A Study of the Control of Electric Power Interchange in the Kentucky-Indiana Power Pool through the Use of Series Capacitors

PETER W. SAUER, GERALD T. HEYDT
Purdue Electric Power Center
School of Electrical Engineering
Purdue University, West Lafayette, Indiana 47907

Abstract

Electric power interchange into a power system may be controlled in many ways. The method proposed in this paper involves the use of series capacitors to shunt bulk power away from heavily loaded key transmission system components. This technique allows the control of power interchange sources. The control of electric power interchange and the improvement of simultaneous interchange capacity of a power system through the use of series capacitors is discussed and applied to a simplified model of the Kentucky-Indiana power pool. Some conclusions and economic comparison of series capacitive compensation are presented. In addition, introductory remarks concerning electric power interchange are given.

Introduction—Interchanging Power

Interchange of electric power via transmission lines which link electric power systems has become a way of life for the modern electric utility company. Typically, an electric power system is well tied to neighbors by transmission lines so that electric power generation may be imported; interties permit more reliable and economic system operation. The United States and Canada are divided into nine electric reliability councils within which extensive interchange occurs. Interchange between and across reliability councils also occurs on a large scale. By way of example, one of the nine reliability councils, the East Central Area Reliability Coordination Agreement (ECAR), extends over nine Midwestern and Central Atlantic states (Michigan, Indiana, Kentucky, Ohio, Pennsylvania, Maryland, West Virginia, Virginia, and a small portion of Northeast Tennessee); within this region, 26 member utility companies supply bulk generation in an amount in excess of 60,000 MW (1973) in generating stations ranging in size up to 1,300 MW. Interchange into the ECAR system exceeded 2,000 MW in 1973 (15).

Control of electric power interchange is performed in a variety of ways including reactive power scheduling at generating stations, transmission line switching, and quadrature phase shifter placement in transmission circuits. On-line control of tie line flows is nonetheless difficult and limited in a wide variety of cases. The ability of an electric power system to accept power via interties is limited by finite ratings of transmission system components. In both the case of control of interchange power and improvement of simultaneous interchange capacity, the insertion of capacitors in series with transmission circuits offers an alternative which warrants serious consideration in some cases. In this paper, the use of series capacitive compensation is considered and applied to a subsystem of the ECAR council—the Kentucky-Indiana Pool (KIP) which extends roughly over the southern half of Indiana and virtually the entire state of Kentucky. The paper briefly reviews the mathematical model which describes interchange power flow and the mechanism by which the model is modified to permit the study of series capacitor insertion.

The Power Flow Model in Linearized Form

At this juncture, it is necessary to comment on the power flow problem which is stated in terms of a mathematical model of simultaneous, algebraic, nonlinear equations relating system voltages, currents, and injected power; the problem is, in essence, concerned with the calculation of how power flows from the sources to the loads. The volt-ampere response of the system is

$$V_{bus} = Z_{bus} I_{bus}$$

where V_{bus} and I_{bus} are n-vectors of bus voltages and injected currents throughout the system and Z_{bus} is an n by n matrix of impedance coefficients (the bus impedance matrix). The number of system nodes or busses is n. Also, at all system busses except one, the injected power is known (or the load is known),

$$S_{\text{injected, bus } i} = v_i i_i^*$$

where $(.)^*$ denotes complex conjugation. At one bus, the swing bus, it is assumed that $v_1 = \underline{1/0}$ volts. These equations comprise 2n scalar real equations in 2n scaler, real unknowns. Ward and Hale (16) and others (2, 13) further describe this problem and alternate methods of solution. The solution is the line power flows given the bus demands.

