The Barker Effect: The Case of the Circular Cylinder

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In the front stagnation region of a viscous fluid flow around a blunt body there occurs a kinetic energy conversion into pressure whose rate is affected by the viscosity of the fluid.

The earliest mention of viscosity influence on the pressure at the stagnation point of a sphere is made in a paper by Miss M. Barker (3). The author, having noticed in experiments undertaken upon suggestion of G.I. Taylor that velocity measurements with a Pitot-tube consistently gave, at low velocities, higher readings than expected from the Bernoulli theorem, that is

$$p_s - p_\infty = \frac{1}{2}\rho U_\infty^2, \qquad [1]$$

(where ρ is the liquid density, U_∞ is the free stream velocity, p_s is the stagnation pressure, and p_∞ is the pressure at infinity) tried to approach the problem by calculating the stagnation pressure by means of Stokes law.

For very small Reynolds numbers $Re = \rho U_\infty R/\mu$ the stagnation pressure on a sphere of radius R, given by Stokes law is

$$p_s - p_\infty = \frac{3}{2}\frac{\mu U_\infty}{R}, \qquad [2]$$

where μ is the viscosity of the liquid and R the radius of the sphere. For high Reynolds numbers the stagnation pressure is the same as for an ideal fluid, since viscosity effects, mostly in terms of boundary layer thickness, are negligible. Miss Barker argued that, since for low Reynolds numbers

$$\frac{1}{2}\rho U_\infty^2 << \frac{3}{2}\frac{\mu U_\infty}{R}$$

and for high Reynolds numbers, on the contrary,

$$\frac{1}{2}\rho U_\infty^2 >> \frac{3}{2}\frac{\mu U_\infty}{R}$$

both for low Re and for high Re an expression like

$$p_s - p_\infty = \frac{1}{2}\rho U_\infty^2 + \frac{3}{2}\frac{\mu U_\infty}{R} \qquad [3]$$

should approximate the data. While the mid Re are somewhat more difficult to explain by means of this intuitive law, which can be rewritten as

$$C_{ps} = \frac{p_s - p_\infty}{\rho U_\infty^2/2} = 1 + \frac{3}{Re}, \qquad [4]$$

the author presents some data, about which she admits: "The accuracy with which the points lie on this line is probably fortuitous, but a viscosity effect of a type suggested is clearly demonstrated". While Miss Barker's formulation of [4] was intuitive, the fact that the experimental points were falling on its graphical representation was not fortuitous. In fact, Homann (9) demonstrated that, for large Re, [4] is the relationship between C_{ps} and Re.

We point out that [4] was proposed for the stagnation pressure in front of a sphere, and not of a cylinder. Since there is no equivalent of Stokes law for the case of the cylinder, (10), the closest approach to an equation similar to [4] should be done with an Oseen type solution, or Lamb's solution (14), whose results are, however, valid only close to the cylinder's surface and for small Reynolds numbers.

The first author to conduct a comprehensive and systematic study of stagnation pressure on the front generator of a cylinder is Thom (17), followed by Thom (18). In the first paper the author, without mentioning the experimental finds of Miss Barker, proposes different ways for the calculation of the viscous overpressure at the stagnation point on a cylinder. By using a boundary layer type of reasoning he arrives at the expression

$$C_{ps} = 1 + \frac{2}{Re} \sqrt{- \frac{1}{2} \frac{d^2}{d\theta^2} \left(\frac{p - p_s}{\rho U_\infty^2/2} \right) \Bigg|_s} \quad , \qquad [5]$$

being θ the angular polar coordinate. He recognizes that the radicand equals 4 for ideal fluids, that it should be less than 4 for the most general case, and suggests a value 3.5 for high Reynolds numbers, but he fails to give a general expression for the radicand in terms of Reynolds numbers.

Nevertheless, the realization the $C_{ps} = 1 + 4/Re$ (Thom's law) is an upper limit for the actual value of C_{ps}, allows him to come up with experimental data that satisfy that inequality.

For smaller Reynolds numbers, Thom assumes that the stream function for the flow around the cylinder is of the form $\psi = \sin\theta\, S(r,\theta)$, where r is the radial polar coordinate, and, for small values of θ, he is able to integrate numerically the resulting equation by finite differences and, since of the conditions to be satisfied, two are on one end of the range and two are on the other, he adopts a trial and error technique, whereby, from an assumed tabulated relationship S(r), an improved function is obtained.

