against the optimal storages S_t^* and inflows I_t to provide general operating rules.

$$\hat{R}_t = aI_t + bs_t + c \tag{5}$$

where: a, b, and c are constants defined by a multiple regression. The consistency and the efficiency of these rules can be tested by simulation of the rules in operation of the reservoir over a long time horizon.

To refine an operating rule, the dynamic program can be used. If an operating rule already available would require the release at time t to equal $\hat{R}_t$, then the allowable release in the dynamic program can be bounded between plus and minus some percentage of this general operating rule:

$$(1 - \text{BOUND})\hat{R}_t < R_t < (1 + \text{BOUND})\hat{R}_t \tag{6}$$

where: BOUND = fraction of general operating rule that actual outflow may deviate from the rule in the dynamic program.

The solution of this dynamic program can be used to define a new, refined, general operating rule. This cycle—dynamic program, regression, simulation, dynamic program, ...—can be continued with the value of BOUND decreasing until it equals zero.

Case Study

In order to test the algorithm and its ability to produce good general operating rules, several cases were studied including different reservoir sizes and different streamflow sites. The streamflow sequence used in the dynamic program was the historical streamflow record. The simulation model was constructed for a 1,000 year time horizon. The streamflow sequence used in the simulation model was generated by the autoregressive (AR) or auto-regressive-moving-average (ARMA) model with the best AIC value (1).

The loss function used for all cases was a discontinuous exponential function. As the outflow increases from zero to a safe range of outflow, this function exponentially decreases to zero; and after a safe range, it increases exponentially.

Table 1 is a sample result which clearly expresses the significant value of the algorithm in selecting reservoir operating rules.

TABLE 1. *Blacksmith River, Reservoir Capacity Equals 1.1* 10^8m^3*

Dynamic Program Results		General Operating Rules				Simulation Results		
BOUND	losses/year	a	b	c	R^2	losses/year	% Time Reservoir Empty	% Time Reservoir Filled
>0.36	12,299.	0.441	0.066	59,343.	0.521	74,932.	19.0	17.6
0.18	16,606.	0.478	0.053	55,769.	0.655	76,019.	18.0	17.6
0.09	26,285.	0.524	0.015	52,848.	0.774	78,901.	19.0	18.0
0.06	31,619.	0.553	0.106	44,238.	0.790	75,035.	12.9	13.2
0.03	47,546.	0.593	0.262	30,261.	0.914	74,122.	3.9	6.1
0.01	107,362.	0.764	0.677	−14,429.	0.838	107,756.	0.0	0.0

Discussion of Results

1. As the value of BOUND decreases, R^2 (goodness of fit) increases in general. This is expected because as BOUND goes to zero all releases are required to be closer and closer to the regression line.
2. As the BOUND decreases, the losses/year in the dynamic program increase. Once again this is expected because as the value of BOUND decreases, there is a tightening of the constraints of the dynamic program and, therefore, the objective function suffers more.
3. As the value of BOUND decreases to zero the general operating rule changes drastically. In general the value of c decreases and the values of a, and b increase. These changes signify a shift in the rule from a relatively constant release to one that depends more on inflow and storage.
4. The loss/year from the dynamic program is less than the loss/year from the simulation. This is due to the optimal selection of outflows in the dynamic program.
5. As the BOUND decreases from infinity to zero, in most of the cases, the simulation's loss/year first decreases then increases.
6. In the simulation, the percentage of time that the reservoir is either filled or empty (implying that the actual outflow deviated from the operating rule) is inversely correlated to the losses/year. In all test cases the minimum percentage of filled or empty time occurred within minimum BOUND value of 0.01.

Choosing the best operating rule for a reservoir involves many criteria, comparisons and trade-offs. Two criteria that are explicitly considered here are the simulation's losses/year and the percentage of time that actual outflow is unequal to the outflow defined by the operating rule (which equals the percentage of time that the reservoir is filled or empty). The assumption is that each criterion should be minimized. This analysis demonstrates that the two criteria are conflicting and it provides the information needed to choose that rule with the best compromise between the two criteria.

Rules with a form other than the linear one described here could also be used. However, as the form of the rule becomes more complex (e.g. non-linear), the computational effort of the algorithm increases. Bhaskar and Whitlach (2) have found a simple, linear operating rule is as good as or better than the more complex rules in many cases.

Conclusion

An algorithm to generate reservoir operating rules has been proposed and tested. The algorithm is easy to use and each component of the algorithm (deterministic dynamic program, multiple regression, simulation) is relatively simple and well documented in the literature. The algorithm can be used for complex multiple reservoir systems and for seasonal or annual operation of the reservoirs. All the test cases with different reservoir capacities and different streamflow characteristics show significant improvements in the operating rules are possible when the operating rules are refined using the proposed algorithm.

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