In a typical power system, the bus voltage profile vector is near to the generated voltage (1.00 on a per-unitized system), and therefore the injected power is very nearly the complex conjugate of the injected current. If $\bar{S}_{ij}$ is the complex line power flow in line ij metered at j, and S_k is the bus injection at k, the line flow, as a function of bus injection, is approximately,

$$\frac{\partial \bar{S}_{ij}}{\partial S_k} = \frac{\bar{I}_{ij}^*}{I_k}.$$

Furthermore, the line current $\bar{I}_{ij}$ is related to bus voltages at i ind j by Ohm's law (with $\bar{y}_{ij}$ as the primitive line admittance),

$$\frac{\partial \bar{S}_{ij}}{\partial S_k} = [\frac{\partial}{\partial I_k} \bar{y}_{ij} (v_i - v_j)]^* .$$

The partials $\frac{\partial v_i}{\partial I_k}$ and $\frac{\partial v_j}{\partial I_k}$ are elements of the bus impedance matrix, z_{ik} and z_{jk} respectively. Therefore, the line flow is approximately related to the swing in power demand or bus injection as

$$\frac{\partial S_{ij}}{\partial S_k} = \bar{y}_{ij}^*(z_{ij} - z_{jk})^* . \quad (1)$$

The right hand side of Equation (1) is called a distribution factor, and Limmer and others discuss many additional details in (3, 7, 8, 9). Figure 1 pictorially shows the approximate relationship between transmission line loading and change in bus demand or generation.

The result of the foregoing discussion is that the power flow problem is linearized and reduced to the following form:

$$\overline{S}^{B} = \overline{S}^{A} + \sum_{i=1}^{k} (\text{distribution factors}) \Delta S_i$$

where $\overline{S}^{B}$ and $\overline{S}^{A}$ are line loads under different demand/generation schedules, and the ΔS_i are the differences in the bus demands under the two schedules. In the foregoing development, different schedules occur on account of different bus power injections. Consider now different schedules on account of different transmission system configuration. In particular consider the insertion of series impedence into line ij; in this case, the base loading schedule A is modified to schedule B on account of the change of the volt-ampere response of the system. In this case using the same linearization as shown above, Sauer and Heydt have shown (12),

$$\overline{S}^{B}_{kl} = \overline{S}^{A}_{kl} + D_{kl,ij}\,\overline{S}^{A}_{ij} \qquad (2)$$

where,

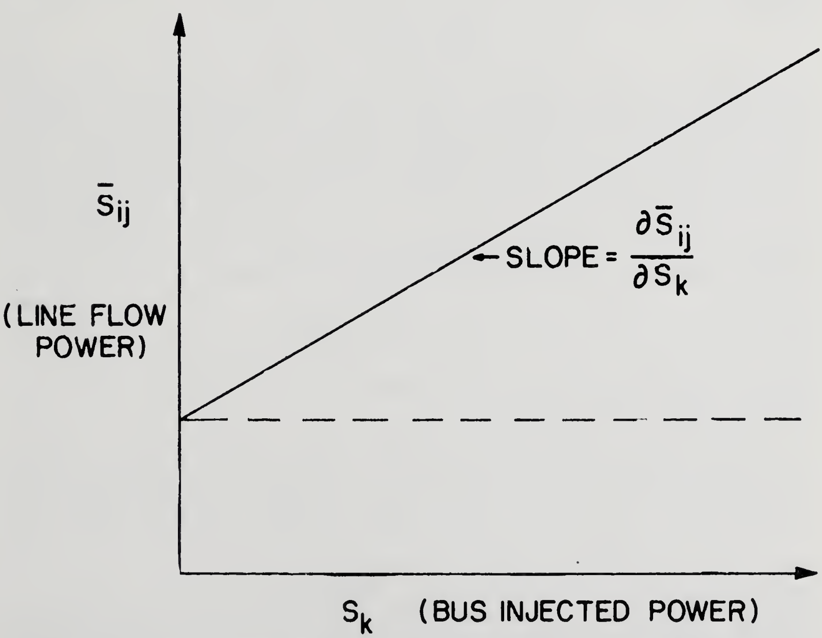

FIGURE 1. *Transmission Line Loading As A Function of Bus Demand Change.*

$$D_{kl\,ij} = \frac{\rho^{B}_{kl,j} - \rho^{A}_{kl,j}}{\rho^{A}_{ij,j}} \qquad (3)$$

Superscript "A" indicates base case values before insertion, superscript "B" indicates values after insertion, and the power transfer distribution factors are the ρ's given by

$$\rho_{ij,k} = \frac{\partial \overline{S}_{ij}}{\partial S_k} = \frac{z_{ik} - z_{jk}}{\overline{z}_{ij}}^{*} . \tag{4}$$

Evaluation of Equation (4) for schedule "B" is accomplished by the modification of the bus impedance matrix of schedule "A" to reflect the insertion of series impedance (12).