The derivation of Thom's law can be performed by following Homann (9) who suggests to calculate the pressure at the front stagnation generator by integrating the x component of the Navier-Stokes equations from $-\infty$ to -R, in the Cartesian plane, along the x direction of the uniform stream, that is

$$\frac{1}{\tilde{\rho}} \frac{\partial p}{\partial x} = \nu \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] - u \frac{\partial u}{\partial x} , \qquad [6]$$

where $\tilde{\rho}$ is the fluid density, y the conjugate Cartesian coordinate, and u the velocity of the flow along the x direction. The integration yields

$$\frac{1}{\tilde{\rho}}(p_s - p_\infty) = \nu \frac{\partial u}{\partial x}\Bigg|_{-\infty}^{-R} + \nu \int_{-\infty}^{-R} \frac{\partial^2 u}{\partial y^2}\,dx - \frac{u^2}{2}\Bigg|_{-\infty}^{-R} \qquad [7]$$

Since $\partial u/\partial x$ is zero both at infinity and at the stagnation point, we can rewrite [7] as

$$\frac{p_s - p_\infty}{\tilde{\rho}} = \frac{U^2_\infty}{2} + \upsilon \int_{-\infty}^{-R} \frac{\partial^2 \upsilon}{\partial y^2}\,dx$$

or [8]

$$C_{ps} = 1 + \frac{2}{Re} \int_{-\infty}^{-1} \frac{\partial^2 \nu}{\partial \eta^2}\,d\xi ,$$

where $\eta = y/R$, $\xi = x/R$, $\upsilon = u/U_\infty$.

For the particular case of ideal fluid flow ($Re \to \infty$) the expression for υ can be shown to be

$$\upsilon = 1 + \frac{\eta^2 - \xi^2}{(\xi^2 + \eta^2)^2} , \qquad [9]$$

which yields

$$\frac{\partial^2 \upsilon}{\partial \eta^2}\Bigg|_{\eta = 0} = \frac{6}{\xi^4} , \qquad [10]$$

If we substitute [10] into [8] we obtain, with Homann [9]

$$C_{ps} = 1 + \frac{4}{Re} , \qquad [11]$$

which is obviously valid only for $Re \to \infty$. For any finite value of Re it is always $\frac{\partial^2 \upsilon}{\partial \eta^2} \leq \frac{6}{\xi^4}$ because the velocity reduction, caused by viscosity effects, tends to seperate the streamlines further away than in the ideal case along the approach x axis. We can in fact assume that the streamfunction around the approach x axis can be written as $\psi_i\left(\xi, \frac{\eta}{\eta_o(\xi)}\right)$ if $\psi_i(\xi, \eta)$ is the ideal fluid steamfunction and $\eta_o(\xi)$ is a "spread" function, always larger than 1 for any ξ. Then,

$$\frac{\partial^2 \upsilon}{\partial \xi^2}\Bigg|_{\eta = 0} = \frac{\partial^3 \psi}{\partial \eta^3}\Bigg|_{\eta = 0} = \frac{\partial^3 \psi_i}{\partial \eta^3}\Bigg|_{\eta = 0} \frac{1}{\eta_o^3(\xi)} \leq \frac{\partial^3 \psi_i}{\partial \eta^3}\Bigg|_{\eta = 0}$$

Therefore [11] is a limit law for C_{ps}: for no value of Re one can find a value of C_{ps} larger than one given by [9].

As a concluding remark, referring to Figure 1, where the viscous over-pressure coefficient C_{ps} -1 is plotted against Re we can observe that most reliable

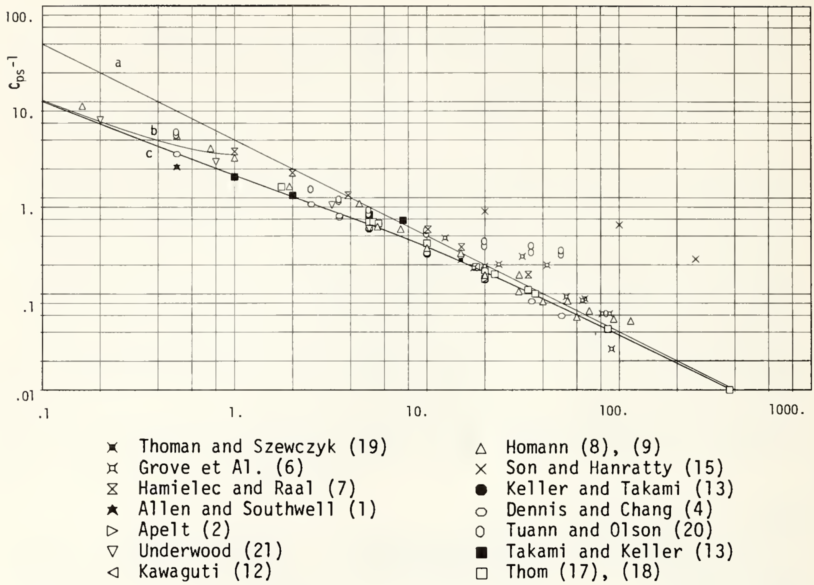

FIGURE 1. *The coefficient (C_{ps}-1) as a function of Re. Curve a, Thom's law (C_{ps}-1 = 4/Re); curve b, after Lamb solution's pressure coefficient; curve c, proposed fittings(5).*

data follow Lamb's solution for low Re and Thom's law for high Re, while Thom's law is an upper limit for any Re. The curve c fitting the data has been derived by applying the result of an analytic model, which is the subject of another paper just submitted for publication (5).

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