Series Capacitive Compensation

The modification of the system volt-ampere response results in modification of system power flow. One method of control of power flow, then, is modification of system configuration. Series insertion of capacitors into a transmission line of impedance $\overline{z}$ results in modification to $\overline{z}'$,

$$\overline{z}' = \overline{z} - jx_c ,$$

where $- jx_c$ is the impedance of the series capacitor. The effects of series capacitors in high voltage transmission lines is discussed in reference (4). The most common application of series capacitors is compensation to improve power transfer capability, approximated as (14)

$$P_t = \frac{E_S E_R}{x_L} \text{ sine } \theta , \tag{6}$$

where E_S is the standing-end voltage, E_R is the receiving-end- voltage, x_L is the series inductive reactance between the two terminals, and θ is the phasor angle by which E_S leads E_R. The negative reactance of series capacitors serves to reduce the reactance x_L in equation (6), and thereby increases the amount of power which can be transmitted over the line. The ability of a capacitor to lower line reactance can be used to shift power flow. Since the power flowing in a network is distributed according to network impedances, the power flow may be controlled by varying the system line impedances. If the reactance of a line ij is lowered by the insertion of series capacitance, the power flow in that line will increase whereas lines located in equivalent parallel paths would decrease.

To this point, discussion of network response has been confined to sinusoidal steady state phenomena. Unfortunately, series capacitive compensation will in some cases create resonant circuits which may be excited at sub-synchronous (<60Hz) frequencies. This occurs in non-linear subsystems in which sub-synchronous frequency limit cycles occur; these limit cycles may excite resonant frequencies created by inserted capicitance and transmission line inductance. The result of these transient phenomena is the appearance of sub-synchronous voltages, currents, and generator shaft torques. These oscillations may not be damped—thereby creating a serious problem. In fact, the use of series compensation has been substantially curtailed by these transient phenomena. In this paper, the objective focuses on steady state power flow control and further elaboration on sub-synchronous oscillations is omitted. A complete discussion of the topic appears in (5, 11). Reference (5) presents corrective measures.

Power Flow and Interchange Control Using Series Compensation

The power flow in a network transmission line kl, resulting from a modification of the impedance in line ij, s given by Eq. (2). Consider the modification to be the insertion of series capacitive reactance such that

$$\overline{z}_{ij}^{B} = \overline{r}_{ij} + j\overline{x}_{ij} - jx_{c}\,. \tag{7}$$

The insertion distribution factors ($D_{kl,ij}$) have been formulated in reference (12), for the schedule "B" modification shown in Eq. (7).

A specific application of series capacitive compensation for load flow control is to shift power throughout the network in order to reduce the loading in a certain line. This is accomplished by determining the value of x_c to be inserted in line ij such that $|\overline{S}_{kl}^{B}|^2$ is minimized. The minimization is presented in reference (12), yielding the following functions of x_c, for the case where line kl is not equal to line ij,

$$\frac{(\rho_{kl,j} - \rho_{kl,i})\,(j\overline{z}_{ij}^{*})}{[x_c(\rho_{iji} - \rho_{ijj} - 1) + jz_{ij}^{*}]^2} = 0\,. \tag{8}$$

The solution of Eq. (8) will produce the value of x_c required to extremize the power flow in line ij. The solution may be effected in several ways—further comments relative to this solution are given later. The application in this case is interchange power control. The significance of the method is the presentation of a viable alternative for power flow control.

Extending this approach to multiple line compensation, apply superimposition to distribution factors (9). The result n vector-matrix notation is,

$$\overline{S}^{B} = \overline{S} + D\overline{S}, \tag{9}$$

where $\overline{S}^{B}$ and $\overline{S}$ are l by 1 vectors in an l-line system, and D is l by l. The minimization is therefore a gradient technique. For this purpose, an index of performance reflecting line power flows is written, this index is minimized by allowing the gradient to go to zero. Using the above vector-matrix notation, let IP denote a quality index or performance index which reflects line loading,

$$IP = (\overline{S}^{B})^{H}\, k\, (\overline{S}^{B}), \tag{10}$$

where k is an l by l matrix of weights, and $(.)^{H}$ denotes complex conjugation followed by transposition. Subsequent discussion will relate to diagonal k,

$$k = \text{diagonal}\,(k_{11}\, k_{22} \ldots k_{ll}\,).$$

Write the gradient,

$$\nabla x_c [IP] = \nabla x_c [\bar{S}^{H} k \bar{S}] = 0,$$

and apply Equation (9). The minimization yields an equation of the form [12],

$$F(x) = 0, \tag{11}$$

where F(x) is a set of non-linear simultaneous equations of variable x_{ci}. In an l-line network, F represents l-equations, each formulated as,

$$f_i(x) = \mathrm{Re}\Big[\sum_{\substack{r=1 \\ \neq i}}^{l} \Big[2k_{rr} E_{rii} + \sum_{j=1}^{l} D^*_{jr} k_{jj} E_{ijj} \Big] \bar{S}^*_r \bar{S}_i + \sum_{\substack{c=1 \\ \neq i}}^{l} \Big[\sum_{j=1}^{l} k_{jj} E^*_{jii} D_{ji} \Big] \bar{S}^*_i \bar{S}_c + 2\bar{S}_i \bar{S}^*_i \Big[k_{ii} E_{iii} + \sum_{j=1}^{l} \mathrm{Re}(k_{jj} E^*_{jii} D_{ji}) \Big]\Big] = 0, \tag{12}$$

where

$$E_{jii} = \frac{\partial D_{ji}}{\partial x_{ci}}.$$

Equation (12) represents the equation generated by the ith gradient $\frac{\partial}{\partial x_{ci}}$ operating on the index of performance. For the special case of single line compensation with all weights zeroed except $k_{kk} = 1$, Equation (12) degenerates to,

$$f(x_c) = 2\mathrm{Re}\Big[\frac{dD_{k,i}}{dx_c} \bar{S}_i (\bar{S}_k + D_{k.i} \bar{S}_i)^*\Big] = 0. \tag{13}$$

When Equation (13) is satisfied, x_c is the capacitive reactance required as series compensation in line i to minimize the power flow in line k. In this study, practical cases were examined for the single line compensation case only. Equation (12) simplifies to,

$$f(x_c) = \mathrm{Re}\Big[\sum_{\substack{r=1 \\ \neq i}}^{l} [2k_{rr} E_{rii}]\bar{S}^*_r \bar{S}_i + 2\bar{S}_i \bar{S}^*_i [k_{ii} E_{iii} + \sum_{j=1}^{l} \mathrm{Re}(k_{jj} E_{jii} D_{ji})]\Big] = 0, \tag{14}$$

When Equation (14) is satisfied, x_c is the capacitive reactance required as series compensation in line i to minimize the index of performance. Since the index of performance represents a summation of all network line power flows, lowering its value for a given load schedule serves to add more "capacity" to the system. This point will be discussed further in subsequent analysis.

The power which can be interchanged between networks is limited by many factors. Knowledge of the amount of power which can be imported into a network is useful to power system operators and planners. For import to one system, the "maximum simultaneous interchange capability (SIC)" is that amount of power which can be interchanged between any one system and all other systems without exceeding continuous loading capabilities when all facilities are in service (1, 6). In this regard, the SIC is a measurement of system "capacity". The problem is formulated with the following assumptions:

a. Line power flows respond in a linear manner to bus power injections.
b. Neighboring generation supply is not a limiting factor (although limits of this kind are very easily added).
c. Transmission system ratings are known.
d. Base case line flows are known, and within rated limits.
e. Power import is real.

Using the linearized relationships, the power flowing in line ij subsequent to power injections at designated busses is,

$$\bar{S}^{B}_{ij} = \bar{S}^{A}_{ij} + \sum_{k=1}^{NT} \rho_{ij,k} \triangle S_{k}, \qquad (15)$$

where NT is the number of tie busses used for interchange. For the power flow rating in line ij, R_{ij}, the simultaneous interchange capability problem is defined as follows:

$$SIC = \sum_{k=1}^{NT} \triangle S_{k}, \triangle S_{k} \geqslant 0 \qquad (16)$$

for all

$$|\bar{S}^{B}_{ij}| \leq R_{ij} . \qquad (17)$$

Inequality (17) may be written as,

$$|\rho_{ij,k}| \triangle S_{k} \leqslant R_{ij} - |\bar{S}^{A}_{ij}|. \qquad (18)$$

Equations (16) and (18) may be solved via the simplex method of linear programming (10). When Equation (18) is maximized under the constraints of inequality (17), the SIC is determined. Since the $\rho_{ij,k}$ are functions of the bus impedance matrix, modification of network line impedances will affect the SIC.

Control of Interchange Power in the KIP System

The following application is presented to exhibit the effects of series capacitive compensation on a transmission system. Figure 2 shows a portion of the KIP network in Central Indiana. Defining system parameters are given in reference (12). The system busses used for interchange tie are listed in Table 1. The following examples are provided as a demonstration of the use of series capacitive compensation as discussed in this paper.

TABLE 1. *INTERTIE BUSSES.*

STAUNTON
COLUMBUS
INDIANAPOLIS—EAST
HANNA
NEW CASTLE
BATESVILLE
MADISON
BEDFORD
EDWARDSPORT

Example 1: The simultaneous nterchange capability (SIC) was calculated for the base case network, and for the cases with series capacitive compensation varies from 0 to 125% in one network line.

Example 2: An index of performance defined by Eq. 10 was calculated for the base case with all weights equal to one, and for the cases with series capacitive compensation in one network line varies from 0-125%. This calculation was made using actual load flow solutions for increments of single line compensation.

Example 3: The same index of performance studied in Example 2 was minimzed using the linear approximaton of Eq. (12) for the cases with series capacitive compensation varied from 0-200%.

Example 4: An index of performance was minimized using Eq. (12) for the cases with series capacitive compensation varied from 0-200% with all line weighting factors equal to zero except lines 10, 17, and 23, which are the lines which limited the interchange found in Example 1.

Discussion of Results

Examination of Table 2 shows that the SIC can be increased through the use of series capacitve compensation. The greatest improvement (over 200%) was realized using 85% compensation in line

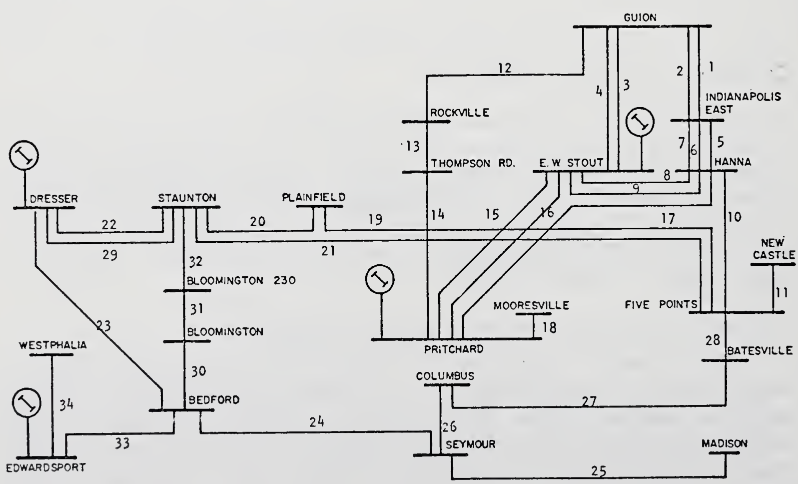

FIGURE 2
EXAMPLE—PART OF THE KIP SYSTEM

TABLE 2. Results of Examples.

Line With Series x_c Added	Example 1		Example 2		Example 3	Example 4
	Maximum SIC	% Compensation Required	Minimum IP	% Compensation Required	% Compensation Required for Min IP	% Compensation Required for Min IP
Base Case	3.91	--	181.52	--	--	---
1	7.47	80	181.49	55	50	126
2	7.47	80	181.49	55	50	126
3	4.30	40	178.89	45	37	149
4	4.33	45	177.98	50	44	139
5	3.91	0	181.46	15	0	0
6	3.91	0	181.46	15	0	0
7	3.91	0	181.46	15	0	0
8	12.21	85	181.52	0	0	145
9	12.21	85	181.52	0	0	145
10	4.19	125	180.00	(*)	0	0
11	3.91	(*)	181.52	(*)	0	0
12	6.41	85	181.52	0	0	200
13	6.02	125	181.52	0	0	200
14	5.84	20	181.52	0	0	200
15	10.93	50	177.42	55	40	135
16	10.93	50	177.42	55	40	135
17	3.91	0	176.63	45	29	0
18	3.91	(*)	181.52	(*)	0	0
19	4.64	125	181.30	(*)	0	200
20	7.68	115	181.04	30	0	200
21	5.19	35	173.87	70	51	160
22	4.84	80	181.46	10	0	184
23	4.02	40	179.87	75	62	0
24	5.41	115	181.49	10	0	0
25	3.91	(*)	181.50	(*)	0	0
26	4.31	125	181.52	0	0	0
27	4.37	125	181.52	0	0	0
28	3.91	(*)	181.52	0	0	0
29	4.84	80	181.46	15	0	184
30	3.91	(*)	181.42	125	125	0
31	3.91	(*)	181.49	125	125	0
32	3.91	(*)	181.52	0	0	200
33	5.05	125	179.88	125	0	0
34	3.91	(*)	181.52	(*)	0	0

* The value calculated does not change significantly in response to compensation in this line.

8 or 9. In Table 2, the lines which when compensated reduce the index of performance can be observed in the columns which refer to Example 2. The Table indicates which lines may be used to affect the power flowing in all system lines. In the columns referring to Example 3, the accuracy of the linearized equation used to generate the percent compensation required to produce a mnimum total system loading is observed. Table 2 in the rightmost columns shows which lines may be used to minimize the loading of lines 10, 17 and 23. The lines with zero percent compensation required are lines which when compensated will increase the loading n lines 10, 17 and 23. The solution to the minimized equation was found by consecutive evaluation rather than minimum value determination. This was done in order to reveal the overall effect of the series compensation in each line.

Economic Considerations

Series capacitive compensation has been shown to be an economic method of improvement of transmission capability in some cases (14). In the case of power flow control, the measure of improvement is typically an index; and therefore the cost-to-benefit ratio is somewhat more difficult to evaluate. Therefore, it is reasonable to obviate the difficulty by considering the objective fixed and comparing costs of alternative approaches.

By considering a fixed objective (e.g. to obtain x units of SIC, or to obtain y percent improvement in performance index), the following factors are ignored:

i. Transient performance
ii. Ancillary active power loss changes
iii. Effective interfacing with existing facilities
iv. Company experience and confidence with particular designs.

Consider two parallel lines with different line reactance as shown in Figure 3a. If the line resistances are neglected, the amount of capacitive reactance required to reduce the power flowing in line 1 to one half the base case value is,

$$x_c = -\frac{x_2^2 + x_1 x_2}{x_2 + 2x_1},$$

where all values are in ohms. Allowing x_2 to be the equivalent "loop" reactance (Fig. 3b) as seen by line 1, the amount of capacitive reactance to be placed in line i or j to reduce the load in line 1 to one half is

$$x_{ci} = -\frac{x_{eq.}^2 + x_1 x_{eq.}}{x_{eq.} + 2x_1}. \qquad (21)$$

An equivalent method of reducing the load in line 1 to one half would be the installation of a parallel line of equal impedance/mi. Assuming that the required value of capacitance can be installed in line i or line j, the cost of the capacitors may be calculated and compared with the cost of the parallel transmission line. For a 345 kv system, typical line reactances are about 0.7 ohms/mi. Using the method outlined above, the total capital investment required to reduce the load in a line to one half is calculated for various values of $x_{eq.}$ and line lengths. The cost comparison is shown for line lengths of 20 to 100 miles in Figure 4. The point where series compensation costs are lower than new line costs vary with $x_{eq.}$ and line length.

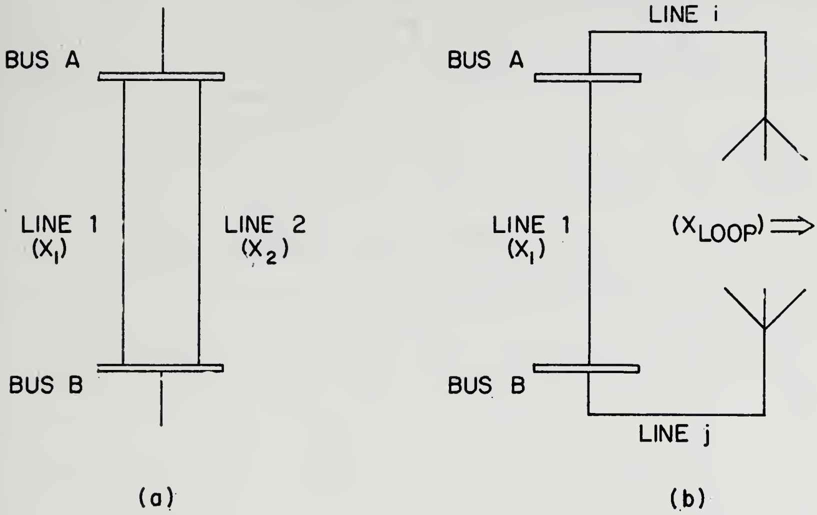

Figure 3. *Economic Example—Single Line Unloading.*

Summary and Conclusions

Electric power flow control and improvement of interchange capacity of a power system may be effected using series capacitors. A useful method of analysis is linearization of the power flow problem. The economics of the application of series capacitors instead of transmission line construction suggests that in some cases, series capacitors may present an economic alternative to transmission installation.

Additional rationale for series compensation involves such non-quantitative factors as more flexible control possibilities and less environmental impact over the transmission line construction solution.

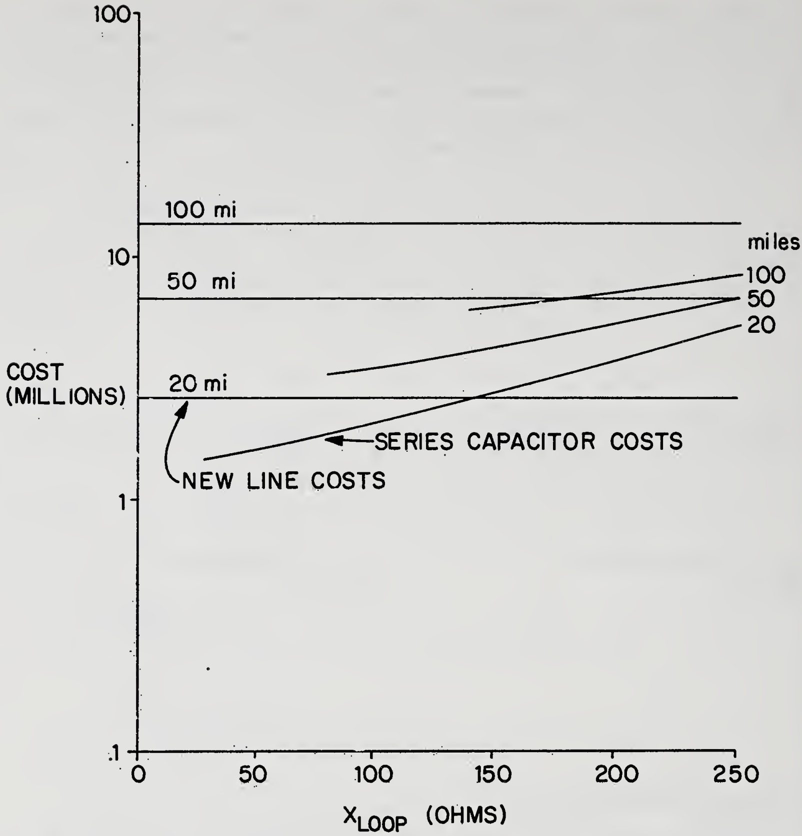

FIGURE 4. *Total Capital Cost Comparison for Series Compensation vs. Line Construction for I_1 = 2000 amps.*

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